December, 2001

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I.D. #: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Recitation Time: ⎯⎯⎯⎯⎯⎯⎯ **Recitation Instructor:** ⎯⎯⎯⎯⎯⎯⎯⎯

Lecturer: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Instructions:

1. Fill in the information requested above.

2. Use a number 2 pencil on the mark-sense sheet.

3. On the mark-sense sheet, fill in your name and student I.D. number, blacken the appropriate spaces, and sign the mark-sense sheet.

4. On the mark-sense sheet mark the division and section number of your class and blacken the corresponding circles. Also fill in the recitation instructor's name and course number.

5. There are 10 pages (including this one) and 25 questions, each worth 8 points. On the mark-sense sheet blacken your choice of answer for each question. Use the exam booklet for working. Turn in both the answer sheet and the exam booklet to your instructor before you leave.

6. No books, notes or calculators may be used.

1. $\sin(\tan^{-1}(2)) =$

A. $\dfrac{\sqrt{3}}{2}$

B. $\dfrac{1}{2}$

C. $\dfrac{\sqrt{3}}{\sqrt{5}}$

D. $\dfrac{2}{3}$

E. $\dfrac{2}{\sqrt{5}}$

2. $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + x} - x \right) =$

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

3. The graph of the function $f(x) = \dfrac{e^x + e^{-x}}{e^x - e^{-2x}}$ has

A. horizontal asymptotes $y = 0$ and
    $y = 1$ and no vertical asymptote

B. horizontal asymptotes $y = 1$ and
    $y = -1$ and vertical asymptote $x = 0$

C. horizontal asymptotes $y = 0$ and
    $y = 1$ and vertical asymptote $x = 0$

D. horizontal asymptotes $y = 1$ and
    $y = -1$ and no vertical asymptote

E. horizontal asymptote only $y = 1$
    and vertical asymptote $x = 0$

4. $\displaystyle \lim_{x \to 0} \frac{e^x - 1 - x}{x^2} =$

    A.   0

    B. $\dfrac{1}{2}$

    C.   1

    D.   2

    E. ∞

5. If $f(x) = \dfrac{3x^2 - x}{x + 2}$, then $f'(1) =$

    A. $\dfrac{13}{9}$

    B.   2

    C. $\dfrac{13}{3}$

    D.   5

    E. $\dfrac{25}{9}$

6. If $f(x) = (x^2 + \ln x)(x^3 + 1)$, then $f'(1) =$

    A. -3

    B.   0

    C.   3

    D.   6

    E.   9

7. If $f(x) = \sqrt{x^2 + \sqrt{x}}$, then $f'(1) =$

    A. $\dfrac{5\sqrt{2}}{8}$

    B. $\dfrac{\sqrt{2}}{2}$

    C. $\sqrt{2}$

    D. $2\sqrt{2}$

    E. $\dfrac{8\sqrt{2}}{5}$

8. If $f(x) = g(e^{2x})$, then $f''(x) =$

    A. $g''(4e^{2x})$

    B. $g'(e^{2x})4e^{4x}$

    C. $4g''(e^{2x})e^{4x} + 4g'(e^{2x})e^{2x}$

    D. $g''(e^{2x})+4g'(e^{2x})e^{2x}+2g(e^{2x})e^{2x}$

    E. $g''(e^{2x})e^{2x} + g'(e^{2x})e^{2x}$

9. The function $y = f(x)$ satisfies $xy^3 + xy = 6$ and $f(3) = 1$. Find $f'(3)$.

    A. $-\dfrac{1}{6}$

    B. 0

    C. $\dfrac{1}{6}$

    D. $\dfrac{11}{3}$

    E. 2

10. If $F(x) = \sin(g(x))$, $g(0) = 0$, $g'(0) = \pi$ then $F'(0) =$

           A.   0

           B. π

           C. -1

           D. $-\pi$

           E.   1

11. The largest interval on which the function $f(x) = x^x$, $x > 0$ is increasing is

           A. $(0, \infty)$

           B. $(0, \frac{1}{e})$

           C. $(1, \infty)$

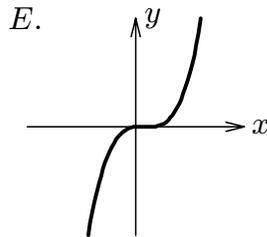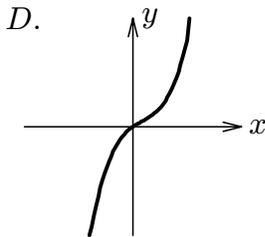   D. $(\frac{1}{e}, 1)$
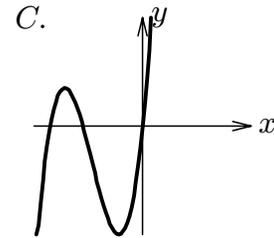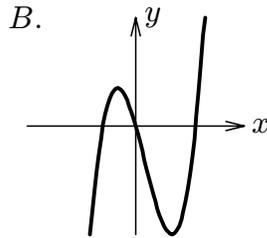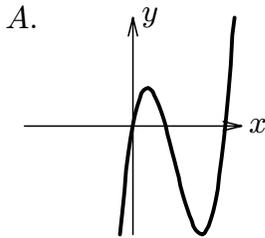
           E. $(\frac{1}{e}, \infty)$

12. If $f(x) = 2\tan x - \tan^2 x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then the length of the largest interval on which $f(x)$ is increasing is

           A.   0

           B. $\dfrac{\pi}{4}$

           C. $\dfrac{\pi}{2}$

           D. $\dfrac{3\pi}{4}$

           E. π

13. $f(x) = x^3 + ax^2 + bx$ has a critical point when $x = 1$ and an inflection point when $x = 2$. Then

    A. $a = -6, \ b = 9$

    B. $a = 1, \ b = 9$

    C. $a = 9, \ b = -6$

    D. $a = 9, \ b = 1$

    E. $a = -6, \ b = 1$

14. Which could be the graph of $y = 2x^3 - 3x^2 - 12x$?

A.

