February 2001

PURDUE UNIVERSITY

Study Guide for the Advanced Placement Exams in One-variable Calculus

Exam 1 and Exam 2 cover respectively the material in Purdue's courses MA 165 (MA 161) and MA 166 (MA 162). These are two separate two hour examinations. Students who pass Exam 1 will receive 4 credit hours for MA 165, and normally will be placed in MA 173. Those who pass Exam 2 will receive 4 credit hours for MA 166, but only if they have credit for MA 165, either by credit examination or other means, and normally will be placed in MA 271. MA 173 and MA 271 are offered in the fall semester only, but students may also register in MA 166 or MA 162 instead of MA 173, and in MA 261 instead of MA 271.

This study guide describes briefly the topics to be mastered prior to attempting these examinations. The material can be studied from many textbooks, almost all of them entitled **Calculus** or **Calculus with Analytic Geometry**. The textbook currently used at Purdue is *Calculus, Early Transcendentals*, 4^{th} edition, James Stewart, Brooks/Cole Publishing Company.

The topics covered on the Credit Exams are listed below. Each exam consists of 25 multiple choice problems, each worth 4 points.

Exam 1: (MA 165) (Two hours)

1. Review of functions and graphs. Trigonometric, exponential and logarithm functions.
2. Limits and continuity.
3. Definition of derivative. Rules of differentiation (sum, product, quotient). Chain rule and implicit differentiation. Derivatives of trigonometric, exponential, and logarithmic functions. Related rates and approximations.
4. Applications of the derivative. Maxima and minima. Mean value theorem. Exponential growth and decay. Curve sketching, concavity, asymptotes. Indeterminate forms and L'Hôpital's Rule.
5. Definition of definite integral. Fundamental Theorem of Calculus. Indefinite integrals. Integration by substitution. Definition of logarithm.
6. Inverse functions. Exponential and logarithm functions. Inverse trigonometric functions and their derivatives.

Exam 2: (MA 166) (Two hours)

1. Cartesian coordinates and vectors in space. Dot product, cross product.
2. Techniques of Integration: Integration by parts, trigonometric integrals, trigonometric substitutions, partial fractions. Improper integrals.
3. Applications of the integral. Volumes by disks and shells. Length of a curve. Area of a surface. Work. Moments and center of gravity.
4. Sequences and series. Convergence. Integral, comparison, ratio and root tests. Alternating series and absolute convergence. Power series and Taylor series.
5. Parametric Equations. Polar coordinates. Conic sections.

Sets of 40 practice problems for each of MA 165 and MA 166 are attached. The correct answers are given on the last page of each set. Naturally, these sets do not cover all points of the courses, but if you have no difficulty with them, you will probably do well in the examination. Additional practice problems are available through the Mathematics Department webpage at www.math.purdue.edu/highSchool/collegePrep/exams-undergrad.html.

The examination, like the set of practice problems, is almost entirely manipulative. This does not mean that you do not need a thorough understanding of the concepts, but rather that you are not asked to quote any definitions or theorems or offer proofs of any theorems. You are expected to perform the manipulations required with a high degree of understanding and accuracy. Two hours are allowed for each examination.

Calculators are not needed for the examination. You may, however, use a non–graphing, non–programmable calculator, if you wish.

Books or notes are not allowed and formulas will not be provided. You should know all the formulas that are needed for solving the practice problems.

IMPORTANT

1. Study all the material thoroughly.
2. Solve a large number of exercises.
3. When you feel prepared for the examination(s), solve the practice problems under exam conditions.

A strong background in algebra, trigonometry, and basic analytic geometry is indispensable for the study of calculus. Successful completion of many of the problems in the examination will depend on your competence in these background areas.

SPECIAL NOTE *A word of advice concerning the taking of the actual examination for credit.* No one does well on an examination when he or she is excessively fatigued. Therefore, you are urged to provide yourself an adequate rest period before taking the actual examination(s). If your trip to the campus necessitates travel into the late hours of the night or an extremely early departure from your home, you should allow for a one night rest in the Lafayette area before taking the examination(s). Many students who are unsuccessful with the examination(s) tell us that failing to take the above precautions contributed strongly to their inability to complete their examination(s) successfully. Most such students find that their first year was somewhat less rewarding than it might have been because of the time spent retracing material studied in high school. Please consult your advanced credit schedule for the actual time and place of the examination(s). They are usually given both morning and afternoon.

MA 165 PRACTICE PROBLEMS

1. $\lim_{x\to 1} \dfrac{x^2-1}{x^2-x} =$

   A. -1 B. 0 C. 1 D. 2 E. Does not exist

2. If $y = (x^2+1)\tan x$, then $\dfrac{dy}{dx} =$

   A. $2x\tan x + (x^2+1)\sec^2 x$ B. $2x\sec^2 x$

   C. $2x\tan x + (x^2+1)\tan x$ D. $2x\tan x + 2x\sec^2 x$ E. $2x\tan x$

3. If $h(x) = \begin{cases} x^2+a, & \text{for } x < -1 \\ x^3-8 & \text{for } x \geq -1 \end{cases}$ determine all values of a so that h is continuous for all values of x.

