MA 161 EXAM III

Spring 2004

Name ________________________________

nine–digit Student ID number ________________________

Division and Section Numbers ________________________

Recitation Instructor ____________________________

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 12 problems, each worth 8 points. You get 4 points for your TA's name.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

1. Let $f(x) = \dfrac{x}{4+x^2}$ for x in the interval $[1, 8]$. Then $f(x)$ attains its absolute maximum at x equal

   a. 1

   b. 2

   c. 4

   d. 8

   e. none of these.

$0 = f'(x) = \dfrac{4-x^2}{(4+x^2)^2} \Rightarrow x = 2$

$f(1) = \frac{1}{5}, \quad f(2) = \frac{1}{4}, \quad f(8) \leq \frac{1}{8}$

2. The function $f(x) = x^3 + 6x^2 + 9x$ is decreasing

   a. just on the interval (1,3).

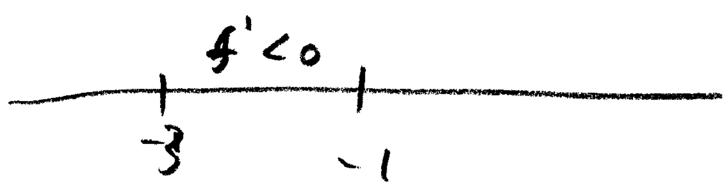
   b. just on the interval (-3,-1).

   c. just on the intervals $(-\infty$,-3) and $(-1, \infty)$.

   d. just on the intervals $(-\infty$,1) and $(3, \infty)$.

   e. nowhere.

$f'(x) = 3x^2 + 12x + 9 = 3(x+1)(x+3)$



3. If $f(x)$ has its derivative satisfying $f'(x) = (x-1)(x-2)^2(x-3)$ then $f(x)$ has

a. a local minimum just at 1 and a local maximum just at 3.

b. local minimums just at 1 and 2, and a local maximum just at 3.

c. local minimums just at 1 and local maximums just at 2 and 3.

d. a local minimum just at 3, and a local maximum just at 1.

e. local minimums just at 1 and 3, and a local maximum just at 2.

f': $+$ | 1 | $-$ | 2 | $-$ | 3 | $+$

4. The function $f(x) = e^{-x^2}$ is concave down

a. on $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

b. on $(-\frac{1}{2}, \frac{1}{2})$.

c. on (-1,1).

d. on $(-\infty, -1)$ and $(1, \infty)$.

e. nowhere.

$f' = -2x e^{-x^2}$

$f'' = e^{-x^2}(-2 + 4x^2)$

f'': $+$ | $-\frac{1}{\sqrt{2}}$ | $-$ | $\frac{1}{\sqrt{2}}$ | $+$

5. The function $f(x) = 8x^2 - x^4$ has inflection point(s) for the x just in the set

a. $\{-2, 2\}$.

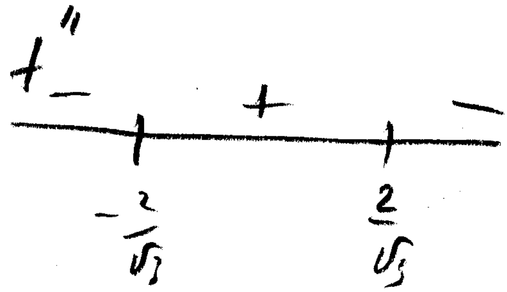
b. $\{-2, -\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, 2\}$.

c. $\{0\}$.

d. $\{-\frac{2}{3}, \frac{2}{3}\}$.

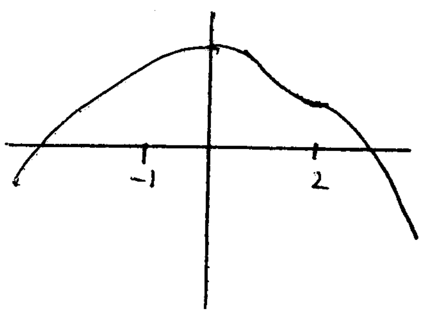
e. $\{-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\}$.

$f'' = 16 - 12x^2$

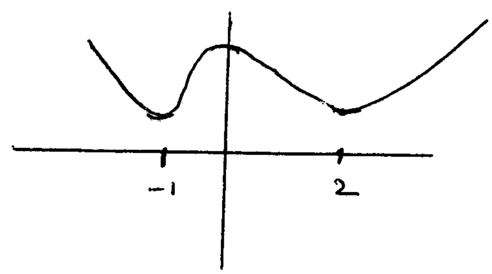


6. If f is differentiable with $f'(x) > 0$ on $(-\infty, -1)$ and (0,2), and $f'(x) < 0$ on (-1,0) and $(2, \infty)$ then the graph for $f(x)$ looks most like

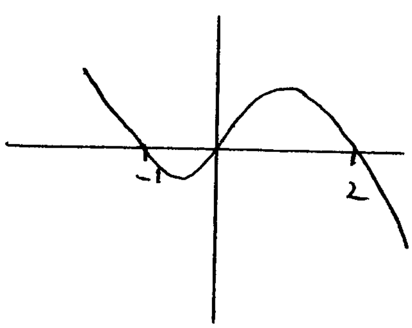
a.



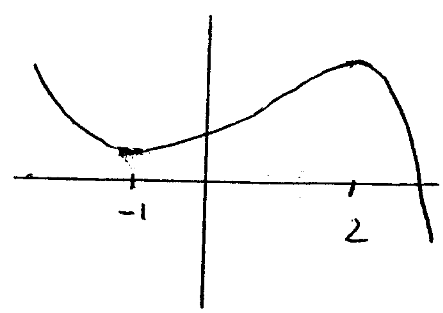
b.