B.

C.

D.

E.

15. The function $f(x) = x^4 - \frac{1}{2} x^2 + \frac{1}{32}$ has critical numbers $x = -\frac{1}{2}, 0, \frac{1}{2}$. How many real solutions are there to the equation $x^4 - \frac{1}{2} x^2 - 1 = 0$?

    A. 0

    B. 1

    C. 2

    D. 3

    E. 4

16. A colony of bacteria started with 1000 bacteria and its population triples every 5 hours. After t hours how many bacteria will there be?

    A. $2^{5000/t}$

    B. $1000 \ 2^{3t/5}$

    C. $1000 \ e^{\frac{\ln 2}{5} t}$

    D. $1000 \ e^{\frac{5}{\ln 3} t}$

    E. $1000 \ 3^{t/5}$

17. The derivative of a function $f(x)$ is given by

    $$f'(x) = x^2(x-1)(x-2)^2(x-3)$$
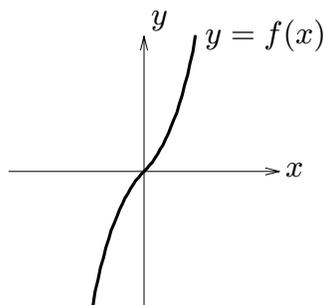
    Consider the following statements.

    I. f has a local minimum when $x = 0$

    II. f has a local maximum when $x = 1$

    III. f has a local minimum when $x = 2$

    IV. f has a local maximum when $x = 3$

    A. all of I, II, III, IV are true

    B. I and III are true, II and IV are false

    C. II is true, I, III and IV are false

    D. II, III and IV are true, I is false

    E. all of I, II, III, IV are false

18. The volume of a cone is $\dfrac{1}{3} \pi r^2 h$. The radius is increasing at 6 cm/sec. and the height is decreasing at 4 cm/sec. At what rate is the volume changing when the radius is 3 cm. and the height is 1 cm.?

A. -6π cm^3/sec

B. 0 cm^3/sec

C. 6π cm^3/sec

D. 9π cm^3/sec

E. 18π cm^3/sec

19. The graph of $f(x) = x\, e^{|x/2|}$ is shown

Let $F(x) = \displaystyle\int_0^x f(t)dt$. Which of the following statements are true?

  I. $F(0) = 0$

 II. $F(x)$ is never negative

III. $F(x)$ is increasing on $(-\infty, \infty)$

IV. $F(-x) = -F(x)$

 V. $F(-x) = F(x)$

A. All

B. only I and II

C. only III and IV

D. only I, II and V

E. only I and IV

20. $\dfrac{d}{dx}\displaystyle\int_{1}^{x} \sec(t^2)\, dt =$

   A. $\tan(x)$

   B. $2x\sec(x^2)$

   C. $2x\sec(x^2)\tan(x^2)$

   D. $\sec(x^2)$

   E. $\tan(x^2)$

21. $\dfrac{d}{dx}\displaystyle\int_{x^2}^{x^3} e^{t^2}\, dt =$

   A. $3x^2 e^{x^6} - 2x e^{x^4}$

   B. $e^{x^6} - e^{x^4}$

   C. $3x^2 e^{x^2} - 2x e^{x^2}$

   D. $e^{x^3} - e^{x^2}$

   E. $6x^5 e^{x^6} - 4x^3 e^{x^4}$

22. $\displaystyle\int_{0}^{\ln(\pi/2)} e^x \cos(e^x)\, dx =$

   A. -1

   B. $\cos(e) - \cos(1)$

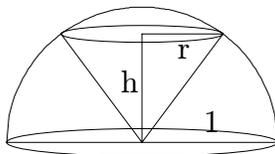   C. $e - \sin(1)$

   D. $\pi/2 - 1$

   E. $1 - \sin(1)$

23. $\displaystyle\int_0^1 \frac{x}{1+x^4}\, dx =$

     A. e^{-2}

     B. $\ln 2$

     C. $1/2$

     D. 2

     E. $\pi/8$

24. $\displaystyle\int 7^x\, dx =$

     A. $(\ln 7)\, 7^x + C$

     B. $\dfrac{1}{\ln 7}\, 7^x + C$

     C. $7^x + C$

     D. $\dfrac{1}{7}\, 7^x + C$

     E. $\dfrac{1}{x+1}\, 7^{x+1} + C$

25. A right circular cone is inscribed in a hemisphere of radius 1 as shown

What value of h will give a cone of maximum volume (the volume of a cone of radius r and height h is $\dfrac{1}{3}\,\pi r^2 h$.)

     A. $\dfrac{2}{3}$

     B. $\dfrac{1}{2}$

     C. $\dfrac{1}{\sqrt{2}}$

     D. $\dfrac{1}{\sqrt{3}}$

     E. $\dfrac{1}{\sqrt{5}}$