1. $\lim_{x\to\infty} (\sqrt{x^2+3x} - x) =$ $\infty - \infty$

$$\left(\sqrt{x^2+3x} - x\right)\left(\frac{\sqrt{x^2+3x}+x}{\sqrt{x^2+3x}+x}\right) = \frac{x^2+3x-x^2}{\sqrt{x^2+3x}+x}$$

$$= \frac{3x}{\sqrt{x^2+3x}+x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \frac{3}{\sqrt{1+\frac{3}{x}}+1} \to \frac{3}{1+1} \text{ as } x\to\infty$$

Therefore $\lim_{x\to\infty}\left(\sqrt{x^2+3x} - x\right) = \frac{3}{2}$

A. 0
B. $\frac{2}{3}$
C. $\frac{3}{2}$
D. 3
E. ∞

2. If $f(x) = x\cos(x)$, then $f''\left(\frac{\pi}{4}\right) =$

$$f'(x) = (1)(\cos(x)) + (x)(-\sin(x))$$
$$= \cos x - x\sin x$$
$$f''(x) = -\sin x - \left((1)(\sin x) + (x)(\cos x)\right)$$
$$= -2\sin x - x\cos x$$

$$f''\left(\frac{\pi}{4}\right) = -2\sin\frac{\pi}{4} - \frac{\pi}{4}\cos\frac{\pi}{4}$$
$$= -2\left(\frac{\sqrt{2}}{2}\right) - \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2}\left(2+\frac{\pi}{4}\right)$$

A. $\frac{1}{\sqrt{2}}$
B. $-\frac{\pi}{4}$
C. $\frac{\pi}{4} - \sqrt{2}$
D. $\frac{\pi}{4}(1-\sqrt{2})$
E. $-\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}+2\right)$

3. If $f(x) = \frac{x}{\sin x}$, then $f'\left(\frac{\pi}{4}\right) =$

$$f'(x) = \frac{(1)(\sin x) - (x)(\cos x)}{\sin^2 x}$$

$$f'\left(\frac{\pi}{4}\right) = \frac{(1)\left(\sin\frac{\pi}{4}\right) - \left(\frac{\pi}{4}\right)\left(\cos\frac{\pi}{4}\right)}{\left(\sin\frac{\pi}{4}\right)^2}$$

$$= \frac{(1)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\pi}{4}\right)\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)^2} = 2\left(\frac{1}{\sqrt{2}} - \left(\frac{\pi}{4}\right)\left(\frac{1}{\sqrt{2}}\right)\right) = \sqrt{2}\left(1-\frac{\pi}{4}\right)$$

A. $1 - \frac{\pi}{2}$
B. $\sqrt{2}\left(1-\frac{\pi}{4}\right)$
C. $\sqrt{2}\left(1+\frac{\pi}{4}\right)$
D. $\sqrt{2}\left(1+\frac{\pi}{2}\right)$
E. $1+\frac{\pi}{4}$

4. Let $g(x) = f(f(x))$ and $f(1) = 2$, $f(2) = -1$, $f'(2) = 7$, $f'(1) = 5$, $f'(-1) = 4$, $f'(4) = 9$, $f'(7) = 3$. Then $g'(1) =$

$$g'(x) = \left(f'(f(x))\right)\left(f'(x)\right)$$
$$g'(1) = \left(f'(f(1))\right)\left(f'(1)\right)$$
$$= f'(2)\, f'(1)$$
$$= (7)(5) = 35$$

A. 35
B. 63
C. 180
D. 189
E. 243

5. If $f(x) = \sqrt{x + \sqrt{x}}$, then $f'(1) =$

$$f'(x) = \left(\frac{1}{2\sqrt{x+\sqrt{x}}}\right)\left(1 + \frac{1}{2\sqrt{x}}\right)$$
$$f'(1) = \left(\frac{1}{2\sqrt{1+\sqrt{1}}}\right)\left(1 + \frac{1}{2\sqrt{1}}\right)$$
$$= \left(\frac{1}{2\sqrt{2}}\right)\left(1 + \frac{1}{2}\right) = \left(\frac{1}{2\sqrt{2}}\right)\left(\frac{3}{2}\right) = \frac{3}{4\sqrt{2}}$$

