1. $\lim_{s\to\infty} \dfrac{2-3s+5s^2}{4-5s^3} =$

$$= \lim_{s\to\infty} \frac{(2-3s+5s^2)\left(\frac{1}{s^3}\right)}{(4-5s^3)\left(\frac{1}{s^3}\right)}$$

$$= \lim_{s\to\infty} \frac{\frac{2}{s^3} - \frac{3}{s^2} + \frac{5}{s}}{\frac{4}{s^3} - \frac{5}{1}} = \frac{0-0+0}{0-5} = \frac{0}{-5} = 0.$$

A. $-\infty$

B. -1

C. 0

D. $\frac{1}{2}$

E. 1

2. If a ball is thrown upwards with a velocity of 52 ft/second, its height in feet after t seconds is $y = 52t - 16t^2$. Find the velocity after 2 seconds.

$$\text{velocity} = v(t) = \frac{dy}{dt} = 52 - 32t$$

$$v(2) = 52 - 32(2) = 52 - 64 = -12$$

A. 20 ft/sec upward

B. 20 ft/sec downward

C. 12 ft/sec upward

D. 12 ft/sec downward

E. 0 ft/sec

3. If $f(x) = x^2 \ln x$, then $f'(x) =$

$$f'(x) = \left(\frac{d}{dx} x^2\right)(\ln x) + (x^2)\left(\frac{d}{dx} \ln x\right)$$

$$= (2x)(\ln x) + (x^2)\left(\frac{1}{x}\right)$$

$$= 2x \ln x + x$$

$$= x(2\ln x + 1)$$

A. 2

B. x

C. $2x \ln x$

D. $2x + \frac{1}{x}$

E. $x(2\ln x + 1)$

4. If $f(x) = \frac{\sin x}{x}$, then $f'(x) =$

$$f'(x) = \frac{\left(\frac{d}{dx}\sin x\right)(x) - (\sin x)\left(\frac{d}{dx}x\right)}{(x)^2}$$

$$= \frac{(\cos x)(x) - (\sin x)(1)}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

A. $-\frac{\cos x}{x^2}$

B. $-\frac{\sin x}{x^2}$

C. $-\frac{\sin x \cos x}{x^2}$

D. $\frac{x\cos x - \sin x}{x^2}$

E. $\frac{x\sin x - \cos x}{x^2}$

5. A spherical balloon is inflated in such a fashion that its radius increases at a rate of 0.5 cm/s. At a certain instant the radius is 3 cm. In cm^3/s, how fast is the volume increasing at that instant? (The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.)

$$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = (4\pi r^2)\left(\frac{dr}{dt}\right)$$

$$\frac{dV}{dt} = (4\pi(3)^2)(0.5)$$

$$= 18\pi$$

A. 12π

B. 18π

C. 24π

D. 30π

E. 36π

6. If $f(x) = (x^3+1)^{10}$, then $f'(1) =$

$$f'(x) = 10(x^3+1)^9(3x^2)$$

$$f'(1) = 10(1^3+1)^9(3\cdot 1^2)$$

$$= 30\cdot 2^9$$

A. $3\cdot 2^9$

B. $6\cdot 2^9$

C. $10\cdot 2^9$

D. $20\cdot 2^9$

E. $30\cdot 2^9$

7. If $f(x) = \sqrt[3]{\sin(x^3)}$, then $f'(2) =$

A. $\cos 2$

B. $\frac{\cos 8}{3\sqrt[3]{\sin^2 8}}$

C. $\frac{4\cos 8}{3\sqrt[3]{\sin^2 8}}$

D. $\frac{4\cos 8}{\sqrt[3]{\sin^2 8}}$

E. $\frac{12\cos 8}{\sqrt[3]{\sin^2 8}}$

$$f(x) = (\sin(x^3))^{1/3}$$
$$f'(x) = \frac{1}{3}(\sin(x^3))^{-2/3}\left(\frac{d}{dx}(\sin(x^3))\right)$$
$$= \frac{1}{3}(\sin(x^3))^{-2/3}((\cos(x^3))(3x^2))$$
$$f'(2) = \frac{1}{3}(\sin 8)^{-2/3}(\cos(8))(12)$$
$$= 4\frac{\cos 8}{(\sin 8)^{2/3}} = 4\frac{\cos 8}{\sqrt[3]{\sin^2 8}}$$

8. If $f'(8) = 5$ and $f'(2) = 7$, evaluate $\frac{d}{dx} f(2x^2)$ at $x = 2$.

A. 10

B. 20

C. 35

D. 40

E. 56

$$\frac{d}{dx} f(2x^2) = (f'(2x^2))(4x)$$
$$\left.\frac{d}{dx} f(2x^2)\right|_{x=2} = (f'(8))(8)$$
$$= 5 \cdot 8 = 40$$

