

**Courses and Seminars of Interest to Graduate Students
offered by the
Mathematics Department
Spring, 2008**

MA 538: Probability Theory I

Instructor: Prof. A. Yip, office: Math 536, phone: 49-47913, e-mail: aar@stat.purdue.edu

Time: MWF 10:30

Prerequisite: MA 504. Some prior knowledge of probability theory (at the level of MA/STAT 519) is helpful but not necessary.

Description: This course is a measure-theoretic introduction to probability theory. Topics include probability space and measure, random variables, integration theory, independence, laws of large numbers, central limit theorem and related topics.

Text: Patrick Billingsley, *Probability and Measure*, 3rd edition, Wiley.

Reference: Richard Durrett, *Probability: Theory and Examples*, 2nd edition, Duxbury Press.

MA 558: Abstract Algebra II

Instructor: Prof. Ulrich, office: Math 618, phone: 49-41972, e-mail: ulrich@math.purdue.edu

Time: MWF 1:30

Prerequisite: Basic knowledge about commutative rings (such as the material of MA 557).

Description: The topics of the course will be introductory homological algebra and commutative algebra. We will study properties of commutative rings and their modules, with some emphasis on homological methods. The course should be particularly useful to students interested in commutative algebra, algebraic geometry, number theory or algebraic topology. Specific topics are: Derived functors, structure of injective modules, flatness, completion, dimension theory, regular sequences, Cohen-Macaulay modules.

Texts: No particular book is required, but typical texts are:

- J. Rotman, *An introduction to homological algebra*, Academic Press.
- H. Matsumura, *Commutative ring theory*, Cambridge.
- W. Bruns and J. Herzog, *Cohen-Macaulay rings*, Cambridge.
- D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, Springer.

MA 572: Introduction to Algebraic Topology

Instructor: Prof. R. Kaufmann, office: Math 710, phone: 49-41205, e-mail: rkaufman@math.purdue.edu

Time: MWF 9:30

Description: The course is an introduction to algebraic topology. The focus will be on homology and cohomology theory. This subject is important to topology, but also to many other fields, such as differential, symplectic and algebraic geometry, number theory, mathematical physics,...

We will treat the classical simplicial and singular homology and cohomology, but we also plan to cover CW complexes and differential forms.

MA 586: Mathematical Logic II

Instructor: Prof. Lipshitz, office: Math 722, phone: 49-46525, e-mail: lipshitz@math.purdue.edu

Time: TTh 10:30-11:45

Description: Topics to be covered will include some of the following, depending on the interests of the class and the instructor:

1. Beginning model theory (including saturated and homogeneous models, omitting types theorems, complete and model-complete theories).
2. Decidable theories (Elimination of quantifiers: \mathbb{N} with addition, \mathbb{R} with semi-algebraic and subanalytic structures, p -adic fields).
3. Undecidable theories (Hilbert's tenth problem. Other undecidable problems).
4. Beginning nonstandard analysis.
5. Models of computation.

MA 598B: Numerical methods for Stochastic Computations**Instructor:** Prof. Xiu, office: Math 434, phone: 49-62846, e-mail: dxiu@math.purdue.edu**Time:** TTh 3:00-4:15**Prerequisite:** Numerical linear algebra; Numerical methods for ordinary and partial differential equations; Programming skill in one language, e.g., C/C++, FORTRAN, or MatLab**Description:** This course focuses on numerical methods for stochastic equations. The equations are the classical ones frequently encountered in various disciplines of sciences and engineering, e.g., ordinary differential equations, elliptic equations, advection-diffusion, linear/nonlinear wave equations, etc. Contrast to traditional numerical approach where precise information is assumed for the simulation inputs, the course will deal with the case when the inputs, e.g., initial condition, boundary condition, parameter values, geometry, etc., are uncertain and modelled as random variables and random processes. Subsequently the governing equations become stochastic equations and efficient numerical methods are required. This type of stochastic equations are important in practice where imprecise knowledge exists in the inputs. In addition to mathematical framework, the course will cover extensively the available numerical techniques. Note that the stochastic computations discussed here are different from those in finance. In fact computational finance will not be covered.

