MATH 161 & 161E – FINAL EXAM – FALL 2002
DECEMBER 9, 2002

STUDENT NAME:

STUDENT ID:

RECITATION INSTRUCTOR :

INSTRUCTIONS:

1. This test booklet has 13 pages including this page.
2. Fill in your name, your student ID number, and your recitation instructor's name above.
3. Use a number 2 pencil on the mark-sense sheet (answer sheet).
4. On the mark-sense sheet, fill in the recitation instructor's name and the course number.
5. Fill in your name and student ID number, blacken the appropriate spaces, and sign the mark-sense sheet.
6. Mark the division and section number of your class and blacken the corresponding circles, including the circles for the zeros. If you do not know your division and section number ask your instructor.
7. There are 25 questions, each worth 8 points. Blacken your choice of the correct answer in the spaces provided. Turn in BOTH the answer sheet and the question sheets to your instructor when you are finished.
8. No books, notes, or calculators may be used.

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1) If

$$f(x) = \ln\left(\frac{(x^2+1)^3(x^4+3)^2}{(x^6+5)^4}\right), \quad \text{then} \quad f'(1) \quad \text{is equal to}$$

A) 0

B) 1

C) 2

D) $\ln 3$

E) $\ln 2$

2) Which of the following is an equation of the line tangent to the graph of $f(x) = \frac{1-x}{1+x}$ at $(0, 1)$?

A) $3y - 6x - 3 = 0$

B) $y + x - 1 = 0$

C) $y - 2x - 1 = 0$

D) $y + 2x - 1 = 0$

E) $3y + 2x - 3 = 0$

3)

$$\lim_{x \to 0^-} \frac{\sin |x|}{x} \text{ is equal to}$$

A) 1

B) -1

C) 2

D) -2

E) The limit does not exist

4) If $f'(x) = \sqrt{x}$, then

$$\frac{d}{dx}\left(f(x^3)\right) \quad \text{is equal to}$$

A) $3x^{2/3}$

B) $3x^{5/2}$

C) $3x^{3/2}$

D) $3x^{9/2}$

E) $3x^{7/2}$

5) If $f(x) = \sin(x^2 + 1)$, then $f''(x)$ is

A) $2\cos(x^2 + 1)$

B) $2\cos(x^2 + 1) - x^2 \sin(x^2 + 1)$

C) $2\cos(x^2 + 1) - x^3 \sin(x^2 + 1)$

D) $2\cos(x^2 + 1) - 4x^2 \sin^2(x^2 + 1)$

E) $\cos(x^2 + 1) - 3x^2 \sin(x^2 + 1)$

6) $\tan(\sec^{-1} 3) =$

A) 8

B) $\sqrt{5}$

C) $\sqrt{3}$

D) $\sqrt{8}$

E) 2

7) A spherical balloon is being inflated in such a way that its radius increases at a rate of $1 \, \text{cm} / \text{s}$. In cm^3 / s, how fast is the volume increasing 3 seconds after the inflation starts? (The volume of a spherical balloon of radius r is $4/3\pi r^3$)

A) 12

B) 12π

C) 36

D) 36π

E) 18π

8) The value of the limit

$$\lim_{x \to 0} \frac{\cos(x^2) - 1}{x^4} \quad \text{is}$$

A) $-1/2$

B) $1/2$

C) 1

D) $2/3$

E) $5/2$

9) Let $f(x) = (x^2 - 1)\tan(x^2 + 1)$. Then $f'(x)$ is equal to

A) $f'(x) = (2x - 1)\sec^2(x^2 + 1) + (2x^3 - 2x)\tan(x^2 + 1)$

B) $f'(x) = 2x\tan(x^2 + 1) + (2x^3 - 2x)\sec(x^2 + 1)$

C) $f'(x) = 2x\tan(x^2 + 1) + (2x^3 - 2x)\sec^2(x^2 + 1)$

D) $f'(x) = 4x^2\sec^2(x^2 + 1)$

E) $f'(x) = 4x^2\tan^2(x^2 + 1)$

10) If $x^2 - xy + y^3 = 8$, defines y as a function of x, which of the following is dy/dx?