   A. $a=-1$ B. $a=-8$ C. $a=-9$ D. $a=-10$ E. There are no values of a.

4. Evaluate $\lim_{x\to 0^+} x\cos(\frac{1}{x})$. (Hint: $-1 \leq \cos(\frac{1}{x}) \leq 1$ for all $x \neq 0$.)

   A. 0 B. 1 C. -1 D. $\dfrac{\pi}{2}$ E. Does not exist

5. If $f(x) = \dfrac{1}{x+3}$, then $\lim_{x\to 1} \dfrac{f(x)-f(1)}{x-1} =$

   A. $\dfrac{1}{4}$ B. $\dfrac{1}{16}$ C. $-\dfrac{1}{16}$ D. $-\dfrac{1}{4}$ E. Does not exist

6. The equation $x^3 - x - 5 = 0$ has one solution for x between -2 and 2. The solution is in the interval.

   A. $(-2,-1)$ B. $(-1,0)$ C. $(0,1)$ D. $(1,2)$ E. $(-1,1)$

7. If $f(x) = \dfrac{1-x}{1+x}$, then $f'(1) =$

   A. -1 B. $-\dfrac{1}{2}$ C. 0 D. $\dfrac{1}{2}$ E. 1

8. If $y = \ln(1-x^2) + \sin^2 x$, then $\dfrac{dy}{dx} =$

   A. $\dfrac{1}{1-x^2} + \cos^2 x$ B. $\dfrac{1}{1-x^2} + 2\sin x\cos x$

   C. $\dfrac{1}{1-x^2} + 2\sin x$ D. $\dfrac{-2x}{1-x^2} + \cos^2 x$ E. $\dfrac{-2x}{1-x^2} + 2\sin x\cos x$

9. Find $f''(x)$ if $f(x) = \dfrac{1-x}{1+x}$

A. $\dfrac{4}{(1+x)^3}$ B. $\dfrac{-4}{(1+x)^3}$

C. $-\dfrac{4x}{(1+x)^3} + \dfrac{2}{(1+x)^2}$ D. $\dfrac{2(1+x)^2 - 2x(1+x)}{(1+x)^4}$ E. -1

10. Assume that y is defined implicitly as a differentiable function of x by the equation $xy^2 - x^2 + y + 5 = 0$. Find $\dfrac{dy}{dx}$ at $(-2, 1)$.

A. 9 B. $\dfrac{-5}{3}$ C. 1 D. 2 E. $\dfrac{5}{3}$

11. Find the maximum and minimum values of the function $f(x) = 3x^2 + 6x - 10$ on the interval $-2 \le x \le 2$.

A. max is 14 min is -10. B. max is -10 min is -13

C. max is 14 min is -13 D. no max. min is -10 E. max is 14 no min.

12. For a differentiable function $f(x)$ it is known that $f(3) = 5$ and $f'(3) = -2$. Use differentials (linear approximation) to get the approximate value of $f(3.02)$.

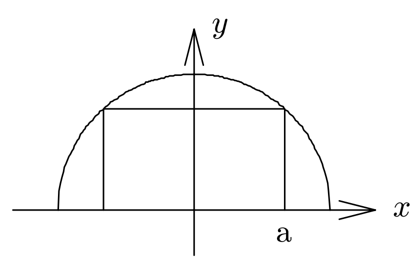
A. 6.02 B. 5.02 C. 5.04 D. 3 E. 4.96.

13. Water is withdrawn from a conical reservoir, 8 feet in diameter and 10 feet deep (vertex down) at the constant rate of 5 ft^3/min. How fast is the water level falling when the depth of the water in the reservoir is 5 ft? ($V = \dfrac{1}{3}\pi r^2 h$).

A. $\dfrac{15}{16\pi}$ ft/min B. $\sqrt{\dfrac{3}{\pi}}$ ft/min C. $\dfrac{2}{\pi}$ ft/min

D. $5\sqrt[3]{3/4\pi}$ ft/min E. $\dfrac{5}{4\pi}$ ft/min.

14. A rectangle is inscribed in the upper half of the circle $x^2 + y^2 = a^2$ as shown at right. Calculate the area of the largest such rectangle.