The course is open to all levels of graduate students in Mathematics, as well as to those in other departments of sciences and engineering. Senior level undergraduate students with outstanding background are also considered. Students, especially first-year graduate students and senior level undergraduate students, are strongly encouraged to consult the instructor for eligibility.

MA 598C: Mixture and Polar Continuum Field Theories**Instructor:** Prof. J. Cushman, office: Math 416, phone: 49-48040, e-mail: jcushman@math.purdue.edu**Time:** TTh 10:30-11:45**Description:** Classical continuum mechanics approaches and principles are reviewed and then extended to multiphase and multi-constituent mixtures with overlaying continua. As in classical mechanics, these theories have three degrees of freedom per continua, corresponding to the spatial coordinates or velocities of material particles. Applications of these theories to drug delivery substrates, contaminant transport in the environment and reservoir engineering will be presented.

When dealing with bodies or processes involving micro-structure such as blood flowing in a micro-capillary bed, granular materials or colloid filtration, mixture theories often fail and a more sophisticated approach is required. One such approach is the theory of polar continua where, in addition to the three degrees of freedom associated with the spatial coordinates, there are nine degrees of freedom representing micro-stretch and micro-shear. This theory will be developed and applied to blood flow in micro-capillaries, i.e. capillaries with diameters similar to the effective diameter of a red blood cell

MA 598D: Porous Media: Theory, Numerical Analysis and Applications**Instructor:** Prof. J. Santos (e-mail: santos@math.purdue.edu), Invited Lecturers: Profs. Osvaldo H. Campanella and Rabi H Mohtar, Department of Agriculture and Biological Engineering, and the Division of Ecological and Environmental Engineering.**Time:** TTh 12:00-1:15**Description:** The course intends to describe the theory of wave propagation and fluid flow in fluid-saturated porous media, with applications to detection and characterization of hydrocarbon reservoirs, groundwater contamination and remediation and characterization of partially frozen foods among other foods.

The equations describing wave propagation and fluid flow in saturated porous media will be solved using finite element techniques that will be described in some detail, including the computer implementation for some particular cases of interest. Numerical upscaling techniques used to represent highly heterogeneous media will also be presented. The mechanical properties of the specific materials will be also reviewed.

MA 598E: Mathematical modeling of nonlinear water waves**Instructor:** Prof. Min Chen, office: Math 418, phone: 49-41964, e-mail: chen@math.purdue.edu**Time:** TTh 1:30-2:45**Description:** This is an advanced course in the modern theory of nonlinear wave propagation. The course is open to all graduate students in mathematics and engineering. Most topics will be developed from scratch, but some knowledges of real analysis, function analysis and Fourier transform will be helpful. The course will cover material taken from the topics listed below. There will be an emphasis on the interaction between modelling and mathematical analysis, augmented by results from numerical simulations and laboratory and field data.**Topics:**

1. Derivation of model equations for long waves.
 - a. One-way models
 - b. Two-way models
 - c. Weakly three-dimensional models
2. Initial-value and initial-boundary value problems
3. Solitary and cnoidal waves and other travelling-wave phenomena.
 - a. explicit solutions
 - b. existence proof with shooting method of topological index theory
 - c. homoclinic orbits and a dynamical system software AUTO
 - d. two-dimensional wave pattern with Lyapunov-Schmidt method
4. Stability of solitary waves
5. Dissipative effects: Long-time asymptotics of solutions
6. Applications in coastal engineering

