1. If $|4 - x^2| \le 5$ then

$\rightarrow -5 \le 4 - x^2 \le 5$
$\rightarrow -9 \le -x^2 \le 1$
$\rightarrow 9 \geq x^2 \geq -1$
$\rightarrow -1 \le x^2 \le 9$
$\rightarrow 0 \le x^2 \le 9$ (x^2 can't be negative)
$\rightarrow -3 \le x \le 3$

A. $-1 \le x \le 3$
B. $-3 \le x$
C. $0 \le x \le 3$
D. $1 \le x \le 3$
E. $-3 \le x \le 3$

2. An equation for the line through the point $(3, 6)$ parallel to the line with equation $2x + y = 1$ is

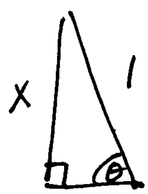
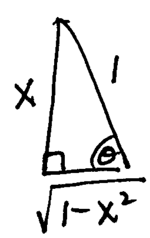
line: $2x + y = 1 \rightarrow y = -2x + 1$
$\rightarrow$ slope $= -2$

line through $(3,6)$ with slope -2 has an equation $y = -2(x-3) + 6$
$\rightarrow y = -2x + 6 + 6$
$\rightarrow y + 2x = 12$

A. $y + x = 9$
B. $2y - x = 9$
C. $y + 2x = 12$
D. $2y + x = 15$
E. $y - 2x = 0$

3. $\cot(\sin^{-1} x) =$

Let $\theta = \sin^{-1} x$. Then $\sin\theta = x = \frac{x}{1}$

 $\rightarrow$  $\rightarrow \cot(\theta) = \frac{\sqrt{1-x^2}}{x}$

A. $\sqrt{1 - x^2}$
B. $\frac{x}{\sqrt{1 - x^2}}$
C. $x\sqrt{1 - x^2}$
D. $\frac{\sqrt{1 - x^2}}{x}$
E. $1 - x^2$

4. The graph of $y = e^x$ is reflected about the y axis and then reflected about the line $y = 1$. The resulting graph is the graph of $y =$

reflect $y = e^x$ about y-axis $\to y = e^{-x}$

To reflect about $y = 1$:

1. translate down 1 unit $\to y = e^{-x} - 1$
2. reflect about $y = 0$ (x-axis) $\to y = -(e^{-x} - 1)$
3. translate up 1 unit $\to y = -(e^{-x} - 1) + 1$

$= -e^{-x} + 2$

A. e^{1-x}

B. $2 - e^x$

C. $1 - e^{-x}$

D. $2 - e^{-x}$

E. $1 - e^x$

5. A radioactive isotope has a half-life of 12 days. Initially there are 7 grams of the isotope. After t days how much of the isotope remains?

$A(0) = 7 = (\frac{1}{2})^0 \cdot 7 = (\frac{1}{2})^{0/12} \cdot 7$

$A(12) = \frac{1}{2} \cdot 7 = (\frac{1}{2})^1 \cdot 7 = (\frac{1}{2})^{12/12} \cdot 7$

$A(24) = \frac{1}{2} \cdot \frac{1}{2} \cdot 7 = (\frac{1}{2})^2 \cdot 7 = (\frac{1}{2})^{24/12} \cdot 7$

$A(36) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 7 = (\frac{1}{2})^3 \cdot 7 = (\frac{1}{2})^{36/12} \cdot 7$

Thus $A(t) = (\frac{1}{2})^{t/12} \cdot 7 = (2^{-1})^{t/12} \cdot 7 = 2^{-t/12} \cdot 7$