A. $\dfrac{a^2}{2}$ B. $3a\sqrt{2}$ C. $2a^2$ D. $4a^2$ E. a^2.

15. Given that $f(x)$ is differentiable for all x, $f(2) = 4$, and $f(7) = 10$, then the Mean Value Theorem states that there is a number C such that

A. $2 < C < 7$ and $f'(C) = \dfrac{6}{5}$ B. $2 < C < 7$ and $f'(C) = \dfrac{5}{6}$

C. $4 < C < 10$ and $f'(C) = \dfrac{6}{5}$ D. $2 < C < 7$ and $f'(C) = 0$

E. $4 < C < 10$ and $f'(C) = 0$.

16. Suppose that the mass of a radioactive substance decays from 18gms to 2 gms in 2 days. How long will it take for 12 gms of this substance to decay to 4 gms?

A. $\dfrac{\ln 3}{\ln 2}$ days B. 1 day C. $\dfrac{\ln 2}{\ln 3}$ days D. 2 days E. $(\ln 3)^2$ days

17. Which of the following is/are true about the function $g(x) = 4x^2 - 3x^4$?

(1) g is decreasing for $x > 1$.

(2) g has a relative extreme value at $(0, 0)$.

(3) the graph of g is concave up for all $x < 0$.

A. (1), (2) and (3) B. only (2) C. only (1) D. (1) and (2) E. (1) and (3).

18. Find where the function $f(x) = 2/\sqrt{1 + x^2}$ is increasing.

A. all x B. no x C. $x < 0$ D. $x > 0$ $x = 0$.

19. Let f be a function whose <u>derivative</u>, f', is given by $f'(x) = (x - 1)^2(x + 2)(x - 5)$. The function has

A. a relative maximum at $x = -2$ and a relative minimum at $x = 5$.

B. a relative maximum at $x = 5$ and a relative minimum at $x = -2$.

C. relative maxima at $x = 1$, $x = -2$ and a relative minimum at $x = 5$.

D. a relative maximum at $x = 5$ and relative minima at $x = 1$, $x = -2$

E. a relative maximum at $x = 1$ and relative minima at $x = -2$, $x = 5$.

20. Find $\dfrac{d}{dx} \displaystyle\int_1^{2x} \sqrt{t^2 + 1}\,dt$ at $x = \sqrt{2}$.

A. 6 B. 3 C. $\sqrt{2}$ D. $\sqrt{4x^2 + 1}$ E. $\dfrac{1}{2\sqrt{3}}$.

21. $\int_3^4 x\sqrt{25-x^2}dx =$ A. 0 B. -37 C. $\frac{37}{3}$ D. $-\frac{74}{3}$ E. $\frac{7}{12}$

22. $\lim\limits_{x\to\infty} \frac{x^2+2x}{3x^2+4} =$ A. 1 3/7 C. 1/4 D. 0 E. 1/3.

23. Calculate the upper sum $U_f(P)$ for the definite integral $\int_{-2}^{2}(x^2+1)dx$ with the partition $P = \{-2,-1,0,1,2\}$

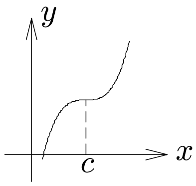
A. $5+2+1+1$ B. $5+2+1+5$ C. $5+2+2+5$

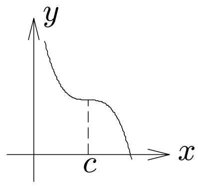
D. $5+1+2+5$ E. $5+1+2+2$

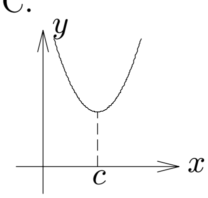
24. Suppose that a function f has the following properties:

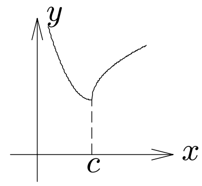
$$f''(x) > 0 \text{ for } x < c, \qquad f'(c) = 0 \qquad \text{and} \qquad f'(x) < 0 \text{ for } x > c.$$

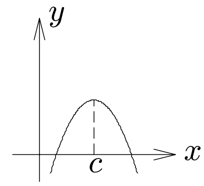
Which of the following could be the graph of f?

A.


B.


C.


D.


E.


25. Find $\lim\limits_{x\longrightarrow 0} \frac{x^2}{1-\cos(3x)}$.

A. 0 B. $\frac{1}{9}$ C. $\frac{2}{9}$ D. $\frac{2}{3}$ E. $-\frac{1}{3}$

26. Find $\lim\limits_{x\longrightarrow\infty} (1+\frac{2}{x})^{3x}$

A. ∞ B. 1 C. e^6 D. e^3 E. e^2

27. $\frac{d}{dx}\left[e^{2x}\ln\sqrt{1+x}\right] =$ A. $e^{2x}\ln(1+x)+\frac{e^{2x}}{2(1+x)}$ B. $\frac{e^{2x}}{\sqrt{1+x}}+2e^{2x}\ln\sqrt{1+x}$

C. $\frac{1}{2}e^{2x}\ln(1+x)+\frac{e^{2x}}{2(1+x)}$ D. $\frac{2e^{2x}}{\sqrt{1+x}}$ E. $\frac{e^{2x}}{1+x}$

28. $\frac{d}{dx}x^{\sin x} =$ A. $(\cos x)x^{\sin x}$ B. $(\sin x)x^{\sin x - 1}$ C. $x^{\cos x}$