MA 598T: Bridge to Research Seminar**Instructor:** Prof. F. Milner, office: Math 628, phone: 49-41967, e-mail: milner@math.purdue.edu**Time:** M 4:30**Description:** The seminar has two main goals, both aimed at helping students early in their graduate career find their place in the department. The first is to help students discover what area of mathematics they might be interested in researching, as well as who they might like to work with. The second is to provide students with an opportunity to interact with faculty in a casual setting. This is achieved by having professors from the department give brief talks about their research area at a level that is accessible to those in their first and second year of graduate study.**MA 611: Methods of Applied Mathematics I****Instructor:** Prof. Plamen Stefanov, office: Math 742, phone: 49-67330, e-mail: stefanov@math.purdue.edu**Time:** MWF 11:30**Prerequisite:** MA 511, 544. Authorized equivalent courses or consent of instructor may be used in satisfying course prerequisites**Description:** Banach and Hilbert spaces; linear operators; weak topologies; compactness; spectral theory of compact linear operators; Fredholm theory; applications to linear differential and integral equations.**Reference:** Michael Reed and Barry Simon *Methods of Modern Mathematical Physics, Functional Analysis, Vol. 1*, 2nd edition, ISBN 978-0-12-585050-6, Elsevier Science and Technology Books**MA 615: Numerical Methods for Partial Differential Equations I (meets with CS 615)****Instructor:** Prof. B. Lucier, office: Math 400, phone: 49-41979, e-mail: lucier@math.purdue.edu**Time:** MWF 10:30**Prerequisite:** MA 514, MA 523. Authorized equivalent courses or consent of instructor may be used in satisfying course prerequisites.**Description:** Finite element method for elliptic partial differential equations; weak formulation; finite-dimensional approximations; error bounds; algorithmic issues; solving sparse linear systems; finite element method for parabolic partial differential equations; backward difference and Crank-Nicholson time-stepping; introduction to finite difference methods for elliptic, parabolic, and hyperbolic equations; stability, consistency, and convergence; discrete maximum principles.**Texts:** 1. (required) Stig Larsson and Vidar Thome *Partial Differential Equations with Numerical Methods*, Springer, 2005, ISBN: 35400177202. (recommended) Susanne C. Brenner and L. Ridgway Scott *The Mathematical Theory of Finite Element Methods*, Second Edition, Springer, 2002, ISBN: 0-387-95451-13. (recommended) V. Manis and J. Little, *The Schematics of Computation*, Prentice Hall, ISBN: 0-13-834284-9

MA 639: Stochastic Process II**Instructor:** Prof. A. Yip, office: Math 432, phone: 49-41941, e-mail: yip@math.purdue.edu**Time:** MWF 12:30**Prerequisites:** MA/STAT 638. But students with good knowledge of stochastic differential equations and analysis are also welcome.**Description:** This is a continuation of 638. Depending on the progress of 638, topics include:

- (i) Markov and diffusion processes;
- (ii) long time behavior and stability properties of stochastic differential equations (SDEs);
- (iii) exit time problems and large deviations;
- (iv) convergence of Markov chains to diffusion processes and limit theorems;
- (v) introduction to numerical methods of SDEs.

Text: Ioannis Karatzas, Steven E. Shreve, *Brownian Motion and Stochastic Calculus*, (2nd edition), Springer-Verlag**Reference:** Bernt Oksendal, *Stochastic Differential Equations: An Introduction with Applications*, 6th edition, Springer-Verlag**MA 643: Methods Of Partial Differential Equations II****Instructor:** Prof. D. Phillips, office: Math 706, phone: 49-41939, e-mail: phillips@math.purdue.edu**Time:** MWF 9:30**Prerequisite:** MA 642. Authorized equivalent courses or consent of instructor may be used in satisfying course prerequisites.**Description:** Continuation of MA 642. Topics to be covered are L_p theory for solutions of elliptic equations, including Moser's estimates, Aleksandrov maximum principle, and the Calderon-Zygmund theory. Introduction to evolution problems for parabolic and hyperbolic equations, including Galerkin approximation and semigroup methods. Applications to nonlinear problems.**References:** 1. Gilbarg and Trudinger *PDE's of 2nd Order*2. L. C. Evans *Partial Differential Equations***MA 661: Modern Differential Geometry****Instructor:** Prof. H. Donnelly, office: Math 716, phone: 49-41944, e-mail: hgd@math.purdue.edu**Time:** MWF 10:30**Prerequisite:** MA 562.**Description:** A foundational course in Riemannian geometry. Topics include the Levi-Civita connection, geodesics, normal coordinates, Jacobi fields. Emphasis on curvature and its relation with topology. Familiarity with differentiable manifolds, tensor fields, and differential forms is assumed.

The course will conclude with an exposition of the fundamentals of the Ricci flow. This technique was introduced by R. Hamilton in the 1980's. It is the principal tool in Perelman's recent solution of the Poincaré conjecture.

