1. $\frac{d}{dx}\sin^{-1}\sqrt{x}=$

$$= \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{d}{dx}\sqrt{x}$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\cdot\sqrt{x-x^2}}$$

A. $\frac{\cos\sqrt{x}}{2\sqrt{x}\sin^2\sqrt{x}}$

B. $\frac{\cos^{-1}\sqrt{x}}{2\sqrt{x}}$

C. $\frac{1}{2\sqrt{x-x^3}}$

D. $\frac{1}{\sqrt{1-x}}$

E. $\frac{1}{2\sqrt{x-x^2}}$

2. An equation of the line tangent to the graph of $x^2+y^4=2xy^2+1$ at (2,1) is

differentiate w.r.t. x $\longrightarrow$

$$2x + 4y^3\frac{dy}{dx} = 2y^2 + 4xy\frac{dy}{dx}$$

x=2 and y=1 $\longrightarrow$

$$4 + 4\frac{dy}{dx} = 2 + 8\frac{dy}{dx}$$

$$\rightarrow \frac{dy}{dx} = \frac{1}{2}$$

tangent line is $y-1=\frac{1}{2}(x-2) \rightarrow y=\frac{1}{2}x$

A. $y=\frac{x}{2}$

B. $y=2-\frac{x}{2}$

C. $y=2x-3$

D. $y=2x+3$

E. None of the above

3. When $x = \frac{\pi}{3}$, $\frac{d^2}{dx^2}\cos 3x =$

A. 6
B. 9
C. 12
D. 0
E. $-6$

let $y = \cos 3x$.

$\rightarrow \frac{dy}{dx} = -3\sin 3x$

$\rightarrow \frac{d^2y}{dx^2} = -9\cos 3x$

$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{\pi}{3}} = -9\cos\pi = -9(-1) = 9.$$

4. If $y = \frac{e^x\sqrt{x}}{(x-1)^2}$, then $y' =$

A. $\frac{e^x}{4\sqrt{x}(x-1)}$

B. $\frac{\sqrt{x}}{(x-1)^4}$

C. $1+\frac{1}{2x}-\frac{2}{x-1}$

D. $e^x\left(\frac{\sqrt{x}}{(x-1)^2}+\frac{1}{2\sqrt{x}}\right)$

E. $\left(1+\frac{1}{2x}-\frac{2}{x-1}\right)\frac{e^x\sqrt{x}}{(x-1)^2}$

$\ln y = \ln e^x + \ln\sqrt{x} - \ln(x-1)^2$

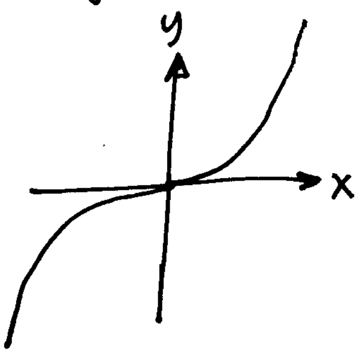
$= x + \frac{1}{2}\ln x - 2\ln(x-1)$

$\rightarrow \frac{\frac{dy}{dx}}{y} = 1 + \frac{1}{2x} - \frac{2}{x-1}$

$\rightarrow \frac{dy}{dx} = y\left(1+\frac{1}{2x}-\frac{2}{x-1}\right)$

5. $\lim_{x \to \infty} \sinh x =$

$y = \sinh x$




A. $-\infty$

B. $-1$

C. $0$

D. $1$

E. $\infty$

6. The half-life of radioactive calculium-161 is 3 years. If a sample has a mass of 6 lb, how much will remain after 8 years?

Find k: $A(t) = A(0) e^{kt}$

$A(3) = \frac{1}{2} \cancel{A(0)} = \cancel{A(0)} e^{3k}$

$\rightarrow \frac{1}{2} = e^{3k} \rightarrow k = \frac{1}{3} \ln \frac{1}{2}$

$A(8) = A(0) e^{(\frac{1}{3} \ln \frac{1}{2}) t}$

$= 6 \cdot e^{(\frac{1}{3} \ln \frac{1}{2}) 8} = 6 \cdot e^{(\ln \frac{1}{2})(\frac{8}{3})}$

$= 6 \cdot \left(\frac{1}{2}\right)^{8/3} = \frac{6}{2^{8/3}}$

A. $\frac{6}{2^{8/3}}$ lb

B. 2 lb

C. $\frac{3}{2^{4/3}}$ lb

D. $\frac{6}{3^{1/4}}$ lb

E. $\frac{8}{3^3}$ lb

7. A particle moves along the curve $y = (5 + x^2)^{3/2}$. As it reaches the point (2,27) the $x$-coordinate is increasing at a rate of 2 in/s. How fast (in in/s) is the $y$-coordinate of the point increasing at that instant?

