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PURDUE UNIVERSITY
Study Guide for the Credit Exam in Multivariate Calculus (MA 261)

Students who pass this exam will receive 4 credit hours for MA 261

This study guide describes briefly the topics to be mastered prior to attempting to obtain credit by examination for MA 261. The material covered is the calculus of several variables, and it can be studied from many textbooks, almost all of them entitled CALCULUS or CALCULUS WITH ANALYTIC GEOMETRY. The textbook currently used at Purdue is CALCULUS — Early Transcendentals, Stewart, Brooks/Cole. See also http://www.math.purdue.edu/academic/courses for recent course information and materials.

The exam consists of 25 multiple choice questions, with two hour time limit. No books, notes, calculators or any other electronic devices are allowed.

IMPORTANT:

1. Study all the material thoroughly.
2. Solve a large number of exercises.
3. When you feel prepared for the examination, solve the practice problems.
4. Come to the examination rested and confident.

The subject matter of the calculus of several variables extends the student's ability to analyze functions of one variable to functions of two or more variables. Graphs of functions of two variables or of equations involving three variables may be thought of as surfaces in three dimensions. Tangents, normals, and tangent planes to these surfaces are part of the subject matter of the calculus of several variables. The concept of volume is defined for three–dimensional solids. These new concepts require the introduction of partial derivatives and multiple integrals.

Most of the problems to be solved require the repeated application of ideas and techniques from the calculus of one variable. Accordingly, a good grasp of the notions of differentiation and integration for functions of one variable is a necessary prerequisite for the study of the calculus of several variables. Several of the concepts from plane analytic geometry are also generalized in the course, leading to a brief study of three dimensional analytic geometry, including such topics as planes and the quadric surfaces (whose cross sections are conics), and three dimensional coordinate systems, including rectangular, cylindrical and spherical coordinates, and the relationships among these.

The topics to be studied prior to attempting the attached practice problems are listed below

1. Analytic Geometry of Three Dimensions

   Angle between two vectors, scalar product, cross product, planes, lines, surfaces, curves in 3 dimensional space.

2. Partial Differentiation

   Functions of several variables, partial derivatives, differential of a function of several variables, partial derivatives of higher order, chain rule, extreme value problems, directional derivatives, gradient, implicit functions.

3. Multiple Integrals

   Double integrals, iterated integrals in rectangular and polar coordinates, applications, surface integrals, triple integrals in rectangular, cylindrical and spherical coordinates.

4. Line and surface integrals, independence of path, Green's theorem, divergence theorem.

A set of 52 practice problems is attached. Naturally, this set does not cover all points of the course, but if you have no difficulty with it, you will probably do well in the examination. The correct answers are given on the last page.

The examination consists of 25 multiple choice questions and, like the set of practice problems, is almost entirely manipulative. This does not mean that you do not need a thorough understanding of the concepts, but rather that you are not asked to quote any definitions or theorems or offer proofs of any theorems. You are expected to perform the manipulations required with a high degree of understanding and accuracy.

SPECIAL NOTE *A word of advice concerning the taking of the actual examination for credit.* No one does well on an examination when he or she is excessively fatigued. Therefore, you are urged to provide yourself an adequate rest period before taking the actual examination. If your trip to the campus necessitates travel into the late hours of the night or an extremely early departure from your home, you should allow for a one night rest in the Lafayette area before taking the examination. Many students who are unsuccessful with the examination tell us that failing to take the above precautions contributed strongly to their inability to complete their examination successfully. Most such students find that their first year was somewhat less rewarding than it might have been because of the time spent retracing materials studied in high school. Please consult your advanced credit schedule for the actual time and place of the examination. It is usually given both morning and afternoon.

MA 261 PRACTICE PROBLEMS

1. If the line ℓ has symmetric equations

   $\frac{x-1}{2} = \frac{y}{-3} = \frac{z+2}{7}$,

   find a vector equation for the line ℓ' that contains the point $(2, 1, -3)$ and is parallel to ℓ.

