Name: Solution Key **I.D. #:**

Rec. Instr.: **Rec. Time:** **Lecturer:**

Instructions:

1. On the mark sense sheet
   a. Fill in instructor's name and course number.
   b. Fill in your name, student identification number and division and section number, and fill in the appropriate spaces with a pencil.
   c. Fill in the appropriate letter for you response to each question on your mark–sense answer sheet.
   d. Hand in both the answer and question booklet to your recitation instructor when you are done.
2. Verify that you have all the pages (there are 11 pages including this page).
3. Calculators, books and notes are not allowed.
4. In the text booklet circle the letter of your response to each question.
5. Correct answers without justification may be counted as incorrect.

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1. Given that $f(\pi) = \sqrt{2}$, then $\lim_{x\to\pi} (f(x) - f(\pi))$

A. $= 0$.

B. $= \pi - \sqrt{2}$.

C. $= \sqrt{2} - \pi$.

D. cannot be determined.

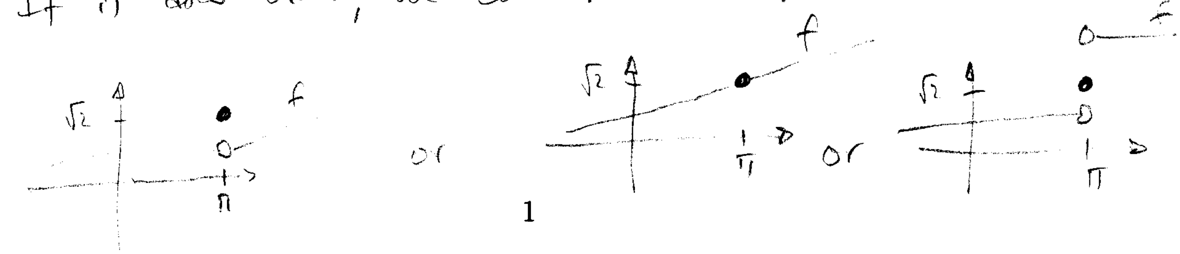
E. does not exist.

$= \lim_{x\to\pi} (f(x) - \sqrt{2})$

$\lim_{x\to\pi} f(x)$ may not exist.

If it does exist, we don't know its value.

ex:



2. An equation of the line that is perpendicular to $2x - 4y + 2 = 0$ and that contains the point $(3, 2)$ is

$-4y = -2x - 2$
$y = \frac{1}{2}x + \frac{1}{2} \rightarrow m_{\perp} = -2$

tangent line is $y - 2 = -2(x-3)$
$y - 2 = -2x + 6$
$y = -2x + 8$
$y + 2x - 8 = 0$

A. $-4y + x + 5 = 0$
B. $2y - x - 1 = 0$
C. $y + 2x - 8 = 0$
D. $2y + x - 7 = 0$
E. $y - 2x + 4 = 0$

3. If

$$f(x) = \begin{cases} \frac{-3}{2}x + 2, & \text{for } x < -1, \\ 2x - \frac{1}{2}, & \text{for } x \geq -1, \end{cases}$$

then $\lim_{x \to -1^+} f(x)$ is

$= \lim_{x \to -1^+} 2x - \frac{1}{2} = -2 - \frac{1}{2} = \frac{-5}{2}$

A. $\frac{-5}{2}$
B. $\frac{7}{2}$
C. $\frac{5}{2}$
D. $\frac{-7}{2}$
E. none of these.

4. The domain for $f \circ g$, where $f(x) = \dfrac{x}{x-1}$ and $g(x) = \dfrac{1}{x+2}$.

A. $x \neq -1$

B. $x \neq 1$ and $x \neq -1$

C. $x \neq 1$ and $x \neq -2$

D. $x \neq -1$ and $x \neq -2$

E. $x \neq 0$ and $x \neq -2$

$$f(g(x)) = f\left(\frac{1}{x+2}\right) = \frac{\frac{1}{x+2}}{\frac{1}{x+2} - 1}$$

$$= \frac{\frac{1}{x+2}}{\frac{1-x-2}{x+2}} = \frac{1}{-x-1}$$

$$= \frac{1}{-(x+1)} \longrightarrow x \neq -1$$

domain of g is $x \neq -2$

5. Given $y \sin 2x = -2y^2$, then $\dfrac{dy}{dx}$ for $(x, y) = \left(\dfrac{\pi}{12}, \dfrac{-1}{4}\right)$ is

A. $\dfrac{\sqrt{3}}{6}$

B. $\dfrac{-\sqrt{3}}{2}$

C. $\dfrac{-\sqrt{3}}{6}$

D. $\dfrac{\sqrt{3}}{2}$

E. none of these.

$$y' \sin 2x + y \cos 2x (2) = -4yy'$$

$$y' \sin \frac{\pi}{6} + \left(-\frac{1}{4}\right)\left(\cos \frac{\pi}{6}\right)(2) = y'$$