9. If $f(x) = \tan x$ then $f'''(x) =$

A. $4\sec^2 x \tan^2 x + 2\sec^4 x$

B. $4\sec^2 x \tan^2 x + \tan^4 x$

C. $2\sec^2 x \tan^2 x + 4\sec^4 x$

D. $4\sec^4 x \tan^2 x$

E. $2\sec^4 x$

$$f'(x) = \sec^2 x$$
$$f''(x) = (2\sec x)(\sec x \tan x)$$
$$= 2\sec^2 x \tan x$$
$$f'''(x) = 2((2\sec x \sec x \tan x)(\tan x) + (\sec^2 x)(\sec^2 x))$$
$$= 4\sec^2 x \tan^2 x + 2\sec^4 x$$

10. The slope of the line tangent to the curve $x^2 + x^3y^2 - y^4 = 11$ at the point $(2,1)$ is

differentiate w.r.t. x, assuming y' exists:

$\rightarrow 2x + (3x^2y^2 + x^3 \cdot 2yy') - 4y^3y' = 0$

$(x,y) = (2,1) \rightarrow 4 + (12 + 16y') - 4y' = 0$

$\rightarrow 12y' = -16$

$\rightarrow y' = \frac{-16}{12} = -\frac{4}{3}$

A. $-\frac{1}{3}$

B. $-\frac{2}{3}$

C. -1

D. $-\frac{4}{3}$

E. 4

11. A particle moves along a straight line with equation of motion $s = t^3 - t^2$. Find the value of t at which the acceleration is zero.

Velocity $= v = \frac{ds}{dt} = 3t^2 - 2t$

acceleration $= a = \frac{dv}{dt} = 6t - 2$

$6t - 2 = 0 \rightarrow t = \frac{1}{3}$

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

E. $\frac{3}{2}$

12. Radium has a half life of 1600 years. How long does it take for 90% of a given amount of radium to decay?

$A(t) = A(0)e^{kt}$. Half life of 1600 years $\rightarrow$

$\frac{1}{2}\cancel{A(0)} = \cancel{A(0)}e^{1600k} \rightarrow k = \frac{1}{1600}\ln\frac{1}{2}$.

Solve $\frac{1}{10}A(0) = A(0)e^{(\frac{1}{1600}\ln\frac{1}{2})t}$ for t.

$\rightarrow \frac{1}{10} = e^{(\frac{1}{1600}\ln\frac{1}{2})t} \rightarrow \ln\frac{1}{10} = (\frac{1}{1600}\ln\frac{1}{2})t$

$\rightarrow t = 1600\frac{\ln\frac{1}{10}}{\ln\frac{1}{2}} = 1600\frac{-\ln 10}{-\ln 2} = 1600\frac{\ln 10}{\ln 2}$

A. $\frac{1600 \ln 10}{\ln 2}$

B. $\frac{1600 \ln 2}{\ln 10}$

C. $1600 \ln 5$

D. $1600 \ln 10$

E. $\frac{1600(\ln 10 - \ln 9)}{\ln 2}$

13. If $f(x) = e^{-3x} + 2x^{27} + 3x^2$. then the nineteen derivative of f evaluated at 0, $f^{(19)}(0) =$

The terms $2x^{27}$ and $3x^2$ will contribute zero to $f^{(19)}(0)$. Consider only $f(x) = e^{-3x}$.

Note that: $f'(x) = -3e^{-3x}$

$f''(x) = (-3)^2 e^{-3x}$

$f'''(x) = (-3)^3 e^{-3x}$

and $f^{(n)}(x) = (-3)^n e^{-3x}$

Thus $f^{(19)}(0) = (-3)^{19} e^0 = (-3)^{19}$

A. 0

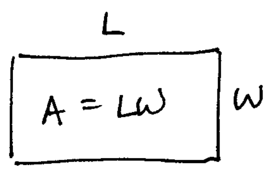
B. 1

C. -3^{19}

D. $-3^{19}e^{-3}$

E. $-3^{-19}e^{-3}$

14. The length of a rectangle is increasing at a rate of 2 feet per second, while the width is increasing at a rate of 1 foot per second. When the length is 5 feet and the width is 3 feet, how fast is the area increasing?



Given: $\frac{dL}{dt} = 2 \frac{ft}{sec}$

$\frac{dW}{dt} = 1 \frac{ft}{sec}$

Want: $\frac{dA}{dt}$ at time t when L=5 and W=3,

$$\frac{dA}{dt} = \frac{dL}{dt} W + L \frac{dW}{dt} = (2)(3) + (5)(1) = 11$$

A. 2 ft^2/s

B. 6 ft^2/s

C. 10 ft^2/s

D. 11 ft^2/s

E. 13 ft^2/s