Seminars and Advanced Graduate Courses offered by the Mathematics Department Fall, 1999

MA 546	MA 556	MA 557	MA 598A	MA 598F	MA 598F
MA 637	MA 638	MA 642	MA 650	MA 651	MA 672
MA 690B	MA 690C	MA 692A	MA 693B	MA 693C	MA 696A

Seminars

MA 546: Introduction to Functional Analysis
Instructor: Prof. L. Brown, office: Math 704, phone: 49--41938, e-mail: lgb@math.purdue.edu
Time: MWF 11:30
Prerequisite: MA 544
Description: This is a course in Banach and Hilbert spaces and bounded linear operators on such spaces, culminating in the spectral theorem for bounded self--adjoint linear operators on Hilbert space. Other topics include dual spaces and the Hahn-Banach theorem, compact operators and their spectral theory, spectrum and holomorphic functional calculus for general bounded operators, Fredholm operators and index, reflexivity, and separation of convex sets and the double polar theorem.
Text: Schechter, *Principles of Functional Analysis,* Academic Press

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MA 556: Introduction to the Theory of Numbers
Instructor: Prof. F. Shahidi, office: Math 802, phone: 49--41917, e-mail: shahidi@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 162 and 262 or MA 351 or MA 265
Description: This is an introduction to number theory which by its elementary nature requires reasonably light prerequisites. I will follow the book {\it Number Theory with Computer Applications} by Kumanduri and Romero, Prentice Hall, 1998, and cover the following material: Divisibility and Primes; modular arithmetic and its fundamental theorems; some primality testing and factoring; quadratic reciprocity law, Arithmetical function and Dirichlet series; and if time permits "Distribution of primes." The last two chapters are more analytic and my goal there is to prove certain special cases of Dirichlet's Theorem that: given a pair of relatively prime positive integers a and b there are infinitely many positive integers n for which $a+bn$ is a prime. There is no rigid schedule and I may replace the analytic parts with more algebraic ones such as "continued fractions" and "Diophantine equations", or others, according to the preparation and the mood of participants.
Text: R. Kumanduri and C. Romero, *Number Theory with Computer Applications*, Prentice Hall, 1998.

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MA 557: Abstract Algebra I

Instructor: Prof. J. Lipman, office: Math 750, phone: 49--41994, e-mail: lipman@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA 553
Description: Topics: Rudiments of Algebraic Geometry and Algebraic Number Theory; Introduction to Homological Algebra and Group Cohomology; Semisimple rings and Wedderburn's theorem; Linear Representations and Characters of Finite Groups.
Text: Dummit and Foote, *Abstract Algebra,* Second Edition, Chapters 15--19.

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MA 598A: Abstract Algebra
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49--41980, e-mail: heinzer@math.purdue.edu
Time: MWF 12:30
Description: This course will cover fundamentals on groups, rings, and fields (excluding galois theory). It is intended to prepare students who don't have enough background in algebra for the faster-paced Ph.D. qualifier courses MA 553 and 554, as well as to introduce those whose goals may not include the algebra qualifiers to some of the basic concepts.
Text: Dummit and Foote, *Abstract Algebra,* Prentice Hall

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MA 598D: How to Design a Course
Instructor: Dr. R. Saerens, office: MATH 826, phone: 49--41906, email: saerens@math.purdue.edu
Time: M 1:30
Prerequisite: Though open to all graduate students, more senior TAs who have taught their own class and have experience writing exams, will benefit most. A minimum of 7 students is needed for the course to run.
Goal: Using a concrete example, participants will be guided through the various aspects of designing an undergraduate mathematics course (junior level), from choosing a textbook, to deciding on a lesson plan, writing ground rules, etc. The emphasis will be on the participants designing the course themselves and on participants critiquing each others' efforts in a constructive manner. "Lecturing" will be kept to an absolute minimum. Participants are expected to complete assignments by given deadlines. A commitment on the part of the participants to attend all the meetings and to do the assignments is expected.