A. $\frac{1}{2\sqrt{2}}$
B. $\frac{1}{\sqrt{2}}$
C. $\sqrt{2}$
D. $2\sqrt{2}$
E. $\frac{3}{4\sqrt{2}}$

6. If $x^2 - xy + y^3 = 14$ then $\frac{dy}{dx} =$

$$2x - \left((1)(y) + (x)\left(\frac{dy}{dx}\right)\right) + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x + y}{-x + 3y^2}$$

A. $-\dfrac{2xy}{x + 3y^2}$
B. $-\dfrac{3x^2 + y}{2x - y}$
C. $\dfrac{y - 2x}{3y^2 - x}$
D. $\dfrac{x + y}{x^2 + 2y}$
E. $\dfrac{xy}{x^2 - y^2}$

7. The function $f(x)$ has derivative $f'(x) = x(x+1)^3(x-1)^2$. Consider the following statements

   I. f has a local maximum at $x = -1$

   II. f has a local minimum at $x = -1$

   III. f has a local maximum at $x = 0$

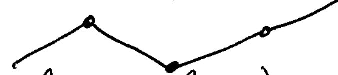
   IV. f has a local minimum at $x = 0$

   V. f has a local maximum at $x = 1$

   VI. f has a local minimum at $x = 1$

	$-\infty$ to -1	-1 to 0	0 to 1	1 to ∞
x	$-$	$-$	$+$	$+$
$(x+1)^3$	$-$	$+$	$+$	$+$
$(x-1)^2$	$+$	$+$	$+$ ϕ	$+$
$f'(x)$	$+$	$-$	$+$	$+$

f



A. I, III, VI are true

B. I and IV are true, V and VI are false

C. I and V are true, III and VI are false

D. III and VI are true, I and II are false

E. I, IV and VI are true

8. Find the absolute maximum of the function $f(x) = x^3 - x^2 - x$ on the interval $-10 \le x \le 1$.

$f'(x) = 3x^2 - 2x - 1$

$f'(x) = 0 \rightarrow (3x+1)(x-1) = 0 \rightarrow x = -\frac{1}{3}, 1$

x	$x^3 - x^2 - x$
-10	$-1000 - 100 - 10 = -1110$
$-1/3$	$-1/27 - 1/9 + 1/3 = \frac{-1-3+9}{27} = \frac{5}{27} \leftarrow$ MAX
1	$1 - 1 - 1 = -1$

A. $-\frac{2}{9}$

B. $\frac{5}{27}$

C. -1

D. $\frac{7}{27}$

E. $\frac{2}{9}$

9. What is the length of the longest interval on which the function $f(x) = \dfrac{x}{x^2+1}$ is increasing?

$$f'(x) = \frac{(1)(x^2+1) - (x)(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2} > 0 \rightarrow 1 - x^2 > 0 \rightarrow x^2 < 1$$

Thus f increasing on interval $-1 < x < 1$, which has length 2

A. 0

B. 1

C. 2

D. 4

E. ∞

10. Determine where the function $f(x) = x + \frac{1}{x^2}$ is concave upward.

$f'(x) = 1 - \frac{2}{x^3}$

$f''(x) = \frac{6}{x^4} > 0 \to x \neq 0$

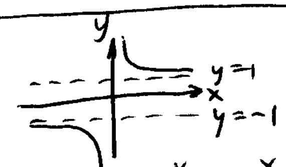
A. $(-\infty, 0)$ and $(0, \infty)$

B. $(-1, 0)$

C. $(0, \infty)$

D. $(0, 1)$

E. nowhere



$\lim_{x \to 0} \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{2}{0} = \pm\infty \to x = 0$ is a vert. asymptote

11. Given the function $f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$, consider the following statements

I. $y = 1$ is a horizontal asymptote of f

II. $y = -1$ is a horizontal asymptote of f

III. $x = 0$ is a vertical asymptote of f

A. I, II, III are false

B. I is true, II and III are false

C. I and II are true, III is false

D. I and III are true, II is false

E. I, II and III are true

$\lim_{x \to \infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} = 1 \to y = 1$ is horizontal asympt.