D. $x^{\sin x}[\frac{\sin x}{x}+(\cos x)\ln x]$ E. $(\ln x)x^{\sin x}$

29. $\frac{d}{dx}\tan^{-1}e^{3x} =$ A. $\frac{1}{1+e^{3x}}$ B. $\frac{e^{3x}}{1+e^{3x}}$ C. $\frac{3e^{3x}}{1+e^{6x}}$ D. $\frac{3e^{3x}}{1+e^{9x^2}}$ E. $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$

30. $\displaystyle\int_0^1 x^2 e^{3x^3}\,dx =$

A. $\frac{2}{9}e^3$ B. $\frac{1}{9}+\frac{2}{9}e^3$ C. 1 D. $\frac{1}{9}$ E. $\frac{1}{9}(e^3-1)$

31. $\displaystyle\int_{-2}^{1}\frac{1}{x^2+4x+13}dx =$ A. $\frac{\pi}{4}$ B. $\frac{\pi}{12}$ C. $\frac{1}{3}\tan^{-1}3$ D. $\frac{\pi}{6}$ E. $\frac{\pi}{36}$

32. $\displaystyle\int_0^1 \frac{e^x}{1+e^x}dx =$ A. $\ln\frac{1+e}{2}$ B. $\ln(1+e)$ C. $\frac{1}{2}$ D. $1-\ln 2$ E. e

33. An equation of the parabola with horizontal axis, vertex $(-2,3)$, and containing the point $(1,2)$ is

A. $(y-3)^2=\frac{1}{12}(x+2)$ B. $(y+2)^2=-8(x-3)$ C. $(y-3)^2=\frac{1}{3}(x+2)$

D. $(x+2)^2=-9(y-3)$ E. $(x+2)^2=-\frac{9}{4}(y-3)$

34. The ellipse $16(x-3)^2+25(y-7)^2=400$ has one focus at

A. $(6,7)$ B. $(7,7)$ C. $(3,10)$ D. $(3,11)$ E. $(3,12)$

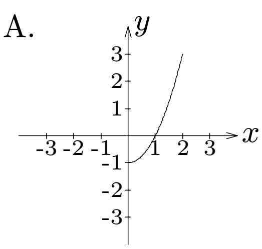
35. The asymptotes of the hyperbola $9x^2-4y^2-36x-8y-4=0$ have equations:

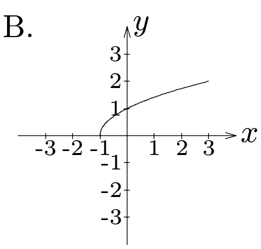
A. $2x-3y-7=0, 2x+3y-1=0$ B. $3x-2y-8=0, 3x+2y-4=0$

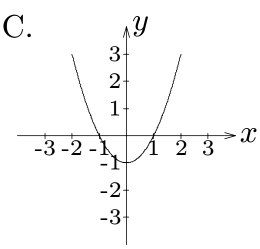
C. $3x-2y=0, 3x+2y=0$ D. $3x-2y+7=0, 3x+2y-1=0$

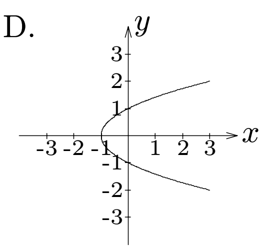
E. $2x-3y+8=0, 2x+3y-4=0$

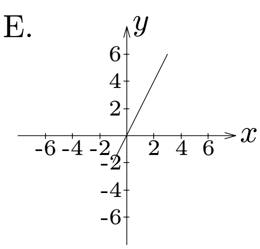
36. If $f(x)=x^2-1$, $0\le x\le 2$, then the graph of $y=f^{-1}(x)$ is

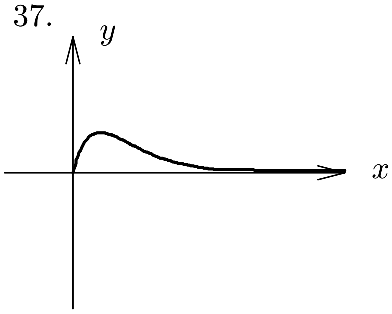
A.


B.


C.


D.


E.


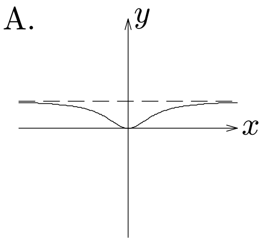
37.