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MA 598F: Mathematics of Finance
Instructor: Prof. V. Weston, office: Math 746, phone: 49--41959 e-mail: weston@math.purdue.edu
Time: MWF 8:30
Prerequisite: Knowledge of Ordinary Differential Equations (MA 262, MA 366 or equivalent), Multivariate Calculus (curves, line integrals, double integrals, and the chain rule for partial derivatives). is required, and some knowledge of elementary Probability Theory would be useful (only needed for very small part of course.)
Course Objective: For students in mathematics and related disciplines ,to introduce them to the mathematical models of Financial Derivatives, and the theory and technique for solving the associated partial differential equations. Course is self contained in that it will include an introduction to the terminology of mathematical finance and supplementary background material.
Course Outline:
1. Mathematical Models: basic definitions, European and American options, Mathematical model of assets (brief introduction to random walk), Stochastic differential equation for options, Ito's Lemma, developement of Black-Scholes equation, terminal and boundary conditions.
2. Diffusion Equation: well-posedness, initial and terminal boundary conditions, explicit solutions to the Black Scholes equation.
3. Greens Function: brief introduction to distribution theory and weak solutions, delta and Heaviside function, fundamental solution and Greens function representation

of the initial or terminal boundary-value problem.
4. American Options: Free boundary problems, formulation in terms of Greens functions, local analysis using self-similar solutions
5. Diffusion Equation with Time Dependent Parameters: formulation and solutions of Black- Scholes equation with time dependent parameters, formulation when dividends are paid out in discrete time intervals .
6. Variational Formulation of the American Option: variational formulation of free -boundary problems.
7. Exotic Options: models of look-back ,barrier, Asiatic options, etc., solutions of the corresponding equations.
8. Bond models: Vasicek and other equations for modeling bond pricing.
9. Binomial approach to option pricing.
Text: P. Wilmott, J. Dewynne, and S. Howison, *Mathematics of Fincial Derivatives, a Student Edition,* Oxford Financial Press, 1996.
References
1. *Option Pricings, Mathematical Models and Computation,* by P. Wilmott, J. Dewynne, S. Howison (Oxford Financial Press, 1997)
2. *Options, Futures and Other Derivative Securities,* by J. C. Hull (Prentice--Hall, 2nd edition)

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MA 637: Stochastic Integration
Instructor: Prof. P. Protter, office: Math 700, phone: 49--41936, e-mail: protter@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA/STAT 539 or equivalent or consent of instructor.
Description: We cover Stochastic Integration Theory for continuous semimartingales. We begin with a review of martingale theory and martingale inequalities; next we cover main topics in stochastic integration theory with an eye towards those needed for Finance Theory; this includes the Meyer-Ito formula, Girsanov's theorem, Kazamaki's version of Novikov's theorem, Martingale Representation, and a new elementary proof of the Doob-Meyer decomposition theorem. Snell envelopes (which are useful when understanding American Options in Finance theory) are covered. Local times and the Stop-Loss paradox of Carr-Jarrow are presented. Stochastic differential equations completes the course. There will be no required text, but rather notes will be available.

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MA/STAT 638: Stochastic Processes I
Time: MWF 8:30
Prerequisite: MA/STAT 539
Description: Advanced topics in probability theory which may include stationary processes, independent increment processes, Gaussian processes; martingales, Markov processes, ergodic theory.

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MA 642: Methods of Linear and Nonlinear Partial Differential Equations I
Instructor: Prof. P. Bauman, office: Math 718, phone: 49--41945, e-mail: bauman@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 523 and 611.
Description: Second order elliptic equations including maximum principles, Harnack inequality, Schauder estimates, and Sobolev estimates. Applications of linear theory to nonlinear equations.
Text: Gilbarg and Trudinger *Elliptic Partial Differential Equations of Second Order,* 2nd Ed., Springer Verlag.

