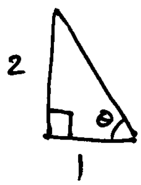
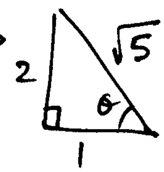
1. $\sin(\tan^{-1}(2)) =$



Let $\tan^{-1} 2 = \theta$

$\Rightarrow$



$\Rightarrow \sin(\tan^{-1} 2) = \sin\theta = \dfrac{2}{\sqrt{5}}$

A. $\dfrac{\sqrt{3}}{2}$

B. $\dfrac{1}{2}$

C. $\dfrac{\sqrt{3}}{\sqrt{5}}$

D. $\dfrac{2}{3}$

E. $\dfrac{2}{\sqrt{5}}$

2. $\lim\limits_{x\to\infty} (\sqrt{x^2+x} - x) =$

$$= \lim_{x\to\infty} \left(\left(\sqrt{x^2+x} - x\right) \left(\frac{\sqrt{x^2+x} + x}{\sqrt{x^2+x} + x} \right) \right)$$

$$= \lim_{x\to\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \left(\frac{x}{\sqrt{x^2+x}+x} \cdot \frac{1/x}{1/x} \right) = \lim_{x\to\infty} \frac{1}{\sqrt{1+1/x}+1} = \frac{1}{2}$$

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

3. The graph of the function $f(x) = \dfrac{e^x + e^{-x}}{e^x - e^{-2x}}$ has

$$\lim_{x\to\infty} \left(\frac{e^x + e^{-x}}{e^x - e^{-2x}} \cdot \frac{e^{-x}}{e^{-x}} \right)$$

$$= \lim_{x\to\infty} \left(\frac{1 + e^{-2x}}{1 - e^{-3x}} \right) = \frac{1}{1} = 1$$

$\Rightarrow y = 1$ is a horizontal asymptote

$$\lim_{x\to-\infty} \left(\frac{e^x + e^{-x}}{e^x - e^{-2x}} \cdot \frac{e^x}{e^x} \right)$$

$$= \lim_{x\to-\infty} \left(\frac{e^{2x} + 1}{e^{2x} - e^{-x}} \right) = \frac{0+1}{0-\infty} = 0 \Rightarrow y = 0 \text{ is a horizontal asympt.}$$

$$\lim_{x\to 0} \frac{e^x + e^{-x}}{e^x - e^{-2x}} = \frac{2}{0} \Rightarrow x = 0 \text{ is a vertical asymptote.}$$

A. horizontal asymptotes $y = 0$ and $y = 1$ and no vertical asymptote

B. horizontal asymptotes $y = 1$ and $y = -1$ and vertical asymptote $x = 0$

C. horizontal asymptotes $y = 0$ and $y = 1$ and vertical asymptote $x = 0$

D. horizontal asymptotes $y = 1$ and $y = -1$ and no vertical asymptote

E. horizontal asymptote only $y = 1$ and vertical asymptote $x = 0$

4. $\lim\limits_{x\to 0} \dfrac{e^x - 1 - x}{x^2} =$ $\dfrac{1-1-0}{0} = \dfrac{0}{0}$

$\overset{\frac{0}{0}}{=} \lim\limits_{x\to 0} \dfrac{e^x - 1}{2x} = \dfrac{1-1}{0} = \dfrac{0}{0}$