This could be the graph of the function

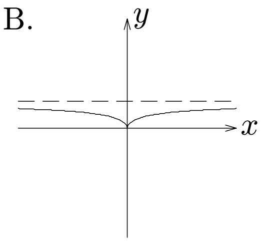
A. $e^{-2x}-e^{-3x},\ x>0$
B. $1/(1+x),\ x>0$
C. $x/(1+x),\ x>0$
D. $1/\ln x,\ x>0$
E. $xe^x,\ x>0$

38. Which could be the graph of $f(x) = \dfrac{x^3}{1+|x|^3}$

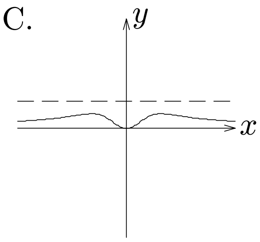
A.



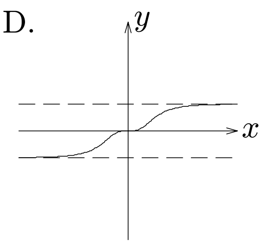
B.



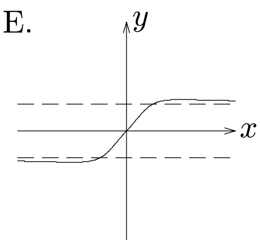
C.



D.



E.



39. If $f(x) = x^5 + 4x$ then $(f^{-1})'(5)$ is

A. 1 B. 1/4 C. 1/5 D. 1/9 E. 1/20

40. The horizontal asymptotes to the graph of $f(x) = \dfrac{e^x + 5e^{-x}}{e^x + 3e^{-x}}$ are

A. $y = 1,\ y = \dfrac{5}{3}$ B. $y = 0$ C. $y = -\dfrac{1}{2}$ and $y = \dfrac{3}{5}$ D. $y = 1$

E. This graph does not have any horizontal asymptotes

Answers: 1.D, 2.A, 3.D, 4.A, 5.C, 6.D, 7.B, 8.E, 9.A, 10.E, 11.C, 12.E, 13.E, 14.E, 15.A, 16.B, 17.C, 18.C, 19.A, 20.A, 21.C, 22.E, 23.C, 24.B, 25.C, 26.C, 27.A, 28.D, 29.C, 30.E, 31.B, 32.A, 33.C, 34.A, 35.B, 36.B, 37.A, 38.D, 39.D, 40.A

MA 166 PRACTICE PROBLEMS

1. Calculate the projection of the vector $\vec{b} = 2\vec{i} - \vec{k}$ onto the vector $\vec{a} = \vec{i} + \vec{j} + \vec{k}$.
   A. $\frac{1}{3}\vec{a}$ B. $\frac{1}{\sqrt{3}}\vec{a}$ C. $\frac{1}{\sqrt{5}}\vec{a}$ D. $\frac{1}{\sqrt{3}}\vec{b}$ E. $\frac{1}{3}\vec{b}$

2. Find the angle between the vectors $\vec{a} = -\vec{i} + 2\vec{j}$ and $\vec{b} = \vec{i} + 3\vec{j}$
   A. $\frac{3\pi}{4}$ B. $\frac{\pi}{4}$ C. $\frac{2\pi}{3}$ D. $\frac{5\pi}{6}$ E. $\frac{11\pi}{12}$

3. Find the area of the triangle with vertices at the points $(1, 0, 2)$, $(2, 4, -3)$ and $(1, 2, 1)$.
   A. $\frac{1}{2}\sqrt{41}$ B. $\sqrt{41}$ C. $\sqrt{10}$ D. $\frac{\sqrt{2}}{2}\sqrt{21}$ E. $\frac{41}{2}$

4. The area of the region bounded by $y = x^2$ and $y = x + 2$ is given by

   A. $\left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Big|_{-1}^{2}$ B. $\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)\Big|_{-1}^{2}$ C. $\left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Big|_{-2}^{1}$
   D. $\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)\Big|_{-2}^{1}$ E $\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{-1}^{2}$

5. Find the area of the region bounded by $y = 2 - x - x^2$ and $y = 2$.

   A. 9/2 B. 25/6 C. 13/6 D. 5/6 E. 1/6

6. $\displaystyle\int_0^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}}\, dx =$ A. $\frac{\pi}{2}$ B. $\frac{\pi}{6}$ C. $\sin^{-1}\sqrt{3}$ D. $\frac{\pi}{3}$ E. 1

7. $\displaystyle\int_0^{\pi/2} \cos^3 x dx =$
   A. $\frac{\pi}{2} - \frac{1}{3}$ B. $\frac{\pi}{2} + \frac{1}{3}$ C. 0 D. $-\frac{2}{3}$ E. $\frac{2}{3}$

8. For the integral $\int (1 - x^2)^{3/2} dx$, (i) choose a trigonometric substitution to simply the integral and (ii) give the resulting integral

