

- If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{b} = 2\vec{i} - \vec{k}$, find the vector projection of \vec{b} onto \vec{a} , $\text{proj}_{\vec{a}} \vec{b}$.
 A. $\frac{1}{3}\vec{a}$ B. $\frac{1}{\sqrt{3}}\vec{a}$ C. $\frac{1}{\sqrt{5}}\vec{a}$ D. $\frac{1}{\sqrt{3}}\vec{b}$ E. $\frac{1}{3}\vec{b}$
- Find the angle between the vectors $\vec{a} = -\vec{i} + 2\vec{j}$ and $\vec{b} = \vec{i} + 3\vec{j}$.
 A. $\frac{3\pi}{4}$ B. $\frac{\pi}{4}$ C. $\frac{2\pi}{3}$ D. $\frac{5\pi}{6}$ E. $\frac{11\pi}{12}$
- Find the area of the triangle with vertices at the points $(1, 0, 2)$, $(2, 4, -3)$ and $(1, 2, 1)$.
 A. $\frac{1}{2}\sqrt{41}$ B. $\sqrt{41}$ C. $\sqrt{10}$ D. $\frac{\sqrt{2}}{2}\sqrt{21}$ E. $\frac{41}{2}$
- If $\vec{a} = \vec{i} - \vec{j}$ and $\vec{b} = 2\vec{j} - \vec{k}$, find a unit vector orthogonal to both \vec{a} and \vec{b} .
 A. $\frac{1}{\sqrt{2}}(\vec{i} + \vec{j})$ B. $\frac{1}{\sqrt{6}}(\vec{i} + \vec{j} + 2\vec{k})$ C. $\frac{1}{\sqrt{5}}(\vec{j} + 2\vec{k})$ D. $\vec{i} + \vec{k}$ E. $\frac{1}{\sqrt{5}}(\vec{i} + 2\vec{k})$
- The radius of the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z = 3$ is
 A. $3 + \sqrt{13}$ B. $\sqrt{13}$ C. $\sqrt{65}$ D. $3 + \sqrt{56}$ E. $\sqrt{17}$
- The area of the region enclosed by the curves $y = x^2 + 1$ and $y = 2x + 9$ is given by
 A. $\int_{-2}^4 (x^2 + 1 - 2x - 9)dx$ B. $\int_{-2}^4 (2x + 9 - x^2 - 1)dx$ C. $\int_{-2}^2 (2x + 9 - x^2 - 1)dx$
 D. $\int_{-4}^2 (2x + 9 - x^2 - 1)dx$ E. $\int_{-4}^2 (x^2 + 1 - 2x - 9)dx$
- The volume of the solid obtained by rotating about the x -axis the region in the first quadrant bounded by the graphs of $y = 1 - x^2$, $y = 2x$, and $x = 0$ is given by
 A. $\int_0^{\sqrt{2}-1} (1 - x^2 - 2x)dx$ B. $\int_0^{\sqrt{2}-1} \pi(1 - x^2 + 2x)dx$ C. $\int_{-\sqrt{2}-1}^0 \pi[(1 - x^2)^2 - (2x)^2]dx$
 D. $\int_0^{\sqrt{2}-1} \pi[(1 - x^2)^2 - (2x)^2]dx$ E. $\int_0^{\sqrt{2}-1} [(2x)^2 - (1 + x^2)^2]dx$
- Let R be the region bounded by the curves $y = x^2$ and $y = 2x$. Using the method of cylindrical shells, the volume of the solid generated by rotating R about the x -axis, is given by
 A. $\int_0^2 \pi(2x - x^2)dx$ B. $\int_0^2 2\pi(2x - x^2)^2dx$ C. $\int_0^2 \pi x^2(x^2 - \frac{1}{2}x)dy$ D. $\int_0^4 \pi y^2(\frac{1}{2}y - \sqrt{y})dy$
 E. $\int_0^4 2\pi y(\sqrt{y} - \frac{1}{2}y)dy$
- A right circular conical tank of height 20 ft. and base radius 5 ft. has its vertex at the bottom, and its axis vertical. If the tank is full of water at 62.5 lb./cu. ft., the work required to pump all the water over the top is: (Take the y -axis upwards along the axis of the tank and the origin at its vertex).
 A. $62.5\pi \int_0^{20} (20 - y)(\frac{y}{4})^2 dy$ B. $62.5\pi \int_0^{2\pi} y(\frac{y}{4})^2 dy$ C. $62.5\pi \int_0^{20} (20 - y)^2(\frac{y}{4}) dy$
 D. $62.5\pi \int_0^{20} (20 - y)(\frac{y}{2})^2 dy$ E. $62.5\pi \int_0^{20} (20 - y)(2y)^2 dy$
- A force of 9 lb. is required to stretch a spring from its natural length of 6 in. to a length of 8 in. Find the work required to stretch it from its natural length to 10 in.
 A. 1 ft.-lb. B. 1.5 ft.-lb. C. 2 ft.-lb. D. 3 ft.-lb. E. 4 ft.-lb.
- $\int_0^1 x e^{3x} dx =$
 A. $\frac{2}{9} e^3$ B. $\frac{1}{9} + \frac{2}{9} e^3$ C. 1 D. $\frac{1}{9}$ E. $\frac{1}{9} e^3 - 1$