A. 4
B. 9
C. 18
D. 36
E. 54

$$\frac{dy}{dt} = \left(\frac{3}{2}\right)(5+x^2)^{1/2}(2x)\left(\frac{dx}{dt}\right)$$

$$= \left(\frac{3}{2}\right)(9)^{1/2}(4)(2)$$

$$= 36$$

8. Use differentials (or equivalently, a linear approximation) to estimate $\sqrt[4]{80}$.

A. $3 - \frac{1}{4} \cdot \frac{1}{27}$
B. $3 + \frac{1}{4} \cdot \frac{1}{27}$
C. $3 - \frac{1}{4} \cdot \frac{1}{3}$
D. $3 + \frac{1}{4} \cdot \frac{1}{3}$
E. $4 - \frac{1}{4} \cdot \frac{1}{3}$

Let $f(x) = x^{1/4} \rightarrow f(81) = 3$

Then $f'(x) = \frac{1}{4} x^{-3/4} \rightarrow f'(81) = \frac{1}{4} \cdot \frac{1}{27}$

$$y = 3 + \frac{1}{4} \cdot \frac{1}{27}(x - 81)$$

$$y(80) = 3 - \frac{1}{4} \cdot \frac{1}{27}$$

9. A spotlight on the ground shines on a vertical wall 12 m away. If a man 2 m tall walks from the spotlight toward the wall at a rate of 2 m/s, how fast is the length of his shadow on the wall decreasing when he is 5 m from the wall?

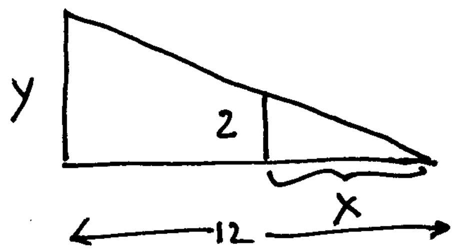
A. 2 m/s

B. $\frac{48}{49}$ m/s

C. $\frac{24}{5}$ m/s

D. $\frac{24}{10}$ m/s

E. $\frac{48}{25}$ m/s




$$\frac{x}{2} = \frac{12}{y} \rightarrow xy = 24$$

$$x = 7 \rightarrow y = \frac{24}{7}$$

differentiate w.r.t. t $\longrightarrow$

$$x\frac{dy}{dt} + \frac{dx}{dt}y = 0$$

$$\rightarrow (7)\left(\frac{dy}{dt}\right) + (2)\left(\frac{24}{7}\right) = 0$$

$$\rightarrow \frac{dy}{dt} = -(2)\left(\frac{24}{7}\right)\left(\frac{1}{7}\right) = -\frac{48}{49}$$

10. A circle has a circumference measured to be 30 in with a possible error of $\frac{1}{4}$ in. Estimate the maximum possible error in the resulting area of the circle (in in$^2$).

A. 30

B. $30\pi$

C. $\frac{30}{\pi}$

D. $\frac{15}{4\pi}$

E. $\frac{15}{2\pi}$

$$\text{Circumference} = C = 2\pi r \rightarrow r = \frac{1}{2\pi}C$$

$$\rightarrow dr = \frac{1}{2\pi}dC = \frac{1}{2\pi}\cdot\frac{1}{4} = \frac{1}{8\pi}$$

$$\text{Area} = A = \pi r^2 \rightarrow dA = 2\pi r\, dr$$

$$dA = 2\pi\left(\frac{15}{\pi}\right)\left(\frac{1}{8\pi}\right) = \frac{15}{4\pi}$$