   A. $\vec{r} = (1+2t)\vec{i} - 3t\vec{j} + (-2+7t)\vec{k}$ B. $\vec{r} = (2+t)\vec{i} - 3\vec{j} + (7-2t)\vec{k}$
   C. $\vec{r} = (2+2t)\vec{i} + (1-3t)\vec{j} + (-3+7t)\vec{k}$ D. $\vec{r} = (2+2t)\vec{i} + (-3+t)\vec{j} + (7-3t)\vec{k}$
   E. $\vec{r} = (2+t)\vec{i} + \vec{j} + (7-3t)\vec{k}$

2. Find parametric equations of the line containing the points $(1, -1, 0)$ and $(-2, 3, 5)$.

   A. $x = 1 - 3t, y = -1 + 4t, z = 5t$ B. $x = t, y = -t, z = 0$
   C. $x = 1 - 2t, y = -1 + 3t, z = 5t$ D. $x = -2t, y = 3t, z = 5t$
   E. $x = -1 + t, y = 2 - t, z = 5$

3. Find an equation of the plane that contains the point $(1, -1, -1)$ and has normal vector $\frac{1}{2}\vec{i} + 2\vec{j} + 3\vec{k}$.

   A. $x - y - z + \frac{9}{2} = 0$ B. $x + 4y + 6z + 9 = 0$ C. $\frac{x-1}{\frac{1}{2}} = \frac{y+1}{2} = \frac{z+1}{3}$
   D. $x - y - z = 0$ E. $\frac{1}{2}x + 2y + 3z = 1$

4. Find an equation of the plane that contains the points $(1, 0, -1)$, $(-5, 3, 2)$, and $(2, -1, 4)$.

   A. $6x - 11y + z = 5$ B. $6x + 11y + z = 5$ C. $11x - 6y + z = 0$
   D. $\vec{r} = 18\vec{i} - 33\vec{j} + 3\vec{k}$ E. $x - 6y - 11z = 12$

5. Find parametric equations of the line tangent to the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ at the point $(2, 4, 8)$

   A. $x = 2 + t, y = 4 + 4t, z = 8 + 12t$ B. $x = 1 + 2t, y = 4 + 4t, z = 12 + 8t$
   C. $x = 2t, y = 4t, z = 8t$ D. $x = t, y = 4t, z = 12t$ E. $x = 2 + t, y = 4 + 2t, z = 8 + 3t$

6. The position function of an object is

   $\vec{r}(t) = \cos t\vec{i} + 3\sin t\vec{j} - t^2\vec{k}$

   Find the velocity, acceleration, and speed of the object when $t = \pi$.

	Velocity	Acceleration	Speed
A.	$-\vec{i} - \pi^2\vec{k}$	$-3\vec{j} - 2\pi\vec{k}$	$\sqrt{1+\pi^4}$
B.	$\vec{i} - 3\vec{j} + 2\pi\vec{k}$	$-\vec{i} - 2\vec{k}$	$\sqrt{10+4\pi^2}$
C.	$3\vec{j} - 2\pi\vec{k}$	$-\vec{i} - 2\vec{k}$	$\sqrt{9+4\pi^2}$
D.	$-3\vec{j} - 2\pi\vec{k}$	$\vec{i} - 2\vec{k}$	$\sqrt{9+4\pi^2}$
E.	$\vec{i} - 2\vec{k}$	$-3\vec{j} - 2\pi\vec{k}$	$\sqrt{5}$

7. A smooth parametrization of the semicircle which passes through the points $(1,0,5)$, $(0,1,5)$ and $(-1,0,5)$ is

A. $\vec{r}(t) = \sin t\vec{i} + \cos t\vec{j} + 5\vec{k}, 0 \le t \le \pi$
B. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 5\vec{k}, 0 \le t \le \pi$
C. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 5\vec{k}, \frac{\pi}{2} \le t \le \frac{3\pi}{2}$
D. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 5\vec{k}, 0 \le t \le \frac{\pi}{2}$
E. $\vec{r}(t) = \sin t + \cos t\vec{j} + 5\vec{k}, \frac{\pi}{2} \le t \le \frac{3\pi}{2}$