12. $\int_0^{\pi/2} \cos^3 x dx =$
 A. $\frac{\pi}{2} - \frac{1}{3}$ B. $\frac{\pi}{2} + \frac{1}{3}$ C. 0 D. $-\frac{2}{3}$ E. $\frac{2}{3}$
13. For the integral $\int (1-x^2)^{3/2} dx$, (i) choose a trigonometric substitution to simplify the integral and (ii) give the resulting integral
 A. (i) $x = \sec \theta$, (ii) $\int \tan^3 \theta d\theta$ B. (i) $x = \sec \theta$, (ii) $\int \tan^4 \theta \sec \theta d\theta$
 C. (i) $x = \sec \theta$, (ii) $\int \tan^3 \theta \sec^2 \theta d\theta$ D. (i) $x = \sin \theta$, (ii) $\int \cos^3 \theta d\theta$ E. (i) $x = \sin \theta$, (ii) $\int \cos^4 \theta d\theta$
14. Give the form of the partial fraction decomposition of $\frac{3x+2}{(x^2+1)(x-1)^2}$.
 A. $\frac{A}{x^2+1} + \frac{B}{(x-1)^2}$ B. $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$ C. $\frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$
 D. $\frac{A}{x^2+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ E. $\frac{Ax+B}{x^2+1} + \frac{C}{(x-1)^2}$
15. $\int_1^2 \frac{1}{x^3+x} dx =$
 A. $\ln 2 + \ln \frac{2}{5}$ B. $\ln 2 + \frac{1}{2} \ln \frac{2}{5}$ C. $\ln 2 + \tan^{-1} 5 - \tan^{-1} 2$ D. $2 + \frac{1}{2} \tan^{-1} 5 - \frac{1}{2} \tan^{-1} 2$ E. $\ln \frac{2}{5}$
16. Indicate convergence or divergence for each of the following improper integrals:

$$(I) \int_2^{\infty} \frac{1}{(x-1)^2} dx \quad (II) \int_0^2 \frac{1}{(x-1)^2} dx$$

- A. (I) converges; (II) diverges B. (I) converges; (II) converges
 C. (I) diverges; (II) converges D. (I) diverges; (II) diverges
17. The length of the graph of $y = x^{3/2}$ for $0 \leq x \leq 1$ is
 A. $\frac{2}{3} \sqrt{\frac{5}{2}}$ B. $\frac{4}{27} \sqrt{13}$ C. $\frac{4}{9} \sqrt{\frac{5}{2}}$ D. $\frac{8}{27} \left(\left(\frac{13}{4} \right)^{3/2} - 1 \right)$ E. $\frac{4}{9} (\sqrt{\frac{5}{2}} - 1)$
18. If R is the semicircular region bounded by the x axis and $y = \sqrt{4-x^2}$, $-2 < x < 2$, the centroid (\bar{x}, \bar{y}) of R is
 A. $(0, \frac{8}{3\pi})$ B. $(\frac{8}{3\pi}, 0)$ C. $(0, 1)$ D. $(1, 0)$ E. $(0, 0)$
19. Evaluate the limit $\lim_{n \rightarrow \infty} \left[1 + \frac{(-1)^n}{n} \right]$.
 A. 0 B. 1 C. -1 D. 2 E. limit does not exist
20. Evaluate the limit $\lim_{n \rightarrow \infty} \left(\sqrt[n]{n} + \frac{1}{n!} \right)$.
 A. 0 B. 1 C. e D. $1/e$ E. limit does not exist
21. If $s = \sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n}$, then $s =$
 A. 3 B. 6 C. 9 D. 2 E. $4/3$
22. If $L = \sum_{n=1}^{\infty} \frac{1}{2^n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$, then $L =$
 A. $1/3$ B. $2/3$ C. 1 D. $4/3$ E. $5/3$
23. $\sum_{n=1}^{\infty} \frac{1}{(n^2+1)^p}$ converges when
 A. $p > 1$ B. $p \leq 1$ C. $p \geq 1$ D. $p > \frac{1}{2}$ E. $p \leq \frac{1}{2}$
24. $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^p$ converges for
 A. $p \leq 1$ B. $p > 1$ C. $p < 0$ D. $p > 0$ E. no values of p