$$-\frac{1}{2} \cdot \frac{\sqrt{3}}{2} = y'\left(1 - \frac{1}{2}\right)$$

$$2\left(-\frac{\sqrt{3}}{4}\right) = y'$$

$$-\frac{\sqrt{3}}{2} = y'$$

6. The $\lim_{x \to \frac{\pi}{4}} \dfrac{\dfrac{\sin x}{x} - \dfrac{2\sqrt{2}}{\pi}}{x - \dfrac{\pi}{4}}$ is the derivative of a function at $x = \dfrac{\pi}{4}$. Its value is

A. 0

B. 1

C. $\dfrac{8\sqrt{2}}{\pi^2}\left(\dfrac{\pi}{4} + 2\right)$

D. $\dfrac{8\sqrt{2}}{\pi^2}\left(\dfrac{\pi}{4} - 1\right)$

E. $\dfrac{16\sqrt{2}}{\pi^2}\left(\dfrac{\pi}{4} - 1\right)$

$$\frac{\sin\frac{\pi}{4}}{\frac{\pi}{4}} = \frac{\sqrt{2}}{2} \cdot \frac{4}{\pi} = \frac{2\sqrt{2}}{\pi}$$

$$\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{(\cos x)(x) - (\sin x)(1)}{x^2}$$

$$\frac{d}{dx}\left(\frac{\sin x}{x}\right)\Bigg|_{\frac{\pi}{4}=x} = \frac{\frac{\sqrt{2}}{2}\frac{\pi}{4} - \frac{\sqrt{2}}{2}}{\frac{\pi^2}{16}} = \frac{16}{\pi^2}\frac{\sqrt{2}}{2}\left(\frac{\pi}{4} - 1\right)$$

$$= \frac{8\sqrt{2}}{\pi^2}\left(\frac{\pi}{4} - 1\right)$$

7. Suppose you have a cache of radium, whose half–life is approximately $2294 \ln 2$ years. How many years would you have to wait for one-twentieth of it to disappear?

A. $2294 \ln\left(\dfrac{20}{19}\right)$

B. $2294 \ln 20$

C. $\dfrac{2294}{\ln 2} \ln\left(\dfrac{20}{19}\right)$

D. $\dfrac{2294}{\ln 2} \ln 20$

E. none of these.

$$P(t) = P(0)e^{kt} \qquad k\,2294 \ln 2$$

$$P(2294 \ln 2) = \frac{1}{2}\cancel{P(0)} = \cancel{P(0)}e^{k\,2294 \ln 2}$$

$$\longrightarrow \ln\frac{1}{2} = k\,2294 \ln 2$$

$$\longrightarrow k = \frac{\ln\frac{1}{2}}{2294 \ln 2} = \frac{-1}{2294}$$

$$P(t) = \frac{19}{20}\cancel{P(0)} = \cancel{P(0)}e^{kt}$$

$$\longrightarrow \ln\frac{19}{20} = t\,\frac{-1}{2294}$$

$$\longrightarrow t = -2294 \ln\frac{19}{20} = 2294 \ln\frac{20}{19}$$

8. Let $F(x) = f(g^3(x) + g^2(x))$. If $g(1) = 2$, $g'(1) = 3$, $f(12) = 1$ and $f'(12) = 2$, then $F'(1) =$

$$F'(x) = f'(g^3+g^2)\,(3g^2g' + 2gg')$$

$$F'(1) = f'(8+4)\,(3\cdot 4\cdot 3 + 2\cdot 2\cdot 3)$$

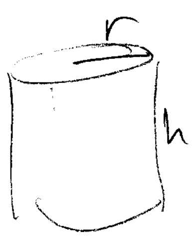
$$= f'(12)\,(36+12)$$

$$= (2)(48)$$

$$= 96$$

A. 12
B. 24
C. 48
D. 96
E. none of these.

9. A circular cylindrical can with top and bottom has volume V. Find the radius of the can with the smallest possible surface area.