8. The length of the curve $\vec{r}(t) = \frac{2}{3}(1+t)^{\frac{3}{2}}\vec{i} + \frac{2}{3}(1-t)^{\frac{3}{2}}\vec{j} + t\vec{k}$, $-1 \le t \le 1$ is

A. $\sqrt{3}$ B. $\sqrt{2}$ C. $\frac{1}{2}\sqrt{3}$ D. $2\sqrt{3}$ E. $\sqrt{2}$

9. The level curves of the function $f(x,y) = \sqrt{1 - x^2 - 2y^2}$ are

A. circles B. lines C. parabolas D. hyperbolas E. ellipses

10. The level surface of the function $f(x,y,z) = z - x^2 - y^2$ that passes through the point $(1,2,-3)$ intersects the (x,z)-plane $(y=0)$ along the curve

A. $z = x^2 + 8$ B. $z = x^2 - 8$ C. $z = x^2 + 5$ D. $z = -x^2 - 8$
E. does not intersect the (x,z)-plane

11. Match the graphs of the equations with their names:

(1) $x^2 + y^2 + z^2 = 4$	(a) paraboloid
(2) $x^2 + z^2 = 4$	(b) sphere
(3) $x^2 + y^2 = z^2$	(c) cylinder
(4) $x^2 + y^2 = z$	(d) double cone
(5) $x^2 + 2y^2 + 3z^2 = 1$	(e) ellipsoid

A. 1b, 2c, 3d, 4a, 5e B. 1b, 2c, 3a, 4d, 5e C. 1e, 2c, 3d, 4a, 5b
D. 1b, 2d, 3a, 4c, 5e E. 1d, 2a, 3b, 4e, 5c

12. Suppose that $w = u^2/v$ where $u = g_1(t)$ and $v = g_2(t)$ are differentiable functions of t. If $g_1(1) = 3$, $g_2(1) = 2$, $g_1'(1) = 5$ and $g_2'(1) = -4$, find $\frac{dw}{dt}$ when $t = 1$.

A. 6 B. 33/2 C. -24 D. 33 E. 24

13. If $w = e^{uv}$ and $u = r + s$, $v = rs$, find $\frac{\partial w}{\partial r}$.

A. $e^{(r+s)rs}(2rs + r^2)$ B. $e^{(r+s)rs}(2rs + s^2)$ C. $e^{(r+s)rs}(2rs + r^2)$
D. $e^{(r+s)rs}(1 + s)$ E. $e^{(r+s)rs}(r + s^2)$.

14. If $f(x,y) = \cos(xy)$, $\frac{\partial^2 f}{\partial x \partial y} =$

A. $-xy\cos(xy)$ B. $-xy\cos(xy) - \sin(xy)$ C. $-\sin(xy)$

D. $xy\cos(xy) + \sin(xy)$ E. $-\cos(xy)$

15. Assuming that the equation $xy^2 + 3z = \cos(z^2)$ defines z implicitly as a function of x and y, find $\frac{\partial z}{\partial x}$.

A. $\frac{y^2}{3-\sin(z^2)}$ B. $\frac{-y^2}{3+\sin(z^2)}$ C. $\frac{y^2}{3+2z\sin(z^2)}$ D. $\frac{-y^2}{3+2z\sin(z^2)}$ E. $\frac{-y^2}{3-2z\sin(z^2)}$

16. If $f(x,y) = xy^2$, then $\nabla f(2,3) =$

A. $12\vec{i} + 9\vec{j}$ B. $18\vec{i} + 18\vec{j}$ C. $9\vec{i} + 12\vec{j}$ D. 21 E. $\sqrt{2}$.

17. Find the directional derivative of $f(x,y) = 5 - 4x^2 - 3y$ at (x,y) towards the origin

A. $-8x - 3$ B. $\frac{-8x^2-3y}{\sqrt{x^2+y^2}}$ C. $\frac{-8x-3}{\sqrt{64x^2+9}}$ D. $8x^2 + 3y$ E. $\frac{8x^2+3y}{\sqrt{x^2+y^2}}$.