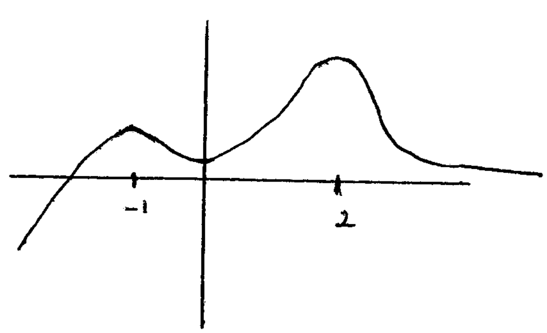
c.



d.



e.



7. The linear approximation of $f(x) = x^{1/2}$ at $a = 16$ is used to find the approximate value for $17^{1/2} - 4$. The approximate value found is

a. $-\frac{1}{8}$.

b. $\frac{1}{4}$.

c. $-\frac{1}{4}$.

d. $\frac{1}{8}$.

e. none of these.

$$17^{1/2} - 4 = f(17) - f(16)$$

$$\approx f'(16)(17-16)$$

$$= df$$

$$= \frac{1}{8} \cdot 1$$

8. $\lim\limits_{x\to\infty} \dfrac{x^2}{e^x}$ equals

a. ∞.

b. 0.

c. 1.

d. 2.

e. none of these.

$$\lim_{x\to\infty} \frac{x^2}{e^x} = \lim_{x\to a} \frac{2x}{e^x}$$

$$= \lim_{x\to a} \frac{2}{e^x} = 0$$

9. $\lim_{x\to 0} \dfrac{(e^x + e^{-x} - 2)}{x^2}$ equals

a. ∞.

b. 0.

c. 1.

d. 2.

e. none of these.

$$\lim_{x\to 0} \frac{(e^x + e^{-x} - 2)}{x^2}$$

$$= \lim_{x\to 0} \frac{e^x - e^{-x}}{2x}$$

$$= \lim_{x\to 0} \frac{e^x + e^{-x}}{2} = 1$$

10. $\lim_{x\to 0+} (1+2x)^{\frac{1}{x}}$ equals

a. e^2.

b. 2.

c. ln2.

d. 0.

e. none of these.

$$\lim_{x\to 0^+} \ln(1+2x)^{1/x}$$

$$= \lim_{x\to 0^+} \frac{\ln(1+2x)}{x}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{1+2x} \cdot 2}{1}$$

$$= 2$$

$$\Rightarrow \lim_{x\to 0} (1+2x)^{1/x} = e^2$$

11. Two sides, of the right triangle pictured, change with time. Find $x'(t)$ when $h'(t) = 8$ in/min and $x(t) = 4$ in.

a. 2 in/min.

b. 5 in/min.

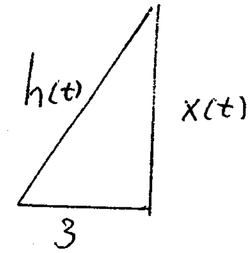
c. 10 in/min.

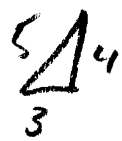
d. 20 in/min.

e. 40 in/min.

$$x = (h^2 - 9)^{1/2}$$

$$x' = \frac{h \cdot h'}{(h^2-9)^{1/2}} = \frac{5}{4} \cdot 8$$





12. A girl walks east on a beach and is observed from a boat 100 ft from shore. Determine $x'(t)$ when $x(t) = 100$ ft and $\theta'(t) = 4$ radians/min.

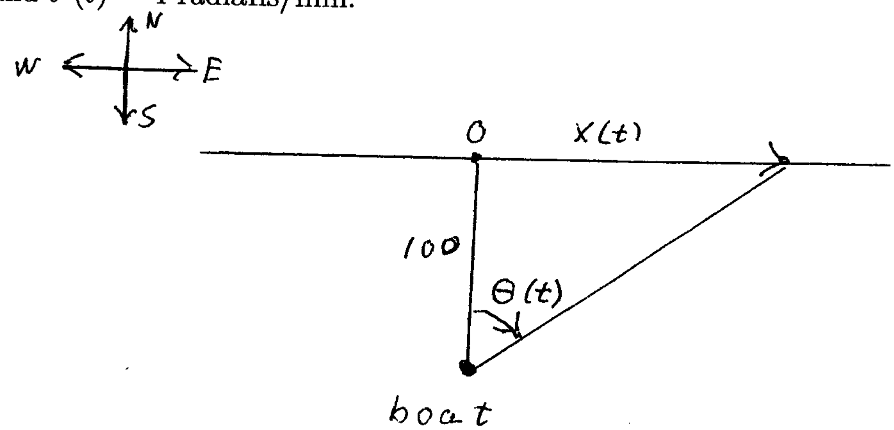
a. 200 ft/min.

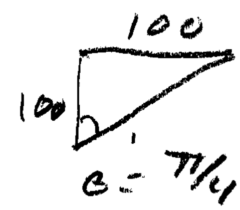
b. 400 ft/min.

c. 800 ft/min.

d. 1200 ft/min.

e. 1600 ft/min.





$$x = 100 \tan\theta$$

$$x' = 100 \sec^2\theta \cdot \theta'$$

$$= 100 \cdot 2 \cdot 4$$