$$\pi r^2 h = V$$

$$\to h = \frac{V}{\pi r^2}$$

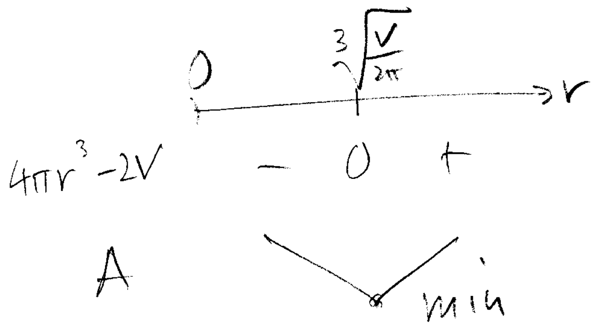
A. $3\sqrt[3]{\pi V^2}$
B. $3\sqrt[3]{2\pi V^2}$
C. $\sqrt[3]{\frac{V}{\pi}}$
D. $\sqrt[3]{\frac{V}{2\pi}}$
E. none of these.

$$\text{Area} = A = 2\pi r^2 + 2\pi r h$$

$$A(r) = 2\pi r^2 + 2\pi r \frac{V}{\pi r^2}$$

$$= 2\pi r^2 + 2V\cdot\frac{1}{r}$$

$$A' = 4\pi r - \frac{2V}{r^2} = \frac{4\pi r^3 - 2V}{r^2} = 0 \to r = \sqrt[3]{\frac{V}{2\pi}}$$



10. Using a linear approximation to $y = x^{2/3}$ at $x = 27$, $(25.5)^{2/3} \approx$

A. $8\frac{1}{3}$

B. $8\frac{2}{3}$

C. $8\frac{7}{9}$

D. $9\frac{1}{3}$

E. $9\frac{2}{3}$

$y = x^{2/3}$ $\quad y(27) = 9$

$y' = \frac{2}{3} x^{-1/3}$ $\quad y'(27) = \frac{2}{3}\left(\frac{1}{3}\right) = \frac{2}{9}$

tangent line: $y = y(27) + y'(27)(x-27)$

$= 9 + \frac{2}{9}(x-27)$

$y(25.5) = 9 + \frac{2}{9}(25.5-27)$

$= 9 + \frac{2}{9}\left(-\frac{3}{2}\right)$

$= 9 - \frac{1}{3}$

$= 8\,\frac{2}{3}$

11. $\lim_{x\to\infty}\left(\sqrt{\frac{x^2}{4}-1}-\frac{3}{2}x\right)$ $= \infty - \infty$

$\approx \frac{x}{2} - \frac{3}{2}x = -x$

A. 0

B. -1

C. -2

D. $-\infty$

E. ∞

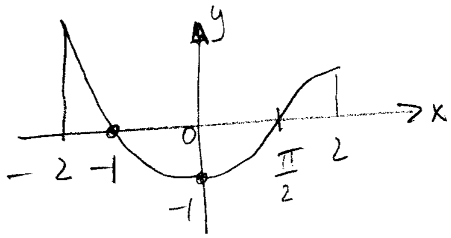
$\left(\sqrt{\quad} - \frac{3}{2}x\right)\dfrac{\sqrt{\quad} + \frac{3}{2}x}{\sqrt{\quad} + \frac{3}{2}x}$

$= \dfrac{\frac{x^2}{4} - 1 - \frac{9}{4}x^2}{\sqrt{\frac{x^2}{4}-1} + \frac{3}{2}x} = \dfrac{-\frac{5}{4}x^2 - 1}{\sqrt{\frac{x^2}{4}-1} + \frac{3}{2}x} \cdot \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \dfrac{-\frac{5}{4} - \frac{1}{x^2}}{\sqrt{\frac{1}{4x^2} - \frac{1}{x^4}} + \frac{3}{2x}} \longrightarrow \dfrac{-\frac{5}{4}}{0^+} = -\infty$

as $x \to \infty$

12. Let $f(x) = \begin{cases} x^2-1, & \text{for } x < 0, \\ -\cos x, & \text{for } x \ge \cancel{x}\,0. \end{cases}$ The area of the region bounded by the graph of f, the x–axis, and the lines $x=-2$ and $x=2$ is



A. $4+\sin 2$
B. $4-\sin 2$
C. $3-\sin 2$
D ~~C.~~ $2-\sin 2$
E ~~D.~~ $2+\sin 2$

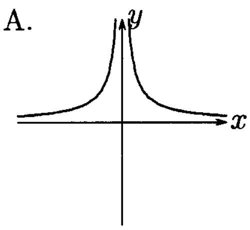
$$\int_{-2}^{-1}(x^2-1)\,dx + \int_{-1}^{0}(-x^2+1)\,dx + \int_0^{\pi/2}\cos x\,dx + \int_{\pi/2}^{2}-\cos x\,dx$$