18. For the function $f(x,y) = x^2y$, find a unit vector $\vec{u}$ for which the directional derivative $D_{\vec{u}}f(2,3)$ is zero.

A. $\vec{i} + 3\vec{j}$ B. $\frac{i+3\vec{j}}{\sqrt{10}}$ C. $\vec{i} - 3\vec{j}$ D. $\frac{i-3\vec{j}}{\sqrt{10}}$ E. $\frac{3\vec{i}-\vec{j}}{\sqrt{10}}$.

19. Find a vector pointing in the direction in which $f(x,y,z) = 3xy - 9xz^2 + y$ increases most rapidly at the point $(1,1,0)$.

A. $3\vec{i} + 4\vec{j}$ B. $\vec{i} + \vec{j}$ C. $4\vec{i} - 3\vec{j}$ D. $2\vec{i} + \vec{k}$ E. $-\vec{i} + \vec{j}$.

20. Find a vector that is normal to the graph of the equation $2\cos(\pi xy) = 1$ at the point $(\frac{1}{6}, 2)$.

A. $6\vec{i} + \vec{j}$ B. $-\sqrt{3}\,\vec{i} - \vec{j}$ C. $12\vec{i} + \vec{j}$ D. $\vec{j}$ E. $12\vec{i} - \vec{j}$.

21. Find an equation of the tangent plane to the surface $x^2 + 2y^2 + 3z^2 = 6$ at the point $(1,1,-1)$.

A. $-x + 2y + 3z = 2$ B. $2x + 4y - 6z = 6$ C. $x - 2y + 3z = -4$

D. $2x + 4y - 6z = 0$ E. $x + 2y - 3z = 6$.

22. Find an equation of the plane tangent to the graph of $f(x,y) = \pi + \sin(\pi x^2 + 2y)$ when $(x,y) = (2,\pi)$.

A. $4\pi x + 2y - z = 9\pi$ B. $4x + 2\pi y - z = 10\pi$ C. $4\pi x + 2\pi y + z = 10\pi$

D. $4x + 2\pi y - z = 9\pi$ E. $4\pi x + 2y + z = 9\pi$.

23. The differential df of the function $f(x,y,z) = xe^{y^2-z^2}$ is

A. $df = xe^{y^2-z^2}dx + xe^{y^2-z^2}dy + xe^{y^2-z^2}dz$
B. $df = xe^{y^2-z^2}dx\,dy\,dz$
C. $df = e^{y^2-z^2}dx - 2xye^{y^2-z^2}dy + 2xze^{y^2-z^2}dz$
D. $df = e^{y^2-z^2}dx + 2xye^{y^2-z^2}dy - 2xze^{y^2-z^2}dz$
E. $df = e^{y^2-z^2}(1 + 2xy - 2xz)$

24. The function $f(x,y) = 2x^3 - 6xy - 3y^2$ has

A. a relative minimum and a saddle point
B. a relative maximum and a saddle point
C. a relative minimum and a relative maximum
D. two saddle points
E. two relative minima.

25. Consider the problem of finding the minimum value of the function $f(x,y) = 4x^2 + y^2$ on the curve $xy = 1$. In using the method of Lagrange multipliers, the value of λ (even though it is not needed) will be

A. 2 B. -2 C. $\sqrt{2}$ D. $\frac{1}{\sqrt{2}}$ E. 4.

26. Evaluate the iterated integral $\int_1^3 \int_0^x \frac{1}{x}\,dydx$.

A. $-\frac{8}{9}$ B. 2 C. $\ln 3$ D. 0 E. $\ln 2$.

27. Consider the double integral, $\iint_R f(x,y)dA$, where R is the portion of the disk $x^2+y^2 \leq 1$, in the upper half-plane, $y \geq 0$. Express the integral as an iterated integral.

A. $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)dydx$
B. $\int_{-1}^0 \int_0^{\sqrt{1-x^2}} f(x,y)dydx$
C. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} f(x,y)dydx$
D. $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)dydx$
E. $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)dydx$.