$\lim_{x \to -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} = -1 \to y = -1$ is also a horizontal asymptote

12. Consider the statements

I. $\lim_{x \to 0^+} \ln|x| = -\infty$ TRUE

II. $\lim_{x \to 0^-} \ln|x| = \infty$ FALSE

III. $\lim_{x \to -\infty} \ln|x| = -\infty$ FALSE

A. I, II, III are false

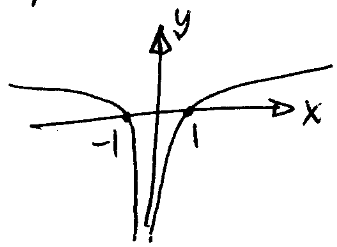
B. I is true, II and III are false

C. I and II are true, III is false

D. I and III are true, II is false

E. I, II and III are true

$y = \ln|x|$ has graph



13. $\lim_{x \to \infty} \cos\left(\frac{1}{x}\right) =$ $\cos(0) = 1$

A. -1

B. 0

C. 1

D. $-\infty$

E. does not exist

14. $\lim_{x \to 0} \frac{1 - \cos x}{x^2} =$ $\frac{1-1}{0} = \frac{0}{0}$

$\lim_{x \to 0} \frac{0 + \sin x}{2x} = \frac{0}{0}$

$\lim_{x \to 0} \frac{\cos x}{2} = \frac{1}{2}$

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

15. A colony of bacteria, undergoing exponential growth, starts with 200 bacteria. One hour later it contains 400 bacteria. How many hours does it take to reach 2000 bacteria?

$P(t) = 200\, e^{kt}$

$P(1) = 200\, e^{k} = 400 \rightarrow e^{k} = 2 \rightarrow k = \ln 2$

$P(t) = 200\, e^{(\ln 2)(t)}$

$2000 = 200\, e^{(\ln 2)(t)} \rightarrow 10 = e^{(\ln 2)(t)} \rightarrow \ln 10 = (\ln 2)(t)$

$\rightarrow t = \frac{\ln 10}{\ln 2}$

A. 5

B. $\ln 1600$

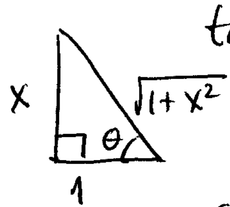
C. $\ln 200$

D. $\frac{\ln 10}{\ln 2}$

E. $\ln 10$

16. $\sin(\tan^{-1} x) =$

$\tan^{-1} x = \theta \rightarrow \tan\theta = x$



$\sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$

A. $\frac{1}{1+x^2}$

B. $\sqrt{1-x^2}$

C. $\frac{1-x^2}{1+x^2}$

D. $\frac{x}{\sqrt{1+x^2}}$

E. $1+x^2$

17. If $f(a) = b$ and $f'(a) = c$, use differentials to approximate $f(a + \frac{1}{2}) - f(a)$.

$$f(a+\tfrac{1}{2}) - f(a) \cong df$$
$$= f'(a) \cdot \tfrac{1}{2}$$
$$= c \cdot \tfrac{1}{2}$$

A. $c - b$
B. $\frac{c}{2}$
C. b
D. $\frac{b}{2c}$
E. bc

18. If $f''(x) = 20x^3 - 6x + 2$, $f'(1) = 2$ and $f(1) = 4$, then $f(-1) =$

$$f'(x) = 5x^4 - 3x^2 + 2x + C_1$$
$$f'(1) = 5 - 3 + 2 + C_1 = 2 \rightarrow C_1 = -2$$
$$\rightarrow f'(x) = 5x^4 - 3x^2 + 2x - 2$$
$$f(x) = x^5 - x^3 + x^2 - 2x + C_2$$
$$f(1) = 1 - 1 + 1 - 2 + C_2 = 4 \rightarrow C_2 = 5$$
$$\rightarrow f(x) = x^5 - x^3 + x^2 - 2x + 5 \rightarrow f(-1) = -1 + 1 + 1 + 2 + 5$$
$$= 8$$

