NAME SOLUTIONS

STUDENT ID # ____________________

RECITATION INSTRUCTOR ____________________

RECITATION TIME ____________________

LECTURER ____________________

INSTRUCTIONS

1. There are 9 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–9.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. Solve the inequality for x:

$$\frac{2x+3}{(x-4)^3} > 0.$$

$x = -\frac{3}{2}\ ,\ 4$

$+ + + + \quad - - - - - - \quad + + + +$

$-\frac{3}{2} \qquad 4 \qquad x$

At $x = -2 \to +$

$x = 0 \to -$

$x = 5 \to +$

A. $x < -\frac{3}{2}$

B. $x > -\frac{3}{2}$

C. $x > -\frac{3}{2}$ and $x \neq 4$

D. $x < -\frac{3}{2}$ or $x > 4$

E. $-\frac{3}{2} < x < 4$

2. $\lim_{x\to 1} \frac{x^2+4x-5}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+5)}{x-1}$

$= \lim_{x\to 1} (x+5) = 6$

A. Does not exist

B. -4

C. 1

D. 5

E. 6

3. $\lim_{x\to 0} \frac{\sin^2 x}{1-\cos x} = \lim_{x\to 0} \frac{1-\cos^2 x}{1-\cos x}$

$= \lim_{x\to 0} \frac{(1+\cos x)(1-\cos x)}{1-\cos x}$

$= \lim_{x\to 0} (1+\cos x) = 2$

A. Does not exist

B. -2

C. 0

D. 2

E. 1

4. The domain of $f(x) = \ln(1-\ln x)$ is

Domain of $\ln x$ is $x > 0$

$x > 0$

$1 - \ln x > 0 \to \ln x < 1 \to x < e$

A. $x > 0$

B. $0 < x < 1$

C. $0 < x < e$

D. $0 < x < \frac{1}{e}$

E. $x > e$

5. The equation of the tangent line to the graph of $y = \ln(2\sin x)$ at $\left(\frac{\pi}{6}, 0\right)$ is

$$\frac{dy}{dx} = \frac{1}{2\sin x} 2\cos x = \frac{\cos x}{\sin x}$$

$$\left.\frac{dy}{dx}\right|_{\frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

A. $y = \sqrt{3}\left(x - \frac{\pi}{6}\right)$

B. $y = \frac{\sqrt{3}}{3}\left(x - \frac{\pi}{6}\right)$

C. $y = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

D. $y = \frac{1}{2}\left(x - \frac{\pi}{6}\right)$

E. $y = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{6}\right)$

6. If $f(x) = \ln(\sin e^x)$, then $f'(x) =$

$$f'(x) = \frac{1}{\sin e^x}\left(\cos e^x\right) e^x$$

A. $\dfrac{\cos e^x}{\sin e^x}$

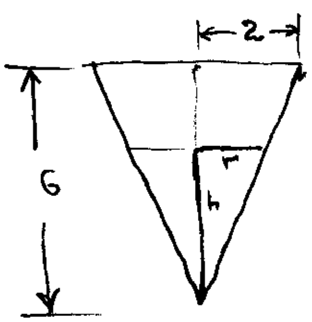
B. $\dfrac{e^x}{\sin e^x}$

C. $\dfrac{e^x \cos e^x}{\sin e^x}$

D. $\dfrac{\cos e^x}{e^x \sin e^x}$

E. $\dfrac{e^x}{\cos e^x \sin e^x}$

7. Water is poured into a conical paper cup so that the height increases at the constant rate of 1 inch per second. If the cup is 6 inches tall and its top has a radius of 2 inches, how fast is the volume of the water in the cup increasing when the height is 3 inches. ($V = \frac{1}{3}\pi r^2 h$).