28. Find a and b for the correct interchange of order of integration:
$\int_0^2 \int_{x^2}^{2x} f(x,y)dydx = \int_0^4 \int_a^b f(x,y)dxdy$.

A. $a = y^2, b = 2y$
B. $a = \frac{y}{2}, b = \sqrt{y}$
C. $a = \frac{y}{2}, b = y$
D. $a = \sqrt{y}, b = \frac{y}{2}$
E. cannot be done without explicit knowledge of $f(x,y)$.

29. Evaluate the double integral $\iint_R ydA$, where R is the region of the (x,y)-plane inside the triangle with vertices $(0,0)$, $(2,0)$ and $(2,1)$.

A. 2 B. $\frac{8}{3}$ C. $\frac{2}{3}$ D. 1 E. $\frac{1}{3}$.

30. The volume of the solid region in the first octant bounded above by the parabolic sheet $z = 1-x^2$, below by the xy plane, and on the sides by the planes $y = 0$ and $y = x$ is given by the double integral

A. $\int_0^1 \int_0^x (1-x^2)dydx$
B. $\int_0^1 \int_0^{1-x^2} x\,dydx$
C. $\int_{-1}^1 \int_{-x}^x (1-x^2)dydx$
D. $\int_0^1 \int_x^0 (1-x^2)dydx$
E. $\int_0^1 \int_x^{1-x^2} dydx$.

31. The area of one leaf of the three-leaved rose bounded by the graph of $r = 5\sin 3\theta$ is

A. $\frac{5\pi}{6}$ B. $\frac{25\pi}{12}$ C. $\frac{25\pi}{6}$ D. $\frac{5\pi}{3}$ E. $\frac{25\pi}{3}$.

32. Find the area of the portion of the plane $x + 3y + 2z = 6$ that lies in the first octant.

A. $3\sqrt{11}$ B. $6\sqrt{7}$ C. $6\sqrt{14}$ D. $3\sqrt{14}$ E. $6\sqrt{11}$.

33. A solid region in the first octant is bounded by the surfaces $z = y^2$, $y = x$, $y = 0$, $z = 0$ and $x = 4$. The volume of the region is

A. 64 B. $\frac{64}{3}$ C. $\frac{32}{3}$ D. 32 E. $\frac{16}{3}$.

34. An object occupies the region bounded above by the sphere $x^2 + y^2 + z^2 = 32$ and below by the upper nappe of the cone $z^2 = x^2 + y^2$. The mass density at any point of the object is equal to its distance from the xy plane. Set up a triple integral in rectangular coordinates for the total mass m of the object.

A. $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$ B. $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

C. $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$ D. $\int_{0}^{4}\int_{0}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

E. $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} xy\,dz\,dy\,dx$.

35. Do problem 34 in spherical coordinates.

A. $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{32}} \rho^3\cos\varphi\sin\varphi\,d\rho\,d\varphi\,d\theta$ B. $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{32}} \rho\cos\varphi\sin\varphi\,d\rho\,d\varphi\,d\theta$

C. $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{32}} \rho^3\sin^2\varphi\,d\rho\,d\varphi\,d\theta$ D. $\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{\sqrt{32}} \rho^3\cos\varphi\sin\varphi\,d\rho\,d\varphi\,d\theta$

E. $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{32}} \rho\cos\varphi\,d\rho\,d\varphi\,d\theta$.

36. The double integral $\int_0^1\int_0^{\sqrt{1-x^2}} y^2(x^2+y^2)^3 dydx$ when converted to polar coordinates becomes

A. $\int_0^{\pi}\int_0^1 r^9\sin^2\theta\,dr\,d\theta$ B. $\int_0^{\frac{\pi}{2}}\int_0^1 r^8\sin^2\theta\,dr\,d\theta$ C. $\int_0^{\pi}\int_0^1 r^8\sin\theta\,dr\,d\theta$

D. $\int_0^{\frac{\pi}{2}}\int_0^1 r^8\sin\theta\,dr\,d\theta$ E. $\int_0^{\frac{\pi}{2}}\int_0^1 r^9\sin^2\theta\,dr\,d\theta$.