A) $\frac{y-2x}{3y^2-x}$

B) $-\frac{2xy}{x+3y^2}$

C) $-\frac{3x^2+y}{2x-y}$

D) $\frac{x^2-y^2}{x^2+y^2}$

E) $\frac{x+y}{x^2+y^2}$

11) A bacteria culture grows at a rate proportional to its size. If there were 6×10^2 bacteria after two hours and 6×10^4 after four hours, then the initial population was :

A) 60

B) 6

C) 12

D) 40

E) 24

12) The length of a rectangle is increasing at a rate of $2\,\text{m}/\text{s}$ while its width is decreasing at $3\,\text{m}/\text{s}$. When the length is $6\,\text{m}$ and the width is $4\,\text{m}$, the area of the rectangle is

A) decreasing at $10\,\text{m}^2/\text{s}$

B) increasing at $10\,\text{m}^2/\text{s}$

C) increasing at $26\,\text{m}^2/\text{s}$

D) decreasing at $26\,\text{m}^2/\text{s}$

E) increasing at $6\,\text{m}^2/\text{s}$

13) The function $f(x) = \frac{1}{x}\ln|x^2 - 1|$ has

A) vertical asymptotes $x = -1$, $x = 1$ and no horizontal asymptotes

B) vertical asymptote $x = -1$ and horizontal asymptote $y = 0$

C) vertical asymptotes $x = -1$, $x = 0$ and $x = 1$ and horizontal asymptote $y = 0$

D) vertical asymptotes $x = -1$, $x = 1$ and horizontal asymptotes $y = 0$

E) vertical asymptotes $x = -1$, $x = 0$ and $x = 1$ and no horizontal asymptotes

14) The absolute minimum value of $f(x) = x^4 - 4x$ in the interval $[0, 2]$ is

A) 0

B) 1

C) −1

D) −3

E) −4

15) The function

$$f(x) = \frac{x}{x^2 + 1} \quad \text{is increasing in}$$

A) $(-1, 1)$

B) $(-1, 2)$

C) $(-2, 1)$

D) $(-2, 2)$

E) $(-\infty, 1)$

16) Which of the following statements are true about the function $f(x) = x^4 - 4x^3$?
(i) $f(x)$ is decreasing on $(0, 3)$
(ii) $f(x)$ is concave down on $(-\infty, 0)$
(iii) $f(x)$ has a local maximum at $x = 0$

A) Only (i)

B) Only (iii)

C) (i) and (ii)

D) (i) and (iii)

E) All three

17) A function $f(x)$ satisfies $f''(x) = x^8 - x^2$. How many inflection points does $f(x)$ have?

A) 4

B) 3

C) 2

D) 1

E) 0

18)

$$\lim_{x \to \infty} \left(x - \sqrt{x^2 - 4x} \right) \quad \text{is equal to}$$

A) 1

B) 2

C) -1

D) -2

E) 4

19) The minimum surface area of a cardbox of $64 \, \text{in}^3$ in volume which has square top and bottom is

A) $16 \, \text{in}^2$

B) $32 \, \text{in}^2$

C) $48 \, \text{in}^2$

D) $64 \, \text{in}^2$

E) $96 \, \text{in}^2$

20) Given $f''(x) = 6x$, $f'(0) = 1$ and $f(0) = 2$, then $f(1)$ is equal to

A) 1

B) 2

C) 0

D) 5

E) 4

21) Find the value of $\int_0^1 (3x^2 + 2x + 1)\, dx$

A) 1

B) 2

C) 3

D) 0

E) 5

22) Let $f(x) = x^2$ on the interval $[0, 2]$. Let the interval be partitioned as follows: $P = \{0, 1, 2\}$. Find the value of the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\Delta x_i$ if each x_i^* is the midpoint of its subinterval.

A) 1/2

B) 2/3

C) 3/4

D) 3/2

E) 5/2

23) If

$$F(x) = \int_{x}^{x^2} \sqrt{1 + t^3}\, dt, \quad \text{then} \quad F'(1) =$$

A) $\sqrt{3}$

B) $\sqrt{2}$

C) 1

D) $2\sqrt{2}$

E) 3

24)

$$\int_0^{\frac{\pi}{2}} \cos^2 x \sin x \, dx =$$

A) 2/3

B) 1/3

C) 1

D) 2

E) 4/3

25)

$$\int_0^2 \frac{x^2}{\sqrt{1+x^3}} \, dx =$$

A) 1

B) 2

C) 2/3

D) 5/3

E) 4/3