Text: John M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, Springer-Verlag, 1997**Reference:** B. Chow and D. Knopf, *Ricci Flow: An Introduction*, AMS, 2004**MA 690A: Topics in Algebra and Algebraic Geometry****Instructor:** Prof. S. Abhyankar, office: Math 600, phone: 49-41933, e-mail: ram@math.purdue.edu**Time:** TTh 3:00-4:15**Description:** We shall discuss several topics in algebra and algebraic geometry. There are no prerequisites. All interested students are welcome.**MA 690B: Topics in Commutative Algebra****Instructor:** Prof. Heinzer, office: Math 636, phone: 49-41980, e-mail: heinzer@math.purdue.edu**Time:** MWF 3:30**Description:** The course is planned to be a continuation of MA 690B of this fall and will cover additional material from the text by W. Bruns and J. Herzog titled *Cohen-Macaulay Rings*, revised edition. Students enrolled in the course will be encouraged to actively participate by presenting material and exercises from the text.

MA 690C: Commutative Algebra II: Rees algebras and integral closures of ideals and modules**Instructor:** Prof. Ulrich, office: Math 618, phone: 49-41972, e-mail: ulrich@math.purdue.edu**Time:** MWF 2:30**Prerequisites:** Basic knowledge about commutative rings (such as the material of MA 557).**Description:** Rees algebras play an important role in the process of resolving singularities. They are also a tool for studying integral closures of ideals and modules. The concept of integral dependence for modules in turn has geometric and analytic significance through its applications in equisingularity theory, for instance.

The course serves in some sense as a continuation of MA 650 and of last semester's MA 690C. However, it should be accessible to anybody with basic knowledge in commutative algebra.

Recommended texts: 1. C. Huneke and I. Swanson, *Integral closure of ideals, rings, and modules*, Cambridge Univ. Press
2. W. Vasconcelos, *Integral closure*, Springer
3. W. Vasconcelos, *Arithmetic of blowup algebras*, Cambridge Univ. Press.**MA 690D: Introduction to Algebraic K-Theory****Instructor:** Prof. Jinhyun Park, office: Math 626 phone: 49-R41423 e-mail: jinhyun@math.purdue.edu**Time:** TTh 1:30-2:45**Prerequisite:** An introductory course on commutative algebra, which is usually covered during the 1st year of graduate study in mathematics.**Description:** This course studies the Grothendieck's K_0 , Quillen's higher K_n groups, and how they are used to study algebraic varieties. The Grothendieck-Riemann-Roch will be proved as our first goal. The second goal is to cover Quillen's classical paper on the higher K-theory in detail. If time permits, we discuss some results on the additive K-theory as well.**References** 1. Yuri Manin, *Lectures on the K-functor in algebraic geometry*. Uspehi Mat. Nauk 24 (1969) no. 5 (149), 3 – 86.
2. Daniel Quillen, *Higher Algebraic K-theory. I. in Algebraic K-theory, I : Higher K-theories* (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972), pp. 85–147. Lecture Notes in Math., Vol. 341, Springer, Berlin 1973.
3. Jonathan Rosenberg, *Algebraic K-theory and its applications*. Graduate Texts in Mathematics, 147. Springer-Verlag, New York, 1994. x+392 pp.
4. Vasudevan Srinivas, *Algebraic K-theory*. Second Edition. Progress in Math. 90. Birkauer Boston, Inc., Boston, MA, 1996. xviii+341 pp.**MA 690E: Local Cohomology, an Introduction****Instructor:** Prof. Walther, office: Math 746, phone: 49-41959, e-mail: walther@math.purdue.edu**Time:** MWF 12:30

graduate students with interest in algebra and algebraic geometry.