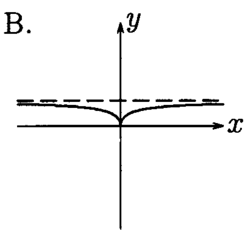
$$= \frac{x^3}{3}-x\Big|_{-2}^{-1} + \left(\frac{-x^3}{3}+x\right)\Big|_{-1}^{0} + \sin x\Big|_0^{\pi/2} + (-\sin x)\Big|_{\pi/2}^{2}$$

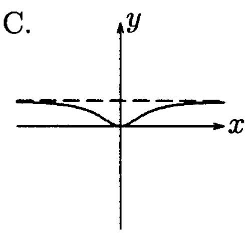
$$= \left[\left(-\frac13+1\right)-\left(-\frac83+2\right)\right] + \left[(0)-\left(\frac13-1\right)\right] + [1-0] + [-\sin 2-(-1)]$$

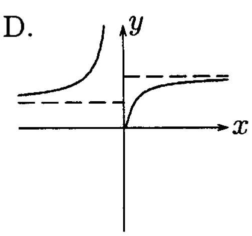
$$\frac23 + \frac23 \qquad + \frac23 \qquad + 1 \qquad -\sin 2 + 1 = 4-\sin 2$$

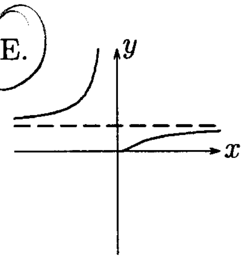
13. The graph of $f(x) = Ke^{-c/\cancel{K}\,x}$, where $K>0$ and $c>0$ resembles most which of the following graphs?

A.


B.


C.


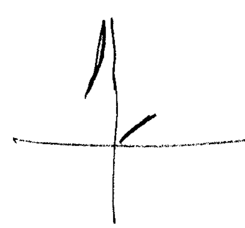
D.


E.


$y = Ke^{-\frac{c}{x}}$ $\qquad y = e^{-\frac{1}{x}}$, $K=1$, $c=1$

$x\to 0^+$, $-\frac1x \to -\infty$, $e^{-1/x}\to 0$

$x\to 0^-$, $-\frac1x \to \infty$, $e^{-1/x}\to \infty$



Also $\lim_{x\to\infty} e^{-1/x} = 1 \longrightarrow$ horizontal asymptote is $y=1$

14. The function $F(x) = \int_1^{x^2} (t - \sqrt{t})\, dt,\ x > 0$ is decreasing for

$F'(x) = (x^2 - x)(2x)$
$= 2x^2(x-1)$

0 1 ∞ x

$x-1$ — 0 +

F dec inc

A. $0 < x < 1/2$

B. $x > 1/2$

C. $0 < x < 1$

D. $x > 1$

E. none of these.

15. If $\int_a^b \sin^{-1} t\, dt - \int_{-1/2}^{1/2} \sin^{-1} t\, dt = \int_{1/2}^{1} \sin^{-1} t\, dt$

$\int_a^b = \int_{1/2}^{1} + \int_{-1/2}^{1/2} = \int_{-1/2}^{1}$

$b = 1$

A. $b = \frac{\pi}{2}$

B. $b = 1$

C. $b = \frac{1}{2}$

D. $b = \frac{-1}{2}$

E. b cannot be determined

16. Let $f(x) = \dfrac{\tan^{-1}(2x)}{\cosh(3x)}$, then $f'\left(\dfrac{1}{2}\right) =$

A. $\dfrac{\cosh\left(\frac{3}{2}\right) - \frac{3\pi}{4}\sinh\left(\frac{3}{2}\right)}{\cosh^2\left(\frac{3}{2}\right)}$

B. $\dfrac{\cosh\left(\frac{3}{2}\right) + \frac{3\pi}{4}\sinh\left(\frac{3}{2}\right)}{\cosh^2\left(\frac{3}{2}\right)}$

C. $\dfrac{\frac{1}{2}\cosh\left(\frac{3}{2}\right) + \frac{\pi}{4}\sinh\left(\frac{3}{2}\right)}{\cosh^2\left(\frac{3}{2}\right)}$

D. $\dfrac{\frac{1}{2}\cosh\left(\frac{3}{2}\right) - \frac{\pi}{4}\sinh\left(\frac{3}{2}\right)}{\cosh^2\left(\frac{3}{2}\right)}$