$\overset{\frac{0}{0}}{=} \lim\limits_{x\to 0} \dfrac{e^x}{2} = \dfrac{1}{2}$

A. 0
B. $\frac{1}{2}$
C. 1
D. 2
E. ∞

5. If $f(x) = \dfrac{3x^2 - x}{x+2}$, then $f'(1) =$

$f'(x) = \dfrac{(6x-1)(x+2) - (3x^2 - x)(1)}{(x+2)^2}$

$f'(1) = \dfrac{(5)(3) - (2)(1)}{(3)^2}$

$= \dfrac{13}{9}$

A. $\frac{13}{9}$
B. 2
C. $\frac{13}{3}$
D. 5
E. $\frac{25}{9}$

6. If $f(x) = (x^2 + \ln x)(x^3 + 1)$, then $f'(1) =$

$f'(x) = \left(2x + \frac{1}{x}\right)(x^3 + 1) + (x^2 + \ln x)(3x^2)$

$f'(1) = (3)(2) + (1)(3)$

$= 9$

A. -3
B. 0
C. 3
D. 6
E. 9

7. If $f(x) = \sqrt{x^2 + \sqrt{x}}$, then $f'(1) =$

$$f'(x) = \frac{1}{2\sqrt{x^2+\sqrt{x}}}\left(2x + \frac{1}{2\sqrt{x}}\right)$$

$$f'(1) = \frac{1}{2\sqrt{2}}\left(2 + \frac{1}{2}\right)$$

$$= \frac{5}{2\sqrt{2}} = \frac{5\sqrt{2}}{8}$$

A. $\frac{5\sqrt{2}}{8}$

B. $\frac{\sqrt{2}}{2}$

C. $\sqrt{2}$

D. $2\sqrt{2}$

E. $\frac{8\sqrt{2}}{5}$

8. If $f(x) = g(e^{2x})$, then $f''(x) =$

$$f'(x) = g'(e^{2x}) \cdot 2e^{2x}$$

$$f''(x) = g''(e^{2x}) \cdot 2e^{2x} \cdot 2e^{2x} + g'(e^{2x}) \cdot 4e^{2x}$$

A. $g''(4e^{2x})$

B. $g'(e^{2x})4e^{4x}$

C. $4g''(e^{2x})e^{4x} + 4g'(e^{2x})e^{2x}$

D. $g''(e^{2x})+4g'(e^{2x})e^{2x}+2g(e^{2x})e^{2x}$

E. $g''(e^{2x})e^{2x} + g'(e^{2x})e^{2x}$

9. The function $y = f(x)$ satisfies $xy^3 + xy = 6$ and $f(3) = 1$. Find $f'(3)$.

$$\Rightarrow (1)(y^3) + (x)\left(3y^2\frac{dy}{dx}\right) + (1)(y) + (x)\left(\frac{dy}{dx}\right) = 0$$

$$x=3,\ y=1 \Rightarrow 1 + 9\frac{dy}{dx} + 1 + 3\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{12} = -\frac{1}{6}$$

A. $-\frac{1}{6}$

B. 0

C. $\frac{1}{6}$

D. $\frac{11}{3}$

E. 2

10. If $F(x) = \sin(g(x))$, $g(0) = 0$, $g'(0) = \pi$ then $F'(0) =$

$$F'(x) = \cos(g(x)) \cdot g'(x)$$
$$F'(0) = \cos(g(0)) \cdot g'(0)$$
$$= \cos(0) \cdot \pi$$
$$= 1 \cdot \pi$$

A. 0

B. π

C. -1

D. $-\pi$

E. 1

11. The largest interval on which the function $f(x) = x^x$, $x > 0$ is increasing is

Let $y = x^x$. Then $\ln y = x \ln x$, and

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln x + 1 \rightarrow \frac{dy}{dx} = x^x(\ln x + 1)$$

$$\frac{dy}{dx} = 0 \rightarrow \ln x + 1 = 0 \rightarrow x = e^{-1}$$

0 e^{-1} ∞ x

$\ln x + 1$ | $-$ ϕ $+$ |

$\Rightarrow f$ increasing on $(\frac{1}{e}, \infty)$

A. $(0, \infty)$

B. $(0, \frac{1}{e})$

C. $(1, \infty)$

D. $(\frac{1}{e}, 1)$

E. $(\frac{1}{e}, \infty)$

12. If $f(x) = 2\tan x - \tan^2 x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then the length of the largest interval on which $f(x)$ is increasing is

$$f'(x) = 2\sec^2 x - 2\tan x \sec^2 x$$
$$= 2\sec^2 x\,(1 - \tan x)$$
$$= 0 \rightarrow x = \pi/4$$