$$\frac{r}{h} = \frac{2}{6} \qquad \frac{dh}{dt} = 1$$

$$r = \frac{1}{3}h$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$$

$$V = \frac{1}{27}\pi h^3$$

$$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$$

When $h = 3$: $\frac{dV}{dt} = \frac{\pi}{9} \cdot 3^2 \cdot 1 = \pi$

A. $\frac{\pi}{2}$

B. 2π

C. π

D. $\frac{\pi}{3}$

E. $\frac{\pi}{4}$

8. If $y = x^2 \cos x$, then the differential $dy =$

$$dy = \frac{dy}{dx}\,dx$$
$$= (2x\cos x - x^2 \sin x)\,dx$$

A. $-2x \sin x \, dx$

B. $(2x\cos x - 2x\sin x)dx$

C. $(2x\cos x - x^2\sin x)dx$

D. $(x^2\cos x - x^2\sin x)dx$

E. $(x^2\cos x - 2x\sin x)dx$

9. The function $f(x) = xe^{-x}$ has its maximum value on the interval $[0, 2]$ at $x =$

$$f'(x) = -xe^{-x} + e^{-x} = e^{-x}(1-x)$$
$$f'(x) = 0: \quad e^{-x}(1-x) = 0 \longrightarrow x = 1$$
$$f(0) = 0$$
$$f(1) = e^{-1} = \frac{1}{e}$$
$$f(2) = 2e^{-2} = \frac{2}{e^2} = \frac{2}{e}\frac{1}{e} < \frac{1}{e}$$
$$<1$$

A. 0

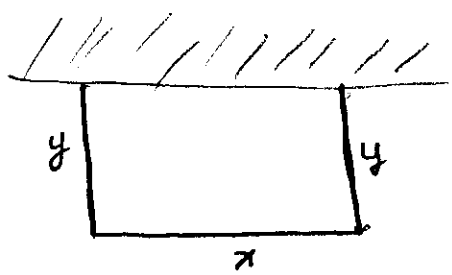
B. $\frac{1}{2}$

C. 2

D. $\frac{3}{2}$

E. 1

10. One side of a rectangular farm is bounded by a straight river and the other three sides are bounded by straight fences. The total length of the fence is 800 ft. Determine the length of the side bounded by the river that makes the area of the farm maximum.



$$x + 2y = 800$$
$$\therefore y = \frac{1}{2}(800 - x)$$

A. 400 ft

B. 300 ft

C. 200 ft

D. $\frac{800}{3}$ ft

E. $\frac{400}{3}$ ft

$$A = xy = x\frac{1}{2}(800 - x)$$
$$A = 400x - \frac{1}{2}x^2, \quad 0 < x < 800$$
$$\frac{dA}{dx} = 400 - x$$
$$\frac{dA}{dx} = 0 \longrightarrow x = 400$$

11. One-third of a radioactive substance decays every 5 years. Assuming exponential decay ($f(t) = f(0)e^{kt}$), the decay constant $k =$

$f(5) = \frac{2}{3} f(0)$

$\frac{2}{3} f(0) = f(0) e^{k5}$

$\ln \frac{2}{3} = k5 \longrightarrow k = \frac{1}{5} \ln \frac{2}{3}$

A. $\ln \frac{3}{5}$

B. $\frac{1}{5} \ln \frac{2}{3}$

C. $5 \ln \frac{1}{3}$

D. $\frac{1}{5} \ln 3$

E. $\frac{1}{3} \ln \frac{1}{5}$

12. Find all values of x for which the graph of the function $f(x) = 3x^5 - 5x^3$ has a point of inflection.

$f'(x) = 15x^4 - 15x^2$

$f''(x) = 60x^3 - 30x = 30x(2x^2 - 1)$

$f''(x) = 0$: $30x(\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0$

$x = 0, \quad x = \pm \frac{1}{\sqrt{2}}$

f'' changes sign at each point.

A. $x = \pm 1$

B. $x = 0, x = \pm\frac{1}{\sqrt{2}}$

C. $x = \pm\frac{1}{\sqrt{2}}$

D. $x = 0, \pm 1$

E. $x = 0, \pm\sqrt{\frac{5}{3}}$

13. Find all values of x in the interval $0 \le x < 2\pi$ where the function $f(x) = \cos^2 x$ has a relative maximum.

$f(x) = \cos^2 x$

$f'(x) = 2\cos x(-\sin x) = -\sin 2x$

$f''(x) = -2\cos 2x$

$f'(x) = 0$: $-\sin 2x = 0$

$2x = 0, \pi, 2\pi, 3\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$f''(0) = -2 < 0$ rel. max.

