MA 162 EXAM #2 **February 28, 2002**

Name _____ Student ID number _____

Lecturer _____ Recitation Instructor _____

Instructions:

1. This package contains 11 problems, each worth 9 points.
2. Please supply all information requested above and on the mark–sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly.
4. No books, notes, or calculator, please.

Let R be the region between the graphs of f and g on $[a, b]$. Then the moments of R about the x and y axes are

$$M_x = \int_a^b \frac{1}{2}(f(x)^2 - g(x)^2)dx$$

$$M_y = \int_a^b x(f(x) - g(x))dx.$$

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1. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^3 x \, dx$.

A. $\frac{2}{15}$

B. $\frac{1}{6}$

C. $\frac{1}{12}$

D. $\frac{1}{7}$

E. $\frac{1}{8}$

2. Which substitution should be used to compute $\int \frac{dx}{\sqrt{9-4x^2}}$

A. $x = \frac{2}{3}\tan\theta$

B. $x = \frac{3}{2}\sin\theta$

C. $x = \frac{3}{2}\tan\theta$

D. $x = \frac{2}{3}\sin\theta$

E. $x = \frac{2}{3}\sec\theta$

3. After making the substitution $x = 2\tan\theta$, the integral $\displaystyle\int \frac{x^2 dx}{\sqrt{4+x^2}}$ transforms into

A. $\displaystyle\int \frac{2\tan^2\theta\, d\theta}{\sec\theta}$

B. $\displaystyle\int \sec\theta\, d\theta$

C. $\displaystyle\int 4\tan^2\theta\, d\theta$

D. $\displaystyle\int 4\tan^2\theta\sec\theta\, d\theta$

E. $\displaystyle\int \frac{\tan^2\theta}{2\sec\theta}\, d\theta$

4. Compute the partial fraction decomposition of $\dfrac{2}{(x+1)(x^2+1)}$.

A. $\dfrac{-1}{x+1}+\dfrac{2}{x^2+1}$

B. $\dfrac{2}{x+1}+\dfrac{x-1}{x^2+1}$

C. $\dfrac{1}{x+1}+\dfrac{-x+1}{x^2+1}$

D. $\dfrac{1}{x+1}+\dfrac{2x-1}{x^2+1}$

E. $\dfrac{-1}{x+1}+\dfrac{2x+1}{x^2+1}$

5. The error estimate for the trapezoidal rule is $|E_T| \leq \dfrac{K(b-a)^3}{12n^2}$, where $K = \text{Max}|f''(x)|$, $a \leq x \leq b$. Find the smallest n such that the above estimate guarantees that the error E_T of the trapezoidal approximation T_n of $\displaystyle\int_2^4 \ln x\, dx$ satisfies $|E_T| \leq \dfrac{1}{300}$.

A. 4

B. 5

C. 6

D. 7

E. 8

6. Evaluate $\displaystyle\int_0^\infty xe^{-x^2} dx$

A. 2

B. 1

C. the integral diverges

D. $\dfrac{1}{2}$

E. $\dfrac{1}{4}$

7. Find the area of the surface generated by rotating the curve $y = \frac{1}{3}x^3$, $0 \leq x \leq 1$ about the x-axis.

A. $\frac{\pi}{9}(2^{3/2} - 1)$

B. $\frac{\pi}{6}(2^{3/2} - 1)$

C. $\frac{\pi}{27}(10^{3/2} - 1)$

D. $\frac{\pi}{54}(10^{3/2} - 1)$

E. $\frac{\pi}{12}(5^{3/2} - 1)$

8. Find the center of mass of the region bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$, and $x = 2$.

A. $\left(\frac{3}{2}, \frac{1}{2}\right)$

B. $\left(\frac{5}{4}, \frac{3}{2}\right)$

C. $\left(\ln 3, \frac{1}{\ln 3}\right)$

D. $\left(\ln 4, \frac{1}{2\ln 2}\right)$

E. $\left(\frac{1}{\ln 2}, \frac{1}{4\ln 2}\right)$

9. Rank the following limits by size:

$$a = \lim_{n\to\infty} \frac{n}{\sqrt{2n^2+1}}$$

$$b = \lim_{n\to\infty} \frac{\ln(n^2)}{n}$$

$$c = \lim_{n\to\infty} 3^{\frac{1}{n}}$$

A. $a > b > c$

B. $c > a > b$

C. $b > a > c$

D. $b > c > a$

E. $a > c > b$

10. Compute $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^{n+1}}{3^n}$.

A. -3

B. 3

C. $-\frac{6}{5}$

D. $\frac{6}{5}$

E. $\frac{3}{5}$

11. Consider the following statements

I. $\sum_{n=1}^{\infty} \frac{n-1}{2n+1}$ converges

II. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges

III. $\sum_{n=1}^{\infty} \frac{5^n}{3^{2n-1}}$ converges

A. Only I and II are true

B. Only I and III are true

C. Only II and III are true

D. Only II is true

E. Only III is true