   A. (i) $x = \sec u$, (ii) $\int \tan^3 u du$ B. (i) $x = \sec u$, (ii) $\int \tan^4 u \sec u du$

   C. (i) $x = \sec u$, (ii) $\int \tan^3 u \sec^2 u du$ D. (i) $x = \sin u$, (ii) $\int \cos^3 u du$

   E. (i) $x = \sin u$, (ii) $\int \cos^4 u du$

9. $\int_1^e x^2 \ln x dx =$

A. $\frac{2}{9}e^3$ B. $\frac{2}{9}e^3 + \frac{1}{9}$ C. $\frac{1}{3}e^3 - \frac{1}{9}$ D. $\frac{1}{2}e^2 + \frac{1}{4}$ E. $e^3 - e$

10. Give the form of the partial fraction decomposition of $\frac{3x+2}{(x^2+1)(x-1)^2}$.

A. $\frac{A}{x^2+1} + \frac{B}{(x-1)^2}$ B. $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$

C. $\frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$ D. $\frac{A}{x^2+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

E. $\frac{Ax+B}{x^2+1} + \frac{C}{(x-1)^2}$

11. Indicate convergence or divergence for each of the following improper integrals:

$$\text{(I)} \int_2^\infty \frac{1}{(x-1)^2}\, dx \qquad \text{(II)} \int_0^2 \frac{1}{(x-1)^2}\, dx$$

A. (I) converges; (II) diverges B. (I) converges; (II) converges

C. (I) diverges; (II) converges D. (I) diverges; (II) diverges

12. $\int_1^2 \frac{1}{x^3+x}\, dx =$

A. $\ln 2 + \ln \frac{2}{5}$ B. $\ln 2 + \frac{1}{2}\ln\frac{2}{5}$ C. $\ln 2 + \tan^{-1} 5 - \tan^{-1} 2$

D. $2 + \frac{1}{2}\tan^{-1} 5 - \frac{1}{2}\tan^{-1} 2$ E. $\ln \frac{2}{5}$

13. The volume of the solid generated by revolving about the x–axis the region in the first quadrant between the graphs of $y = 1 - x^2$ and $y = 2x$ is given by the definite integral

A. $\int_0^{\sqrt{2}-1} (1 - x^2 - 2x)dx$ B. $\int_0^{\sqrt{2}-1} \pi(1 - x^2 + 2x)dx$

C. $\int_{-\sqrt{2}-1}^{0} \pi[(1-x^2)^2 - (2x)^2]dx$ D. $\int_0^{\sqrt{2}-1} \pi[(1-x^2)^2 - (2x)^2]dx$

E. $\int_0^{\sqrt{2}-1} [(2x)^2 - (1+x^2)^2]dx$

14. Let R be the region between the graphs of $y = x^2$ and $y = 2x$. The volume of the solid generated by revolving R about the x–axis, given by the shell method is

A. $\int_0^2 \pi\left(2x - x^2\right) dx$ B. $\int_0^2 2\pi\left(2x - x^2\right)^2 dx$ C. $\int_0^2 \pi x^2\left(x^2 - \frac{1}{2}x\right) dy$

D. $\int_0^4 \pi y^2\left(\frac{1}{2}y - \sqrt{y}\right) dy$ E. $\int_0^4 2\pi y\left(\sqrt{y} - \frac{1}{2}y\right) dy$

15. The length of the graph of $y = x^{3/2}$ for $0 \leq x \leq 1$ is

A. $\frac{2}{3}\sqrt{\frac{5}{2}}$ B. $\frac{4}{27}\sqrt{13}$ C. $\frac{4}{9}\sqrt{\frac{5}{2}}$ D. $\frac{8}{27}\left(\left(\frac{13}{4}\right)^{3/2} - 1\right)$ E. $\frac{4}{9}\left(\sqrt{\frac{5}{2}} - 1\right)$

16. A right circular conical tank of height 20 ft. and base radius 5 ft. has its vertex at the bottom, and its axis vertical. If the tank is full of water at 62.5 lb./cu. ft., the work required to pump all the water over the top is

A. $62.5\pi\int_0^{20}(20 - y)\left(\frac{y}{4}\right)^2 dy$ B. $62.5\pi\int_0^{2\pi} y\left(\frac{y}{4}\right)^2 dy$

C. $62.5\pi\int_0^{20}(20 - y)^2\left(\frac{y}{4}\right) dy$ D. $62.5\pi\int_0^{20}(20 - y)\left(\frac{y}{2}\right)^2 dy$

E. $62.5\pi\int_0^{20}(20 - y)(2y)^2 dy$

17. A force of 9 lb. is required to stretch a spring from its natural length of 6 in. to a length of 8 in. Find the work required to stretch it from its natural length to 10 in.