$f''(\frac{\pi}{2}) = -2\cos\pi = 2 > 0$

$f''(\pi) = -2\cos(2\pi) = -2 < 0$ rel. max

$f''(\frac{3\pi}{2}) = -2\cos(3\pi) = 2 > 0$

A. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

B. $x = \pi$

C. $x = \frac{\pi}{4}, \frac{7\pi}{4}$

D. $x = 0, \pi$

E. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

14. If $F(x) = \int_0^{\sin x} \sqrt{t^2+1}\,dt$, then $F'\left(\frac{\pi}{2}\right) =$

$F'(x) = \sqrt{\sin^2 x + 1} \cdot \cos x$

$F'\left(\frac{\pi}{2}\right) = \sqrt{2} \cdot 0 = 0$

A. 0
B. 1
C. $\frac{1}{2}$
D. $\sqrt{2}$
E. $\frac{\sqrt{3}}{2}$

15. If $f(x) = 3^x$, then $f''(0) =$

$f(x) = 3^x = e^{\ln 3^x} = e^{x \ln 3}$

$f'(x) = (\ln 3)\, e^{x \ln 3}$

$f''(x) = (\ln 3)^2 e^{x \ln 3}$

$f''(0) = (\ln 3)^2$

A. $\ln 3$
B. $3 \ln 3$
C. $2 \ln 3$
D. $(\ln 3)^2$
E. $\ln e^2$

16. The horizontal asymptote of the graph of $f(x) = \dfrac{3x^2 - 2x + 5}{1 + 3x - 2x^2}$ is

$\lim\limits_{x\to\infty} \dfrac{3x^2 - 2x + 5}{1 + 3x - 2x^2} = -\frac{3}{2}$

A. $y = 3$
B. $y = -\frac{3}{2}$
C. $y = \frac{3}{2}$
D. $y = 0$
E. there is no horizontal asymptote

17. If $f(x) = \sin^{-1}(\sqrt{x})$, then $f'\left(\frac{1}{2}\right) =$

$f'(x) = \dfrac{1}{\sqrt{1 - (\sqrt{x})^2}} \; \dfrac{1}{2\sqrt{x}}$

$f'\left(\frac{1}{2}\right) = \dfrac{1}{\sqrt{1 - \frac{1}{2}}} \; \dfrac{1}{2\sqrt{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{\frac{1}{2}}} \; \dfrac{1}{2\sqrt{\frac{1}{2}}} = \dfrac{1}{\frac{1}{2} \cdot 2} = 1$

A. $\frac{\pi}{2}$
B. $\frac{\pi}{4}$
C. $\frac{1}{4}$
D. 1
E. $\sqrt{2}$

18. $\int_0^{\sqrt{\frac{\pi}{3}}} x \sin x^2 dx =$

$\int x \sin x^2 dx = \frac{1}{2}\int \sin u\, du = -\frac{1}{2}\cos u + C$

$u = x^2 \quad du = 2x\,dx$

$= -\frac{1}{2}\cos x^2 + C$

$\int_0^{\sqrt{\frac{\pi}{3}}} x \sin x^2 dx = -\frac{1}{2}\cos x^2 \Big|_0^{\sqrt{\frac{\pi}{3}}}$

$= -\frac{1}{2}\cos\frac{\pi}{3} + \frac{1}{2} = -\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2} = \frac{1}{4}$

A. $-\frac{1}{4}$
B. $\frac{1}{3}$
C. $\frac{1}{4}$
D. $\frac{\sqrt{3}}{2}$
E. $\frac{1}{2}$

19. The substitution $u = e^y$ transforms the definite integral $\int_0^1 e^y \cos^2 e^y dy$ to

$u = e^y \quad du = e^y dy$

$y = 0 \rightarrow u = 1$

$y = 1 \rightarrow u = e$

$\int_0^1 e^y \cos^2 e^y du = \int_1^e \cos^2 u\, du$

A. $\int_0^e \cos^2 u\, du$
B. $\int_0^e \cos^2 u\, du + C$
C. $\int_1^e \cos u\, du$
D. $\int_1^e \cos^2 u\, du$
E. $\int_e^1 u \cos^2 u\, du$

20. If $\int_1^{\sqrt{3}} \frac{1}{(x^2+1)\tan^{-1} x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{u} du = \ln u \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} =$

$u = \tan^{-1} x \quad du = \frac{1}{(x^2+1)} dx$

$x = 1 \rightarrow u = \tan^{-1} 1 = \frac{\pi}{4}$

$x = \sqrt{3} \rightarrow u = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$= \ln\frac{\pi}{3} - \ln\frac{\pi}{4}$