37. Which of the triple integrals converts

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} dz\,dy\,dx$$

from rectangular to cylindrical coordinates?

A. $\int_0^{\pi}\int_0^2\int_r^2 r\,dz\,dr\,d\theta$ B. $\int_0^{2\pi}\int_0^2\int_r^2 r\,dz\,dr\,d\theta$ C. $\int_0^{2\pi}\int_{-2}^2\int_r^2 r\,dz\,dr\,d\theta$

D. $\int_0^{\pi}\int_0^2\int_r^2 r\,dz\,dr\,d\theta$ E. $\int_0^{\frac{2\pi}{2}}\int_{-2}^2\int_r^2 r\,dz\,dr\,d\theta$.

38. If D is the solid region above the xy-plane that is between $z = \sqrt{4-x^2-y^2}$ and $z = \sqrt{1-x^2-y^2}$, then $\iiint_D \sqrt{x^2+y^2+z^2}\,dV =$

A. $\frac{14\pi}{3}$ B. $\frac{16\pi}{3}$ C. $\frac{15\pi}{2}$ D. 8π E. 15π.

39. Determine which of the vector fields below are conservative, i. e. $\vec{F} = \text{grad } f$ for some function f.

1. $\vec{F}(x, y) = (xy^2 + x)\vec{i} + (x^2y - y^2)\vec{j}$.
2. $\vec{F}(x, y) = \frac{x}{y}\,\vec{i} + \frac{y}{x}\,\vec{j}$.
3. $\vec{F}(x, y, z) = ye^z\,\vec{i} + (xe^z + e^y)\vec{j} + (xy + 1)e^z\,\vec{k}$.

A. 1 and 2 B. 1 and 3 C. 2 and 3 D. 1 only E. all three

40. Let $\vec{F}$ be any vector field whose components have continuous partial derivatives up to second order, let f be any real valued function with continuous partial derivatives up to second order, and let $\nabla = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}$. Find the incorrect statement.

A. $\text{curl}(\text{grad } f) = \vec{0}$ B. $\text{div}(\text{curl } \vec{F}) = 0$ C. $\text{grad}(\text{div } \vec{F}) = 0$

D. $\text{curl } \vec{F} = \nabla \times \vec{F}$ E. $\text{div } \vec{F} = \nabla \cdot \vec{F}$

41. A wire lies on the xy-plane along the curve $y = x^2$, $0 \le x \le 2$. The mass density (per unit length) at any point (x, y) of the wire is equal to x. The mass of the wire is

A. $(17\sqrt{17} - 1)/12$ B. $(17\sqrt{17} - 1)/8$ C. $17\sqrt{17} - 1$

D. $(\sqrt{17} - 1)/3$ E. $(\sqrt{17} - 1)/12$

42. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = y\vec{i} + x^2\vec{j}$ and C is composed of the line segments from $(0, 0)$ to $(1, 0)$ and from $(1, 0)$ to $(1, 2)$.

A. 0 B. $\frac{2}{3}$ C. $\frac{5}{6}$ D. 2 E. 3

43. Evaluate the line integral

$$\int_C x\,dx + y\,dy + xy\,dz$$

where C is parametrized by $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + \cos t\vec{k}$ for $-\frac{\pi}{2} \le t \le 0$.

A. 1 B. -1 C. $\frac{1}{3}$ D. $-\frac{1}{3}$ E. 0

44. Are the following statements true or false?

1. The line integral $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy$ is independent of path in the xy-plane.
2. $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy = 0$ for every closed oriented curve C in the xy-plane.
3. There is a function $f(x, y)$ defined in the xy-plane, such that
   $\text{grad } f(x, y) = (x^3 + 2xy)\vec{i} + (x^2 - y^2)\vec{j}$.