A. 12 in.–lb. B. 18 in.–lb. C. 24 in.–lb. D. 36 in.–lb. E. 48 in.–lb.

18. If R is the semicircular region bounded by the x axis and $y = \sqrt{4 - x^2}$, $-2 < x < 2$, the coordinates $(\overline{x}, \overline{y})$ of the center of gravity are

A. $\left(0, \frac{8}{3\pi}\right)$ B. $\left(\frac{8}{3\pi}, 0\right)$ C. $(0, 1)$ D. $(1, 0)$ E. $(0, 0)$

19. The 2nd Taylor polynomial of $f(x) = x^2 + \sin^{-1} x$ is
A. $1 + x + 2x^2$ B. x C. $1 + x + 2x^2$ D. $x + 2x^2$ E. $x + x^2$

20. Evaluate the limit $\lim_{n \to \infty} 1 + \frac{(-1)^n}{n}$.
A. 0 B. 1 C. -1 D. 2 E. limit does not exist

21. Evaluate the limit $\lim_{n\to\infty}\left(\sqrt[n]{n}+\frac{1}{n!}\right)$.

A. 0 B. 1 C. e D. $1/e$ E. limit does not exist

22. If $L=\sum_{n=0}^{\infty}\frac{2^{n+1}}{3^n}$, then $L=$

A. 3 B. 6 C. 9 D. 2 E. 4/3

23. If $L=\sum_{n=1}^{\infty}\frac{1}{2^n}+\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}$, then $L=$

A. 1/3 B. 2/3 C. 1 D. 4/3 E. 5/3

24. $\sum_{n=1}^{\infty}\frac{1}{(n^2+1)^p}$ converges when

A. $p>1$ B. $p\le 1$ C. $p\ge 1$ D. $p>\frac{1}{2}$ E. $p\le\frac{1}{2}$

25. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^p$ converges for

A. $p\le 1$ B. $p>1$ C. $p<0$ D. $p>0$ E. no values of p

26. Which of the following series converge conditionally?

$$\text{(i)}\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\qquad\text{(ii)}\ \sum_{n=2}^{\infty}\frac{(-1)^n n}{\ln n}\qquad\text{(iii)}\ \sum_{n=1}^{\infty}\frac{(-1)^n n}{e^n}$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

27. Which of the following series converge?

$$\text{(i)}\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n^{\frac{1}{4}}}\qquad\text{(ii)}\ \sum_{n=1}^{\infty}\frac{n!}{1\cdot 3\cdot 5\cdots(2n-1)}\qquad\text{(iii)}\ \sum_{n=1}^{\infty}\frac{4}{3}\left(\frac{1}{2}\right)^n$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

28. The interval of convergence for the power series $\sum_{n=2}^{\infty}\frac{3^n x^n}{n\ln n}$ is

A. $-\frac{1}{3}\le x<\frac{1}{3}$ B. $-\frac{1}{3}<x\le\frac{1}{3}$ C. $0\le x\le\frac{1}{3}$

D. $-1\le x\le 2$ E. $-1<x<1$

29. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{nx^n}{2^n}$

A. $-\frac{1}{2} < x < \frac{1}{2}$ B. $-2 < x < 2$ C. $-2 \le x \le 2$

D. $-2 < x \le 2$ E. $-\infty < x < \infty$

30. The fourth term of the Taylor series for $\frac{x^2+3}{x-1}$ about $a = 0$ is

A. $-x^3$ B. $3x^3$ C. $-3x^3$ D. $-4x^3$ E. $4x^3$

31. Use Taylor series to approximate $\int_0^{0.5} e^{-x^2}\,dx$ to within 0.001 $\left(e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}\right)$

A. 0.449 B. 0.452 C. 0.455 D. 0.458 E. 0.461

32. The first three nonzero terms of the Taylor series for $f(x) = (1-x^2)\sin x$ about $a = 0$ are

A. $x - \frac{5}{6}x^3 + \frac{31}{150}x^5$ B. $1 - \frac{3}{2}x^2 + \frac{13}{24}x^4$ C. $x - \frac{7}{6}x^3 + \frac{31}{150}x^5$

D. $x^2 - \frac{7}{6}x^3 + \frac{1}{25}x^5$ E. $x - \frac{7}{6}x^3 + \frac{21}{120}x^5$

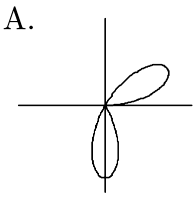
33. A point P has polar coordinates $(3, \frac{\pi}{4})$. Which of the following are also polar coordinates of P?

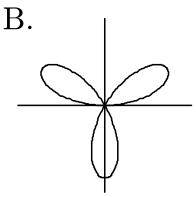
I. $\left(-3, -\frac{\pi}{4}\right)$ II. $\left(-3, \frac{5\pi}{4}\right)$, III. $\left(3, -\frac{7\pi}{4}\right)$ IV. $\left(3, -\frac{5\pi}{4}\right)$.