A. $7 \cdot 2^{t/12}$

B. $12 \cdot 2^{t/7}$

C. $12 \cdot t^{-t/7}$

D. $7 \cdot 2^{-t/12}$

E. $7 \cdot 2^{-12t}$

6. If $\ln x - \ln(x - 1) = 1$ then $x =$

$\ln x - \ln(x-1) = 1$

$\to \ln\left(\frac{x}{x-1}\right) = 1$

$\to \frac{x}{x-1} = e^1$

$\to x = ex - e$

$\to x - ex = -e$

$\to x(1-e) = -e$

$\to x = \frac{-e}{1-e} = \frac{e}{e-1}$

A. 0

B. 1

C. $\frac{e}{e-1}$

D. $1 - e$

E. $1 - \frac{1}{e}$

7. $\lim_{t\to 0} \frac{\sqrt{3+t}-\sqrt{3}}{t} =$ $\frac{\sqrt{3}-\sqrt{3}}{0} = \frac{0}{0}$

A. 0

B. $\frac{1}{2\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

E. ∞

$$\frac{\sqrt{3+t}-\sqrt{3}}{t} \cdot \frac{\sqrt{3+t}+\sqrt{3}}{\sqrt{3+t}+\sqrt{3}} = \frac{(3+t)-(3)}{t(\sqrt{3+t}+\sqrt{3})}$$

$$= \frac{t}{t(\sqrt{3+t}+\sqrt{3})} = \frac{1}{\sqrt{3+t}+\sqrt{3}} \text{ if } t \neq 0.$$

Thus $\lim_{t\to 0} \frac{\sqrt{3+t}-\sqrt{3}}{t} = \lim_{t\to 0} \frac{1}{\sqrt{3+t}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$

8. $\lim_{x\to 2^-} \frac{x^2-4}{|x-2|} =$ $\frac{4-4}{|2-2|} = \frac{0}{0}$

A. -4

B. -2

C. 0

D. 2

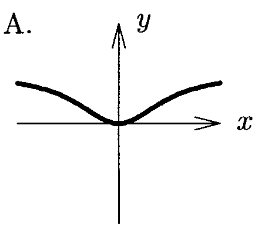
E. 4

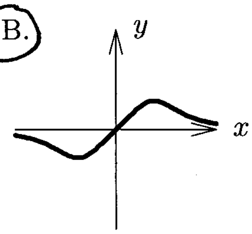
$x \to 2^-$ means $x < 2$ so $|x-2| = -(x-2)$, because $x-2 < 0$. Thus,

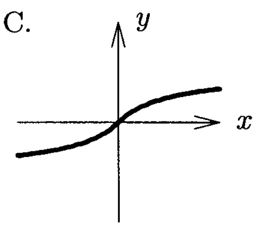
$$\lim_{x\to 2^-} \frac{x^2-4}{|x-2|} = \lim_{x\to 2^-} \frac{x^2-4}{-(x-2)} = \lim_{x\to 2^-} \frac{(x-2)(x+2)}{-(x-2)}$$

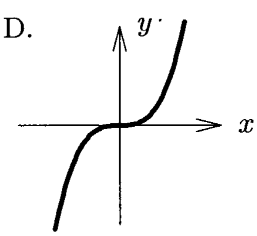
$$= \lim_{x\to 2^-} -(x+2) = -4.$$

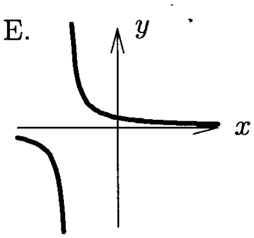
9. The graph of $\frac{x}{1+x^4}$ looks most like

A.


B.


C.


D.


E.


1. $\frac{x}{1+x^2}$ has same sign as x, so A and E are eliminated.

2. $\lim_{x\to\infty} \frac{x}{1+x^4} = 0$, so C and D eliminated.

10. Find all the values of c so that the function

$$f(x) = \begin{cases} c^2 - x^2 & \text{if } x < 2 \\ 2(c+x) & \text{if } x \geq 2 \end{cases}$$

is continuous on all of $(-\infty, \infty)$.

A. $-4, -2$

B. $-2, 4$

C. 2

D. -2

E. $2, 0$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (c^2 - x^2) = c^2 - 4$

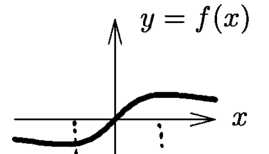
$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2(c+x) = 2c + 4$

f cont. at all $x \neq 2$ because polynomials are cont.

f cont at $x = 2$ if $c^2 - 4 = 2c + 4$.