25. Which of the following series converge conditionally?

$$(i) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \quad (ii) \sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n} \quad (iii) \sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$$

- A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

26. Which of the following series converge?

$$(i) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} \quad (ii) \sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \quad (iii) \sum_{n=1}^{\infty} \frac{4}{3} \left(\frac{1}{2}\right)^n$$

- A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

27. The interval of convergence for the power series $\sum_{n=2}^{\infty} \frac{3^n x^n}{n \ln n}$ is

- A. $-\frac{1}{3} \leq x < \frac{1}{3}$ B. $-\frac{1}{3} < x \leq \frac{1}{3}$ C. $0 \leq x \leq \frac{1}{3}$ D. $-1 \leq x \leq 2$ E. $-1 < x < 1$

28. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{nx^n}{2^n}$$

- A. $-\frac{1}{2} < x < \frac{1}{2}$ B. $-2 < x < 2$ C. $-2 \leq x \leq 2$ D. $-2 < x \leq 2$ E. $-\infty < x < \infty$

29. The fourth term of the Maclaurin series for $\frac{x^2+3}{x-1}$ is

- A. $-x^3$ B. $3x^3$ C. $-3x^3$ D. $-4x^3$ E. $4x^3$

30. The first three nonzero terms of the Maclaurin series for $f(x) = (1-x^2)\sin x$ are

- A. $x - \frac{5}{6}x^3 + \frac{31}{150}x^5$ B. $1 - \frac{3}{2}x^2 + \frac{13}{24}x^4$ C. $x - \frac{7}{6}x^3 + \frac{31}{150}x^5$ D. $x^2 - \frac{7}{6}x^3 + \frac{1}{25}x^5$ E. $x - \frac{7}{6}x^3 + \frac{21}{120}x^5$

31. The fourth term of the Taylor series for $f(x) = \ln x$ centered at $a = 2$ is

- A. $\frac{1}{6}(x-2)^3$ B. $\frac{1}{12}(x-2)^3$ C. $\frac{1}{24}(x-2)^3$ D. $-\frac{1}{3}(x-2)^3$ E. $-(x-2)^3$

32. Using Maclaurin series and the Estimation Theorem for alternating series, we can obtain the approximation

$$\int_0^{0.1} e^{-x^2} dx \approx 0.1 - \frac{1}{3000}, \text{ with error } \leq c$$

The value of c is

- A. 10^{-5} B. 10^{-6} C. $\frac{1}{2} 10^{-6}$ D. $\frac{10^{-7}}{7}$ E. $\frac{10^{-5}}{2}$

33. The parametric equations of a curve C are

$$x = 2 \cos t, \quad y = 3 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

The curve C is

- A. a quarter of a circle B. an ellipse C. a half of an ellipse D. a half of a circle E. a quarter of an ellipse

34. The length of the parametric curve

$$x = \frac{1}{2}t^2, \quad y = 2 + \frac{1}{3}t^3, \quad 0 \leq t \leq \sqrt{3}$$

is

- A. $\frac{21}{4}$ B. $\frac{7}{2}$ C. $\frac{7}{3}$ D. $\frac{14}{3}$ E. $\frac{8}{3}$