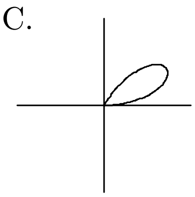
A. I and II only B. I and III only C. I and IV only

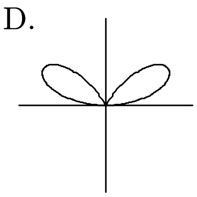
D. II and III only E. II and IV only

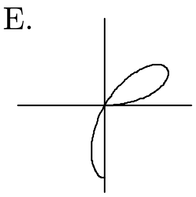
34. Which of the following looks most like the graph of the polar curve $r = \sin(3\theta)$, for $0 \le \theta \le \frac{\pi}{2}$?

A. 

B. 

C. 

D. 

E. 

35. The area inside the curve $r = \sin\theta$ and outside the curve $r = 1 - \sin\theta$ is given by which of the following definite integrals?

A. $\displaystyle\int_{\pi/6}^{5\pi/6} \frac{1}{2}[\sin^2\theta - (1-\sin\theta)^2]d\theta$ B. $\displaystyle\int_{0}^{\pi} \frac{1}{2}[\sin^2\theta - (1-\sin\theta)^2]d\theta$

C. $\displaystyle\int_{\pi/6}^{\pi/2} \frac{1}{2}[\sin^2\theta - (1-\sin\theta)^2]d\theta$ D. $\displaystyle\int_{\pi/6}^{5\pi/6} \frac{1}{2}[\sin\theta - (1-\sin\theta)]^2 d\theta$

E. $\displaystyle\int_{0}^{\pi} \frac{1}{2}[\sin\theta - (1-\sin\theta)]^2 d\theta$

36. The area inside one leaf of the rose curve $r = \sin 2\theta$ is

A. $\dfrac{\pi}{2}$ B. $\dfrac{\pi}{3}$ C. $\dfrac{\pi}{4}$ D. $\dfrac{\pi}{8}$ E. π

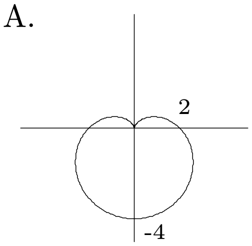
37. The length of the parametrized curve

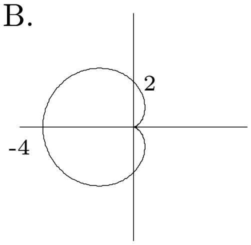
$$x = \frac{1}{2}t^2, \qquad y = 2 + \frac{1}{3}t^3, \qquad 0 \le t \le \sqrt{3}$$

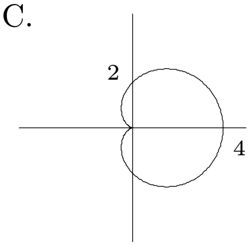
is

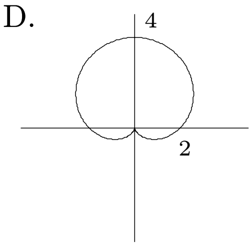
A. $\dfrac{21}{4}$ B. $\dfrac{7}{2}$ C. $\dfrac{7}{3}$ D. $\dfrac{14}{3}$ E. $\dfrac{8}{3}$

38. Which of the following looks most like the graph of the polar curve $r = 2\cos\theta - 2$?

A.


B.


C.


D.


39. The area of the region inside both $r = \sin\theta$ and $r = \sqrt{3}\cos\theta$ is given by

A. $\displaystyle\int_0^{\pi/3} \frac{1}{2}(\sqrt{3}\cos\theta)^2 d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}(\sin\theta)^2 d\theta$ B. $\displaystyle\int_0^{\pi/3} \frac{1}{2}(\sin^2\theta) d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}(\sqrt{3}\cos\theta)^2 d\theta$

C. $\displaystyle\int_0^{\pi} \frac{1}{2}[(\sqrt{3}\cos\theta)^2 - (\sin\theta)^2] d\theta$ D. $\displaystyle\int_0^{\pi/3} \frac{1}{2}[(\sqrt{3}\cos\theta)^2 - (\sin\theta)^2] d\theta$

E. $\displaystyle\int_0^{\pi/2} \frac{1}{2}(\sqrt{3}\cos\theta - \sin\theta)^2 d\theta$

40. Find the length of the curve $x = \sin\theta - \theta\cos\theta$, $y = \cos\theta + \theta\sin\theta$, $0 \le \theta \le \pi$

A. $\dfrac{\pi^2}{2}$ B. $\dfrac{\pi^2}{4}$ C. π^2 D. π E. 2π

Answers: 1–A, 2–B, 3–A, 4–A, 5–E, 6–D, 7–E, 8–E, 9–B, 10–C, 11–A, 12–B, 13–D, 14–E, 15–D, 16–A, 17–D, 18–A, 19–E, 20–B, 21–B, 22–B, 23–E, 24–D, 25–E, 26–E, 27–D, 28–A, 29–B, 30–D, 31–E, 32–E, 33–D, 34–E, 35–A, 36–D, 37–C, 38–C, 39–B, 40–A