A. -1
B. 0
C. 2
D. 4
E. 8

19. $\frac{d}{dx} \int_1^x \sinh(t^2)dt =$

$$\sinh(x^2)$$

Fundamental Theorem of Calculus

A. $\sinh(x)$
B. $\sinh(x^2)$
C. $2x\cosh(x^2)$
D. $2x\sinh(x^2)$
E. $\cosh(x)$

20. Let $F(x) = \int_0^{x^2} \sin(t^2)dt$. Consider the following statements

I. $F(0) = 0$ TRUE $\int_0^0 \sin(t^2)\,dt = 0$

II. $F(0) < F(1)$ TRUE since $\sin(t^2) \geq 0$ for $0 \leq t \leq 1$

III. F is increasing for all values of x

IV. $F(-1) = -F(1)$

A. I, II, III, IV are true

B. I, II, III are true, IV is false

C. I, II are true, III, IV are false

D. I is true, II, III, IV are false

E. I, II, III, IV are false

III FALSE since $F'(x) = 2x \sin x^4 < 0$ for $\pi < x^4 < 2\pi$ (and other x's)

IV FALSE since $F(-1) = \int_0^1 \sin t^2\,dt$ and $F(1) = \int_0^1 \sin t^2\,dt$

21. $\int \frac{x}{1+4x^2}dx =$

A. $\frac{x^2}{1+4x^2} + C$

B. $\frac{1}{2(1+4x^2)^2} + C$

C. $\frac{1}{1+4x^2} + C$

D. $\frac{1}{4}\ln(1+4x^2)+C$

E. $\frac{1}{8}\ln(1+4x^2)+C$

let $u = 1+4x^2$, then $du = 8x\,dx$
and $\frac{1}{8}du = x\,dx$

Substituting, $\int \frac{x\,dx}{1+4x^2} = \int \frac{\frac{1}{8}du}{u}$

$= \frac{1}{8}\ln|u| + C = \frac{1}{8}\ln|1+4x^2| + C$

$= \frac{1}{8}\ln(1+4x^2) + C$

22. $\int_1^2 (1+3x^2+x^3)dx =$

A. $11\frac{3}{4}$

B. $32\frac{3}{4}$

C. 42

D. 57

E. 83

$\left(x + x^3 + \frac{1}{4}x^4\right)\Big|_1^2$

$= \left(2 + 8 + \frac{1}{4}\cdot 16\right) - \left(1 + 1 + \frac{1}{4}\right)$

$= 14 - 2\frac{1}{4}$

$= 11\frac{3}{4}$

23. $\int_1^2 (1 + \frac{1}{x} + \frac{1}{x^2})\, dx =$

$= \left(x + \ln x - \frac{1}{x}\right)\Big|_1^2$

$= \left(2 + \ln 2 - \frac{1}{2}\right) - \left(1 + \ln 1 - 1\right)$

$= \left(\frac{3}{2} + \ln 2\right) - (0)$

A. $-\frac{1}{4}$

B. $\frac{3}{2}$

C. $\ln 2 + \frac{3}{2}$

D. $\ln 2 - \frac{1}{2}$

E. $\ln 2 + 1$

24. The length of a rectangle is decreasing at a rate of 1 foot per second, but the area remains constant. At what rate, in feet per second, is the rectangle's width increasing when its length is 10 feet and its width is 5 feet?

$A = lw$. Know: $\frac{dl}{dt} = -1$, $\frac{dA}{dt} = 0$

Want: $\frac{dw}{dt}$ when $l = 10$, $w = 5$.

$\to \frac{dA}{dt} = l\frac{dw}{dt} + \frac{dl}{dt}w$

$\to 0 = 10\frac{dw}{dt} + (-1)(5) \to \frac{dw}{dt} = \frac{5}{10} = \frac{1}{2}$

A. $\frac{1}{10}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

E. 1

25. Find the shortest distance from the point $(1, 4)$ to the parabola $y^2 = 2x$.

Want to minimize $D = \sqrt{(1-x)^2 + (4-y)^2}$

Since $y^2 = 2x$ then $x = \frac{1}{2}y^2 \Rightarrow D = \sqrt{(1 - \frac{1}{2}y^2)^2 + (4-y)^2}$

$\frac{dD}{dy} = \frac{1}{2\sqrt{\quad}}\left(2(1-\frac{1}{2}y^2)(-y) + 2(4-y)(-1)\right)$

$= \frac{1}{2\sqrt{\quad}}\left(-2y + y^3 - 8 + 2y\right) = 0 \to y^3 - 8 = 0 \to y = 2$

$D(2) = \sqrt{(1-2)^2 + (4-2)^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$

A. $\sqrt{6}$

B. $\sqrt{5}$

C. 2

D. $\sqrt{3}$

E. $\sqrt{2}$