$-\pi/2$ $\pi/4$ $\pi/2$ x

$1 - \tan x$ | $+$ ϕ $-$ |

$\Rightarrow f$ increasing on $(-\frac{\pi}{2}, \frac{\pi}{4})$ which has length $\frac{3\pi}{4}$

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{4}$

E. π

13. $f(x) = x^3 + ax^2 + bx$ has a critical point when $x = 1$ and an inflection point when $x = 2$. Then

$f'(x) = 3x^2 + 2ax + b$

$f''(x) = 6x + 2a$, $f''(2) = 12 + 2a = 0 \rightarrow a = -6$

$a = -6$, $f'(1) = 0 \rightarrow 3(1)^2 + 2(-6)(1) + b = 0$

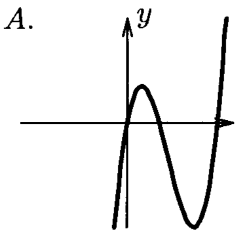
$\rightarrow 3 - 12 + b = 0$

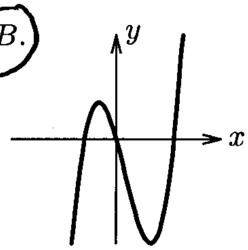
$\rightarrow b = 9$

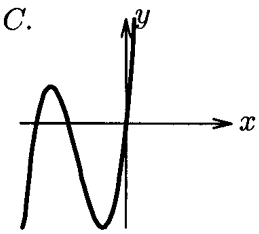
A. $a = -6, \ b = 9$
B. $a = 1, \ b = 9$
C. $a = 9, \ b = -6$
D. $a = 9, \ b = 1$
E. $a = -6, \ b = 1$

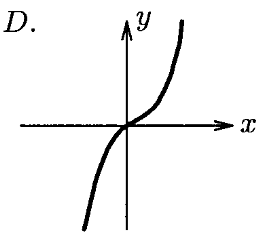
14. Which could be the graph of $y = 2x^3 - 3x^2 - 12x$?

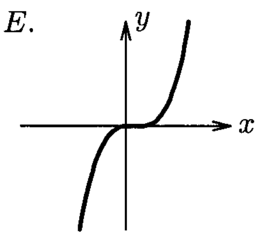
You can investigate either x-intercepts or slope. See below.

A.


B.


C.


D.


E.


x-intercepts, $y = 0 \rightarrow x = \dfrac{3 \pm \sqrt{9 + 96}}{4} \approx \dfrac{13}{4}, \dfrac{-7}{4} \Rightarrow$ B

slope, $y' = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$

$y' = 0 \rightarrow x = -1, 2$

	$-\infty$ to -1	-1 to 2	2 to ∞
$x+1$	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$
y'	$+$	$-$	$+$

rel. max at $x = -1$

rel. min at $x = 2$

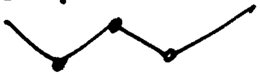
$\Rightarrow$ B

15. The function $f(x) = x^4 - \frac{1}{2}\,x^2 + \frac{1}{32}$ has critical numbers $x = -\frac{1}{2},\ 0,\ \frac{1}{2}$. How many real solutions are there to the equation $x^4 - \frac{1}{2}\,x^2 - 1 = 0$? Let $g(x) = x^4 - \frac{1}{2}x^2 - 1$

$g'(x) = 4x^3 - x = 4x(x^2 - \frac{1}{4})$

	$-\infty$ to $-1/2$	$-1/2$ to 0	0 to $1/2$	$1/2$ to ∞
$4x$	−	−	+	+
$x^2 - \frac{1}{4}$	+	−	−	+
$g'(x)$	−	+	−	+

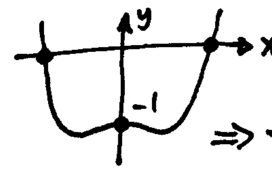
graph of g



$g(\pm\frac{1}{2}) = \frac{1}{16} - \frac{1}{8} - 1 = -\frac{17}{16}$

$g(0) = -1$

graph of g



$\Rightarrow$ there are 2 real sol.