$= \ln \frac{\frac{\pi}{3}}{\frac{\pi}{4}}$

$= \ln\frac{4}{3}$

A. $\ln 3$
B. $\ln 4$
C. $\ln\frac{4}{3}$
D. $\ln 2$
E. $4\ln 3$

21. $\int \frac{dx}{x^2 - 4x + 8} = \int \frac{dx}{(x^2 - 4x + 4) + 4}$

$= \int \frac{dx}{(x-2)^2 + 2^2} = \frac{1}{2}\tan^{-1}\frac{x-2}{2} + C$

A. $\frac{1}{2}\ln(x^2-4x+8)+C$
B. $\ln(x^2-4x+8)+C$
C. $\sin^{-1}\frac{x-2}{2}+C$
D. $\tan^{-1}\frac{x-2}{2}+C$
E. $\frac{1}{2}\tan^{-1}\frac{x-2}{2}+C$

22. The area of the region between the graph of $f(x) = x\sqrt{x-1}$ and the x-axis, from $x = 1$ to $x = 2$ is

$$\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du =$$

$u = x-1 \quad du = dx$

$x = 1 \longrightarrow u = 0$

$x = 2 \longrightarrow u = 1$

$$= \left[\frac{u^{5/2}}{\frac{5}{2}} + \frac{u^{3/2}}{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{5} + \frac{2}{3}$$

$$= \frac{16}{15}$$

A. 1.

B. 2

C. $\frac{16}{15}$

D. $\frac{14}{15}$

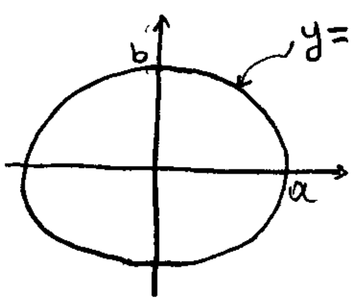
E. $\frac{2}{3}$

23. The area A of the region enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is given by

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right) = \frac{b^2}{a^2}(a^2 - x^2)$$



$y = \frac{b}{a}\sqrt{a^2 - x^2}$

A. $A = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$

B. $A = 4\int_0^a \frac{b^2}{a^2}(a^2 - x^2)dx$

C. $A = 4\int_0^b \frac{b^2}{a}\sqrt{a^2 - x^2}dx$

D. $A = 4\int_0^a \frac{a}{b}\sqrt{b^2 - x^2}dx$

E. $A = 4\int_0^a \frac{b}{a^2}\sqrt{a^2 - x^2}dx$

24. The focus of the parabola

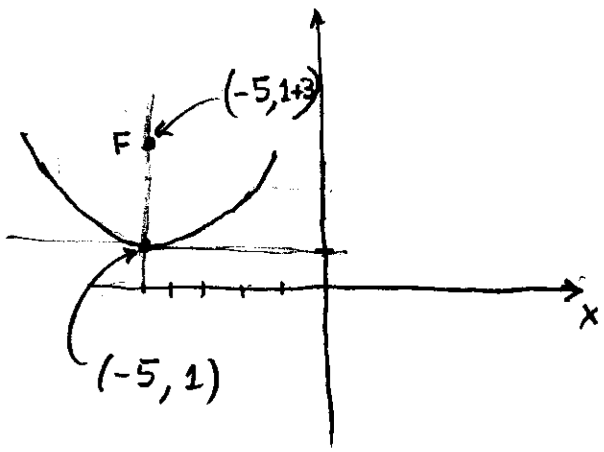
$$(x+5)^2 = 12(y-1)$$

is the point

$4c = 12 \rightarrow c = 3$

vertex $(-5, 1)$

A. $(-5,1)$
B. $(5,-1)$
C. $(-5,4)$
D. $(-2,1)$
E. $(-5,-2)$



25. The hyperbola $9x^2 + 18x - 4y^2 + 16y = 43$ has vertices at the points

$9(x^2+2x+1) - 4(y^2-4y+4) = 43+9-16$

$9(x+1)^2 - 4(y-2)^2 = 36$

$\dfrac{(x+1)^2}{4} - \dfrac{(y-2)^2}{9} = 1$

center $(-1,2)$ $a=2$, $b=3$

A. $(-1,-2)$ and $(-3,-2)$
B. $(2,1)$ and $(-3,2)$
C. $(2,1)$ and $(2,-3)$
D. $(1,2)$ and $(-3,2)$
E. $(-1,2)$ and $(3,2)$