35. A point P has polar coordinates $(2, \frac{2\pi}{3})$. The Cartesian coordinates of P are

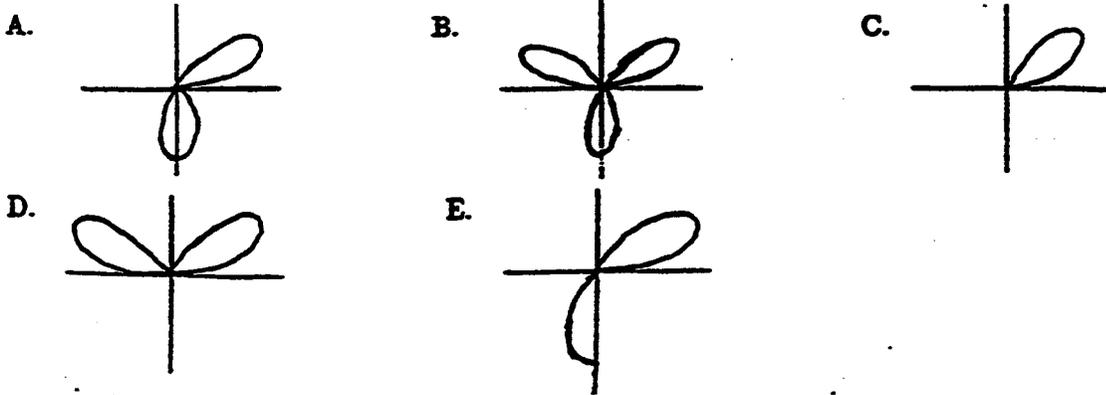
- A. $(-1, \sqrt{3})$ B. $(-2, \sqrt{3})$ C. $(1, \frac{\sqrt{3}}{2})$ D. $(-1, \frac{1}{2})$ E. $(1, -\frac{1}{2})$

36. A point P has polar coordinates $(3, \frac{\pi}{4})$. Which of the following are also polar coordinates of P ?

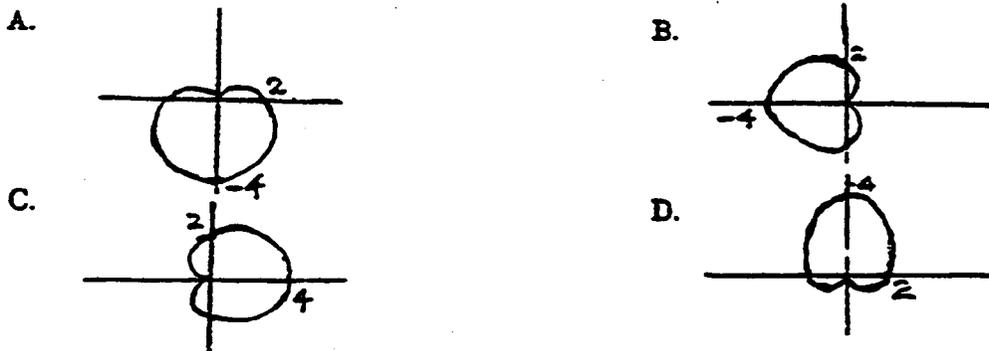
I. $(-3, -\frac{\pi}{4})$ II. $(-3, \frac{5\pi}{4})$, III. $(3, -\frac{7\pi}{4})$ IV. $(3, -\frac{5\pi}{4})$.

- A. I and II only B. I and III only C. I and IV only D. II and III only E. II and IV only

37. Which of the following looks most like the graph of the polar curve $r = \sin(3\theta)$, for $0 \leq \theta \leq \frac{\pi}{2}$?



38. Which of the following looks most like the graph of the polar curve $r = 2 \cos \theta - 2$?



39. The center C and radius a of the circle $r = -2 \sin \theta$ are

- A. $C(0, -\frac{1}{2})$, $a = 1$ B. $C(0, 1)$, $a = 2$ C. $C(-1, 0)$, $a = 2$ D. $C(0, -1)$, $a = 1$ E. $C(-\frac{1}{2}, 0)$, $a = 1$

40. Write the complex number $1 - \sqrt{3}i$ in polar form with argument between 0 and 2π .

- A. $4(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$ B. $2(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})$ C. $4(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$
 D. $2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})$ E. $2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$

Answers: 1-A, 2-B, 3-A, 4-B, 5-E, 6-B, 7-D, 8-E, 9-A, 10-D, 11-B, 12-E, 13-E, 14-C 15-B, 16-A, 17-D, 18-A, 19-B, 20-B, 21-B, 22-E, 23-D, 24-E, 25-E, 26-D, 27-A, 28-B, 29-D, 30-E, 31-C, 32-B, 33-E, 34-C, 35-A, 36-D, 37-E, 38-C, 39-D, 40-D