A. 0

B. 1

C. 2

D. 3

E. 4

16. A colony of bacteria started with 1000 bacteria and its population triples every 5 hours. After t hours how many bacteria will there be?

$A(t) = 1000\, e^{kt}$

$A(5) = 3000 = 1000\, e^{5k}$

$\rightarrow 3 = e^{5k} \rightarrow k = \frac{1}{5} \ln 3$

$\rightarrow A(t) = 1000\, e^{(\frac{1}{5}\ln 3)t} = 1000\, e^{(\ln 3)\frac{t}{5}} = 1000\,(3)^{t/5}$

A. $2^{5000/t}$

B. $1000\ 2^{3t/5}$

C. $1000\ e^{\frac{\ln 2}{5}t}$

D. $1000\ e^{\frac{5}{\ln 3}t}$

E. $1000\ 3^{t/5}$

17. The derivative of a function $f(x)$ is given by

$$f'(x) = x^2(x-1)(x-2)^2(x-3)$$

Consider the following statements.

F I. f has a local minimum when $x = 0$

T II. f has a local maximum when $x = 1$

F III. f has a local minimum when $x = 2$

F IV. f has a local maximum when $x = 3$

	$-\infty$ to 0	0 to 1	1 to 2	2 to 3	3 to ∞
x^2	+	+	+	+	+
$x-1$	−	−	+	+	+
$(x-2)^2$	+	+	+	+	+
$x-3$	−	−	−	−	+
$f'(x)$	+	+	−	−	+

graph of f

A. all of I, II, III, IV are true

B. I and III are true, II and IV are false

C. II is true, I, III and IV are false

D. II, III and IV are true, I is false

E. all of I, II, III, IV are false

18. The volume of a cone is $\frac{1}{3}\pi r^2 h$. The radius is increasing at 6 cm/sec. and the height is decreasing at 4 cm/sec. At what rate is the volume changing when the radius is 3 cm. and the height is 1 cm.?

Know: $\frac{dr}{dt} = +6$ and $\frac{dh}{dt} = -4$.

Want: $\frac{dV}{dt}$ when $r = 3$ and $h = 1$.

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right) = \frac{1}{3}\pi(36-36) = 0$$

A. -6π cm^3/sec

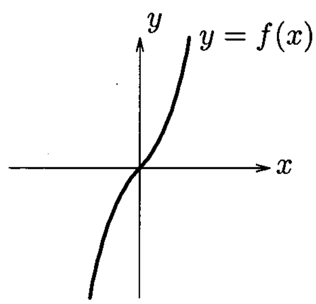
B. 0 cm^3/sec

C. 6π cm^3/sec

D. 9π cm^3/sec

E. 18π cm^3/sec

19. The graph of $f(x) = x\, e^{|x/2|}$ is shown



Let $F(x) = \int_0^x f(t)dt$. Which of the following statements are true?

T I. $F(0) = 0$

T II. $F(x)$ is never negative

F III. $F(x)$ is increasing on $(-\infty, \infty)$

F IV. $F(-x) = -F(x)$

T V. $F(-x) = F(x)$

I. $F(0) = \int_0^0 f(t)\,dt = 0$

II. $x > 0 \rightarrow F(x) > 0$ because $f(t) > 0$ for $t > 0$.

$x < 0 \rightarrow F(x) = \int_0^x f(t)\,dt$

$= \int_x^0 -f(t)\,dt > 0$ because $-f(t) > 0$ for $t < 0$

III. $F'(x) = x e^{|x/2|}$ and $x e^{|x/2|} < 0$ for $x < 0 \rightarrow$ false

A. All

B. only I and II

C. only III and IV

D. only I, II and V

E. only I and IV

IV. & V. f is an odd function, so $\int_0^x f(t)\,dt = \int_0^{-x} f(t)\,dt$ and therefore $F(x) = F(-x)$.

20. $\frac{d}{dx}\int_1^x \sec(t^2)\,dt =$ $\sec(x^2)$

A. $\tan(x)$
B. $2x\sec(x^2)$
C. $2x\sec(x^2)\tan(x^2)$
D. $\sec(x^2)$
E. $\tan(x^2)$

21. $\frac{d}{dx}\int_{x^2}^{x^3} e^{t^2}\,dt =$ $\frac{d}{dx}\left(\int_{x^2}^{a} e^{t^2}dt + \int_a^{x^3} e^{t^2}dt\right)$