A. all three are false B. 1 and 2 are false, 3 is true C. 1 and 2 are true, 3 is false
D. 1 is true, 2 and 3 are false E. all three are true

45. Evaluate $\int_C y^2 dx + 6xy\,dy$ where C is the boundary curve of the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$, in the counterclockwise direction.

A. 0 B. 4 C. 8 D. 16 E. 32

46. If C goes along the x-axis from $(0,0)$ to $(1,0)$, then along $y=\sqrt{1-x^2}$ to $(0,1)$, and then back to $(0,0)$ along the y-axis, then $\int_C \; xy\,dy =$

A. $-\int_0^1\int_0^{\sqrt{1-x^2}} y\,dy\,dx$ B. $\int_0^1\int_0^{\sqrt{1-x^2}} y\,dy\,dx$ C. $-\int_0^1\int_0^{\sqrt{1-x^2}} x\,dy\,dx$
D. $\int_0^1\int_0^{\sqrt{1-x^2}} x\,dy\,dx$ E. 0

47. Evaluate $\int_C \; \vec{F}\cdot d\vec{r}$, if $\vec{F}(x,y)=(xy^2-1)\vec{i}+(x^2y-x)\vec{j}$ and C is the circle of radius 1 centered at $(1,2)$ and oriented counterclockwise.

A. 2 B. π C. 0 D. $-\pi$ E. -2

48. Green's theorem yields the following formula for the area of a simple region R in terms of a line integral over the boundary C of R, oriented counterclockwise. Area of $R=\iint_R dA =$

A. $-\int_C y\,dx$ B. $\int_C y\,dx$ C. $\int_C x\,dx$ D. $\frac{1}{2}\int_C y\,dx - x\,dy$ E. $-\int \; x\,dy$

49. Evaluate the surface integral $\iint_\Sigma x\,dS$ where Σ is the part of the plane $2x+y+z=4$ in the first octant.

A. $8\sqrt{6}$ B. $\frac{8}{3}\sqrt{6}$ C. $\frac{8}{3}\sqrt{14}$ D. $\frac{\sqrt{14}}{3}$ E. $\frac{\sqrt{10}}{3}$

50. If Σ is the part of the paraboloid $z=x^2+y^2$ with $z\le 4$, $\vec{n}$ is the unit normal vector on Σ directed upward, and $\vec{F}(x,y,z)=x\vec{i}+y\vec{j}+z\vec{k}$, then $\iint_\Sigma \vec{F}\cdot\vec{n}\,dS =$

A. 0 B. 8π C. 4π D. -4π E. -8π

51. If $\vec{F}(x,y,z)=\cos z\vec{i}+\sin z\vec{j}+xy\vec{k}$, Σ is the complete boundary of the rectangular solid region bounded by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$ and $z=\frac{\pi}{2}$, and $\vec{n}$ is the outward unit normal on Σ, then $\iint_\Sigma \vec{F}\cdot\vec{n}\,dS =$

A. 0 B. $\frac{1}{2}$ C. 1 D. $\frac{\pi}{2}$ E. 2

52. If $\vec{F}(x,y,z)=x\vec{i}+y\vec{j}+z\vec{k}$, Σ is the unit sphere $x^2+y^2+z^2=1$ and $\vec{n}$ is the outward unit normal on Σ, then $\iint_\Sigma \vec{F}\cdot\vec{n}\,dS =$

A. -4π B. $\frac{2\pi}{3}$ C. 0 D. $\frac{4\pi}{3}$ E. 4π

ANSWERS

1–C, 2–A, 3–B, 4–B, 5–A, 6–D, 7–B, 8–D, 9–E, 10–B, 11–A, 12–E, 13–B, 14–B,

15–D, 16–C, 17–E 18–D, 19–A, 20–C, 21–E, 22–A, 23–D, 24–B, 25–E, 26–B, 27–C,

28–B, 29–E, 30–A, 31–B, 32–D, 33–B, 34–B, 35–A, 36–E, 37–B, 38–C, 39–B, 40–C,

41–A, 42–D, 43–D, 44–E, 45–D, 46–B, 47–D, 48–A, 49–B, 50–E, 51–A, 52–E