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MA 650: Commutative Algebraic
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49--41980, e-mail: heinzer@math.purdue.edu
Time: MWF 3:30
Prerequisite: MA 558
Description: I plan to cover material from the book *Introduction to Commutative Algebra and Algebraic Geometry* by Ernst Kunz. Publisher Birkhauser Boston 1985. Topics include Algebraic varieties, Dimension, Regular and rational functions, Localization, The local-global principle, Regular and singular points, Projective resolutions.

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MA 651: Theory of Rings and Algebras
Instructor: Prof. L. Avramov, office: Math 640, phone: 49--41978, e-mail: avramov@math.purdue.edu
Time: TTh 12:00-1:15
Description: The topic of MA 651 will be Maximal Cohen-Macaulay modules. Finite dimensional vector spaces over a field are described up to isomorphism by a single number -- their dimension. There is no "numerical" description for finitely generated modules over commutative rings that are not finite products of fields. In fact, the problem of describing all their finitely generated modules is in general "wild" in a precise technical sense. About ten years it became clear that a reasonable solution can be obtained for certain classes of modules over local rings. The course will describe the available results and the remaining problems. Only knowledge of basic commutative algebra and a little homological algebra will be required. The basic source will be the be the book of Y. Yoshino, *Cohen-Macaulay modules over Cohen-Macaulay Rings*. LMS Lecture Notes Series 146, Cambridge Univ. Press, 1990.

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MA 672: Algebraic Topology I
Instructor: Prof. J. Smith, office: Math 720, phone: 49--47910, e-mail: jhs@math.purdue.edu
Time: MWF 10:30
Description: This will be a course on Homotopy theory. We will begin by defining the homotopy category of spaces which is obtained from the category of spaces by identifying maps that are homotopic. First we will develop the basic machinery: fibrations, cofibrations, CW-complexes, homotopy groups. Second we will introduce the category of simplicial sets and develop its homtopy theory. We will show that the homotopy theory of simplicial sets agrees with the weak homtopy theory of spaces. Next we will study homotopy limits and colimits. These construction allow one to take spaces apart into simpler pieces. Along the way we will give some applications of the techniques.

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MA 690B: Topics in Classical Number Theory
Instructor: Prof. J. Lipman, office: Math 750, phone: 49--41994, e-mail: lipman@math.purdue.edu
Time: TTh 1:30-2:45
Prerequisite: MA 530, and some background in number theory, including quadratic reciprocity and Dedekind domains, such as found in P. Samuel's *Algebraic theory of numbers* or Zariski-Samuel's *Commutative Algebra, vol. 1,* Chap. 5
Outline: {1.} Binary quadratic forms (reduction, genus, representation of integers \dots), and their relations to ideals in quadratic number fields.
{2.} Analytic methods for determining class numbers in quadratic and cyclotomic number fields (zeta functions, L-series). Applications to Fermat's last theorem.
{3.} (Time permitting) Use of complex multiplication and modular functions to determine ring class fields of orders in quadratic number fields (Kronecker's

"Jugendtraum").
References:
1. Borevich--Shafarevich, *Number Theory,* Chapts. 3 and 5.
2. B. B. Zagier, *Zetafunktionen und quadratische Körper*
3. D. Cox. *Primes of the form* $x^2 + ny^2$

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MA 690C: Introduction to Abelian Varieties
Instructor: Prof. K. Matsuki, office: Math 614, phone: 49--41970, e-mail: kmatsuki@math.purdue.edu
Time: NOTE NEW TIME TTh 10:30-11:45
Prerequisite: Working knowledge of Hartshorne and/or first introductory course in algebraic geometry.
Description: The notions of "group" is fundamental in algebra as the notion of "variety" is in geometry. Simply putting these two fundamental notions together, we obtain "group varieties" or better known as "algebraic groups" in algebraic geometry. Thus an algebraic group G is just a variety with the group structure. G is known to have a splitting (an exact sequence)