$= \frac{d}{dx}\left(-\int_a^{x^2} e^{t^2}dt + \int_a^{x^3} e^{t^2}dt\right)$

$= -e^{x^4}\cdot 2x + e^{x^6}\cdot 3x^2$

A. $3x^2e^{x^6} - 2xe^{x^4}$
B. $e^{x^6} - e^{x^4}$
C. $3x^2e^{x^2} - 2xe^{x^2}$
D. $e^{x^3} - e^{x^2}$
E. $6x^5e^{x^6} - 4x^3e^{x^4}$

22. $\int_0^{\ln(\pi/2)} e^x\cos(e^x)\,dx =$

Let $u = e^x$. Then $du = e^x dx$, and
$u(0) = e^0 = 1$ and $u(\ln\frac{\pi}{2}) = \frac{\pi}{2}$.

$\int_1^{\pi/2} \cos u\,du = \sin u \Big|_1^{\pi/2}$
$= \sin\frac{\pi}{2} - \sin 1 = 1 - \sin 1$

A. -1
B. $\cos(e) - \cos(1)$
C. $e - \sin(1)$
D. $\pi/2 - 1$
E. $1 - \sin(1)$

23. $\displaystyle\int_0^1 \frac{x}{1+x^4}\,dx =$

Let $u = x^2$, then $du = 2x\,dx$ and $u(0)=0$, $u(1)=1$

$$\int_0^1 \frac{x}{1+(x^2)^2}\,dx = \int_0^1 \frac{\frac{1}{2}du}{1+u^2} = \frac{1}{2}\tan^{-1}u\Big|_0^1$$
$$= \frac{1}{2}(\tan^{-1}1 - \tan^{-1}0) = \frac{1}{2}\left(\frac{\pi}{4}-0\right)$$

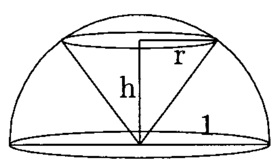
A. e^{-2}
B. $\ln 2$
C. $1/2$
D. 2
E. $\pi/8$

24. $\displaystyle\int 7^x\,dx =$

$$= \int (e^{\ln 7})^x\,dx = \int e^{(\ln 7)x}\,dx$$
(Let $u = (\ln 7)x$, then $du = \ln 7 \cdot dx$)
$$= \int e^u \frac{1}{\ln 7}\,du = \frac{1}{\ln 7}e^u + C = \frac{1}{\ln 7}7^x + C$$

A. $(\ln 7)\,7^x + C$
B. $\dfrac{1}{\ln 7}\,7^x + C$
C. $7^x + C$
D. $\dfrac{1}{7}\,7^x + C$
E. $\dfrac{1}{x+1}\,7^{x+1} + C$

25. A right circular cone is inscribed in a hemisphere of radius 1 as shown



What value of h will give a cone of maximum volume (the volume of a cone of radius r and height h is $\frac{1}{3}\,\pi r^2 h$.)

$h^2 + r^2 = 1$ and $V = \frac{1}{3}\pi r^2 h$, so
$V(h) = \frac{1}{3}\pi(1-h^2)h = \frac{1}{3}\pi(h-h^3)$, $0<h<1$.
We want max V.
$V' = \frac{1}{3}\pi(1-3h^2) = 0 \Rightarrow h = \frac{1}{\sqrt{3}}$

h	0		$1/\sqrt{3}$		1
$1-3h^2$		+	0	−	
V		↗		↘	

$\Rightarrow$ max V when $h = \frac{1}{\sqrt{3}}$

A. $\dfrac{2}{3}$
B. $\dfrac{1}{2}$
C. $\dfrac{1}{\sqrt{2}}$
D. $\dfrac{1}{\sqrt{3}}$
E. $\dfrac{1}{\sqrt{5}}$