E. $\dfrac{\frac{1}{4}\cosh\left(\frac{3}{2}\right) - \frac{\pi}{4}\sinh\left(\frac{3}{2}\right)}{\cosh^2\left(\frac{3}{2}\right)}$

$$f'(x) = \frac{\frac{2}{1+4x^2}\cosh 3x - (\sinh 3x)(3)(\tan^{-1} 2x)}{\cosh^2(3x)}$$

$$f'\left(\tfrac{1}{2}\right) = \frac{\frac{2}{1+1}\cosh\frac{3}{2} - \left(\sinh\frac{3}{2}\right)(3)\left(\frac{\pi}{4}\right)}{\cosh^2\left(\frac{3}{2}\right)}$$

$$= \frac{\cosh\left(\frac{3}{2}\right) - \frac{3\pi}{4}\sinh\left(\frac{3}{2}\right)}{\cosh^2\left(\frac{3}{2}\right)}$$

17. If $f(x) = x^{2/x}$, then $f'(x) =$

$y = x^{2/x} \longrightarrow \ln y = \ln x^{2/x}$

$\longrightarrow \ln y = \frac{2}{x} \ln x$

$\longrightarrow \frac{1}{y}\frac{dy}{dx} = -\frac{2}{x^2}\ln x + \frac{2}{x}\frac{1}{x}$

$\longrightarrow \frac{dy}{dx} = x^{2/x}\left(\frac{-2\ln x}{x^2} + \frac{2}{x^2}\right)$

$= x^{2/x}\left(\frac{1}{x^2}\right)(2 - 2\ln x)$

$= x^{2/x - 2}(2)(1 - \ln x)$

A. $2(1 - \ln x)x^{\frac{2}{x}-2}$

B. $(1 - \ln x)x^{2/x}$

C. $2(1 - \ln x)x^{2/x}$

D. $-2x^{\frac{2}{x}-2}$

E. $x^{\frac{2}{x}-2}$

18. $\frac{1}{3}\int \frac{3 \cdot 1}{4 + 9x^2}\, dx =$ $\frac{1}{3} \cdot \frac{1}{2} \tan^{-1}\frac{3x}{2} + C$

$= \frac{1}{6}\tan^{-1}\frac{3x}{2} + C$

A. $\frac{2}{3}\tan^{-1}(3x) + C$

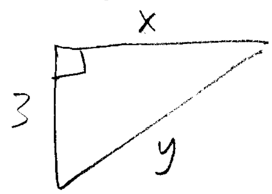
B. $\frac{1}{9}\ln(3x) + C$

C. $\frac{3}{2}\ln\left(\frac{3x}{2}\right) + C$

D. $\frac{2}{3}\tan^{-1}\left(\frac{3x}{2}\right) + C$

E. $\frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right) + C$

19. A helicopter flies parallel to the ground at an altitude of 3 kilometers and at a speed of 2 kilometers per minute. If the helicopter flies along a straight line that passes directly over Hovde Hall, at what rate is the distance between the helicopter and Hovde Hall changing 2 minutes after the helicopter flies over Hovde Hall?



know: $\frac{dx}{dt} = 2 \frac{km}{min}$

want $\frac{dy}{dt}$ when $x = 4$ $(y = 5)$

$x^2 + 9 = y^2 \rightarrow \cancel{2}x\frac{dx}{dt} = \cancel{2}y\frac{dy}{dt}$

$\rightarrow (4)(2) = (5)\left(\frac{dy}{dt}\right)$

$\rightarrow \frac{dy}{dt} = \frac{8}{5}$

A. $\dfrac{8}{\sqrt{20}}\ \dfrac{\text{km}}{\text{min}}$

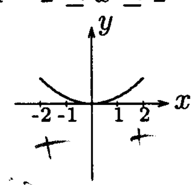
B. $\dfrac{4}{\sqrt{20}}\ \dfrac{\text{km}}{\text{min}}$

C. $\dfrac{2}{5}\ \dfrac{\text{km}}{\text{min}}$

D. $\dfrac{4}{5}\ \dfrac{\text{km}}{\text{min}}$

E. $\dfrac{8}{5}\ \dfrac{\text{km}}{\text{min}}$

20. The following is a graph of f' for $-2 \le x \le 2$

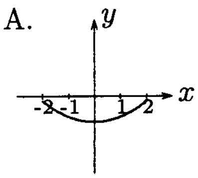


f' + +

f inc inc

which of the following could be a graph of f?

A.



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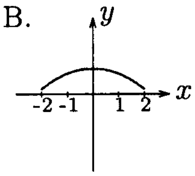
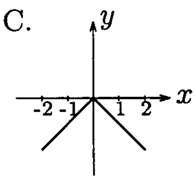
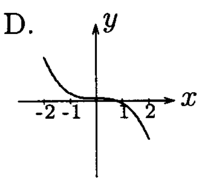


C.



D.



E.