$$O \dashrightarrow L \dashrightarrow G \dashrightarrow A \dashrightarrow O$$

where L is a linear algebraic group (i.e., an affine algebraic group) and where A is a compact algebraic group, called an Abelian variety. Therefore, in principle, if you want to understand G you only have to understand L and A Linear algebraic groups have been studied, e.g., in the course offered by Prof. Roche. We would like to cover the other end studying the subject of abelian varieties. We start analyzing their first properties as compact complex Lie group, structures as algebraic varieties leading to projective embedding and their scheme--theoretic properties. I would like to come back to the subject of elliptic curves, which are nothing but abelian varieties of dimension 1, to demonstrate and illustrate the theory.
Text: D. Mumford, *Abelian Varieties,* Oxford.

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MA 692A: Special Topics in Numerical Analysis
Instructor: Prof. J. Douglas, office: Math 822, phone: 49--41927, e-mail: douglas@math.purdue.edu
Time: TTh 3:00-4:15
Description: Numerical solution of differential equations, modeling of flows and waves in porous media and their numerical approximation, inverse and not--well--posed problems.

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MA 693B: Polynomial Approximation
Instructor: Prof. L. de Branges, office: Math 800, phone: 49--46057, e-mail: branges@math.purdue.edu
Time: MWF 2:30
Description: A variant of the Riemann mapping theorem is presented which is formulated as the solution of a problem in polynomial approximation. The problem is to obtain the closure of the polynomials in the space of functions which are square integrable with respect to a bounded nonnegative measure with compact support on the Borel subsets of the complex plane. An application of the Krein--Milman theorem reduces the problem to the case in which the harmonic polynomials are dense in the space of integrable functions. A theory of bounded polynomial approximations further reduces the problem to one for a measure which is the image of a measure on the closure of the unit disk under a Riemann mapping function. The closure of the polynomials in the space of square integrable functions is then a Hilbert space of analytic

functions which admits reproducing kernel functions. The interpretation of the Riemann mapping theorem is then an invariant subspace computation for multiplication by *z* as a transformation in the space of square integrable functions with respect to a plane measure.

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MA 693C: Advanced Complex Analysis
Instructor: Prof. A. Eremenko, office: Math 612, phone: 49--41975, e-mail: eremenko@math.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: MA 530
Description: Introduction to modern function theory of one complex variable, with emphasis on its geometric aspects. Topics include: Harmonic Measure, Extremal Length, Hyperbolic Metric, Quasiconformal Mappings, Uniformization and some Potential Theory.
Text: L. Ahlfors, *Conformal Invariants. Topics in Geometric Function Theory*
Supplementary Literature:
1. L. Ahlfors, *Lectures on Quasiconformal Mappings*
2. B. Levin, *Lectures on Entire Functions*
3. T. Ransford, *Potential Theory in the Plane*

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MA 696A: Topics in Algebraic Geometry
Instructor: Prof. S. Abhyankar, office: Math 950, phone: 49--41933, e-mail: ram@math.purdue.edu
Time: TTh 3:00-5:15
Description: Various topics of current interest will be discussed. There are no formal prerequisites. All interested persons are welcome.

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Seminars, Fall, 1999

- **Algebraic Geometry Seminar,** Prof. Abhyankar
  Time: Thursday 4:30--6:00
- **Automorphic Forms and Group Representations Seminar,**Prof. Shahidi
  Time: Monday, 2:30-3:30
- **Commutative Algebra Seminar,** Prof. Avramov
  Time: Wednesday 4:30
- **Geometric Analysis Seminar,** Prof. Lempert
  Time: Monday 4:30
- **Linear and Complex Analysis Seminar,** Prof. de Branges
  Time: Thursday 2:30

- **PDE Seminar,** Prof. Bauman
  Time: Tuesdays, 9:30-10:20
- **Probability Seminar,** Prof. Banuelos
  Time: Wednesdays 3:30

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