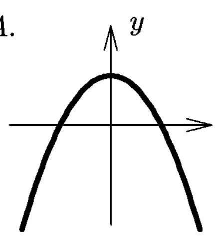
$\rightarrow c^2 - 2c - 8 = 0 \rightarrow (c-4)(c+2) = 0 \rightarrow c = -2, 4$

11. Which could be the graph of $y = f'(x)$ if $y = f(x)$ has graph

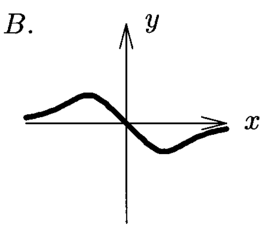


slope of f [$-$ | $+$ | $-$] $\Rightarrow$ elimates B, C, D.

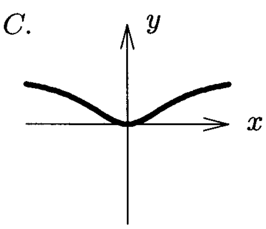
A.



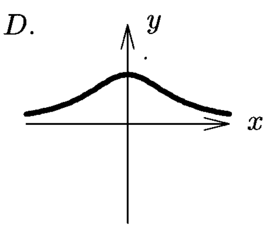
B.



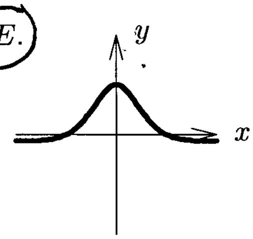
C.



D.



E.



A is incorrect because it indicates steep (large) negative slope while E indicates negative slope near zero.

12. If $f(x) = 1 - \frac{2}{x}$ then $f'(x) =$

$f(x) = 1 - 2x^{-1}$

$f'(x) = 0 - 2(-x^{-2}) = \frac{2}{x^2}$

A. $\frac{2}{x^2}$

B. $\frac{1}{x}$

C. $\frac{1}{2x}$

D. $1 + \frac{1}{x}$

E. $1 + \frac{2}{x^2}$

13. Given the following information

$f(0) = 4,\ f(1) = 2,\ f'(0) = 0,\ f'(1) = 3,\ f'(2) = 5,\ f'(4) = 7,$

an equation of the tangent line to the curve $y = f(x)$ at $(1, 2)$ is

tangent line has slope $f'(1) = 3$.

Tangent line has an equation:

$y = 3(x-1) + 2$

$\rightarrow y - 2 = 3(x-1)$

A. $y - 2 = 3(x - 1)$

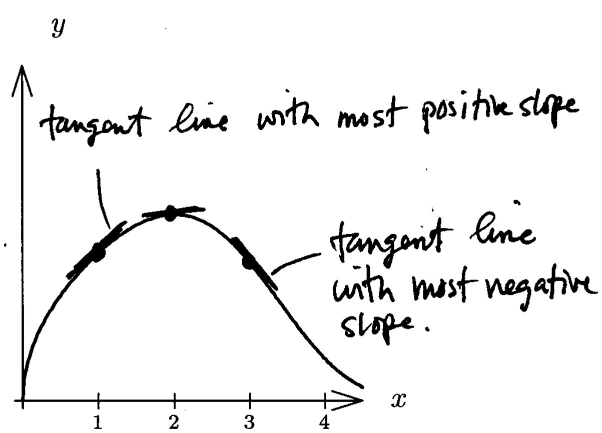
B. $y - 2 = 5(x - 1)$

C. $y - 1 = 7x$

D. $y - 4 = 7(x - 2)$

E. cannot be determined from the information given

14. Given the graph of $y = f(x)$ arrange $f'(1)$ $f'(2)$ and $f'(3)$ in increasing size



A. $f'(1), f'(2), f'(3)$

B. $f'(2), f'(3), f'(1)$

C. $f'(2), f'(1), f'(3)$

D. $f'(1), f'(3), f'(2)$

E. $f'(3), f'(2), f'(1)$