11. The absolute minimum value of the function $f(x) = x^2 + 2/x$ on the interval $\left[\frac{1}{2}, \frac{3}{2}\right]$ is

$$f'(x) = 2x - \frac{2}{x^2} = \frac{2x^3 - 2}{x^2} = 0 \rightarrow x = 1$$

| $x$ | $f(x) = x^2 + \frac{2}{x}$ |
|---|---|
| $\frac{1}{2}$ | $\frac{1}{4} + 4 = 4\frac{1}{4}$ |
| $1$ | $1 + 2 = 3$ ← minimum value |
| $\frac{3}{2}$ | $\frac{9}{4} + \frac{4}{3} = \frac{27+16}{12} = \frac{43}{12} = 3\frac{7}{12}$ |

A. $\frac{9}{4}$

B. $\frac{17}{4}$

C. 3

D. $\frac{43}{12}$

E. $\frac{35}{12}$

12. The function $f(x) = x^4 - 8x^2$ satisfies the three hypotheses of Rolle's Theorem on the interval $[-1, 1]$. How many numbers $c$ are there in this interval that satisfy the conclusion of Rolle's Theorem?

$$f(-1) = 1 - 8 = -7$$

$$f(1) = 1 - 8 = -7$$

$$f'(x) = 4x^3 - 16x$$

$$f'(c) = \frac{f(1) - f(-1)}{1 - (-1)} = 0$$

$$\rightarrow 4c^3 - 16c = 0 \rightarrow 4c(c^2 - 4) = 0 \rightarrow c = 0, 2, -2$$

Only $c = 0$ is in the interval $(-1, 1)$.

A. 0

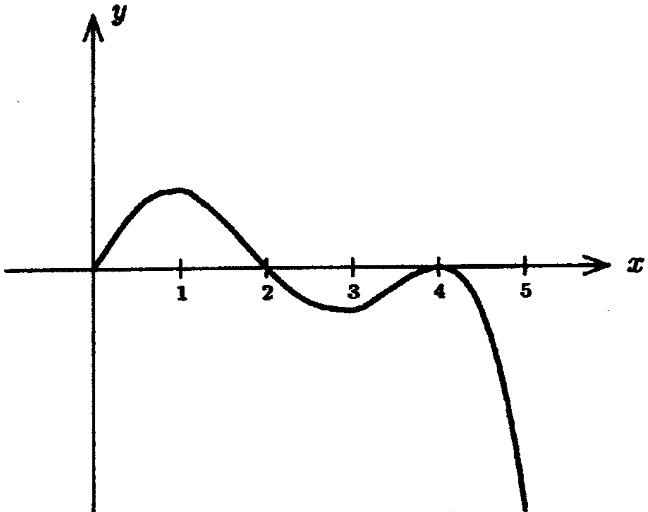
B. 1

C. 2

D. 3

E. None of the above

13. Given the graph of $y = f'(x)$ below, it follows that




0 2 4 x

$f'(x)$: $+$ $\phi$ $-$ $\phi$ $-$

$f$

A. $f$ is decreasing on (1,3) False

B. $f$ is concave upward on (2,4) False

C. $f$ is increasing on (1,2) True

D. $f$ is concave downward on (0,2) False

E. None of the above

Note: $f$ concave upward means $f'$ increasing, and $f$ concave downward means $f'$ decreasing.

14. The function $f(x) = x^4(x-1)^3$ has

$$f'(x) = 4x^3(x-1)^3 + x^4 \cdot 3(x-1)^2$$
$$= x^3(x-1)^2\left(4(x-1) + 3(x)\right)$$
$$= x^3(x-1)^2(7x-4)$$
$$= 0 \rightarrow x = 0, 1, \tfrac{4}{7}$$

A. two critical numbers

B. a relative minimum at $x = 0$

C. a relative minimum at $x = 1$

D. a relative maximum at $x = 1$

E. None of the above

| $x$ | $(-\infty, 0)$ | $(0, 4/7)$ | $(4/7, 1)$ | $(1, \infty)$ |
|---|---|---|---|---|
| $x^3$ | $-$ | $+$ | $+$ | $+$ |
| $(x-1)^2$ | $+$ | $+$ | $+$ | $+$ |
| $7x-4$ | $-$ | $-$ | $+$ | $+$ |
| $f'(x)$ | $+$ | $-$ | $+$ | $+$ |

$f$