Requirements: Interest in commutative algebra. Knowledge of the material of the algebra core course. Having taken Commutative algebra and/or Homological Algebra a plus but not necessarily required. Some ideas of Algebraic Topology are helpful. What one should know: the (at least approximate) meaning of

- * category, functor, complex
- * short/long exact sequence
- * Spec of a ring, variety, sheaf

Description: Local cohomology is the algebraic version of relative usual cohomology. I.e., it is designed to make a certain long sequence exact. As such, the study of local cohomology is related to algebraic topology, but also to the theory of Cohen-Macaulay rings, and algebraic geometry.Otherwise, and in more concrete terms, it measures the difference between a ring R and its localization R_f at some f in R . For a collection f_1, \dots, f_r in R one can form some sort of localization at all of the f_i at once, in terms of a complex, and local cohomology is the cohomology of this complex. One can also do local cohomology to a module, which sort of corresponds to comparing sections of a bundle on some space to the sections of the same bundle on an open subset.

(Non)Vanishing of these modules has to do with the number of generators for an ideal (=number of equations needed to define a variety), with depth, with dimension (of the module), connectedness of varieties and other such things.

There are several equivalent ways of defining local cohomology and we will discuss them to some detail.

Text: Iyengar, et al, *24 Hours of Local Cohomology*, to be published by the AMS.

MA 690F: Topics in Automorphic Forms

Instructor: Prof. Shahidi, office: Math 650, phone: 49-41917, e-mail: shahidi@math.purdue.edu

Time: MWF 9:30

Description: This is a continuation of MA 690S presented in the Fall 2007 semester. It is aimed to give a comprehensive proof of the analytic properties of L -functions obtained from Eisenstein series as well as the cases of functionality established using them. A complete proof requires much more time than the two semesters and we therefore supplement the lectures with lots of references and smooth them out with many examples. We will also discuss their connections with number theory and arithmetic geometry.

References: 1. P. Sarnak and F. Shahidi, editors *Automorphic Forms and Applications*, IAS/Park City Mathematics Series, vol. 12, AMS, 2007.

2. My notes *Eisenstein Series and Automorphic L -functions* which I hope to have it completed by the end of the course.

3. *Automorphic L -functions*, Fields Institute, Monographs, AMS, 2004.

MA 690Y: TAMAGAWA NUMBERS

Instructor: Prof. J-K Yu, office: Math 604, phone: 49-67414 e-mail: jyu@math.purdue.edu

Time: TTh 10:40-11:45

Contents: Integration on real, p -adic and adelic varieties, Tamagawa measures and Tamagawa numbers, Applications to the Minkowski-Siegel formula, Prasad's formula, Gross' motives.

Text: 1. A. Weil *Adeles and algebraic groups*, Birkhauser.

2. V. E. Voskresenskii, *Algebraic Groups and their Birational Invariants*

MA 692A: Geometric Numeric Integration

Instructor: Prof. Leok, office: Math 430, phone: 49-63578, e-mail: mleok@math.purdue.edu

Time: TTh 12:00-1:15

Prerequisite: A strong undergraduate background in linear algebra, and differential equations; Some familiarity with numerical methods, classical mechanics and differential geometry would be helpful, but not essential; Programming experience in any language, e.g., C/C++, FORTRAN, MATLAB.

Target Audience: This course is geared towards graduate students in mathematics, science, and engineering with an interest in long-time simulations of mechanical systems, including applications to robotic motion planning, astrodynamics, rigid-body, molecular and stellar dynamics. The application areas addressed will be tailored to the interests of the course participants. The course is also open to undergraduate students with particularly strong backgrounds.

Description: Many differential equations of interest in the physical sciences and engineering exhibit geometric properties that are preserved by the dynamics. Recently, there has been a trend towards the construction of numerical schemes that preserve as many of these geometric invariants as possible.

Such methods are of particular interest when simulating mechanical systems that arise from Lagrangian or Hamiltonian mechanics, wherein the preservation of physical invariants such as the energy, momentum, and symplectic form can be important when simulating long-time dynamics of such systems.

In applications arising from astrodynamics and robotics, the dynamics evolve on nonlinear manifolds such as Lie groups, and in particular the rotation group, and the special Euclidean group. Numerical schemes that respect the underlying nonlinear manifold structure will also be discussed.

This course will begin with an overview of classical numerical integration schemes, and their analysis, followed by a more in depth discussion of the various geometric properties that are of importance in many practical applications, followed by a survey of the various geometric integration schemes that have been developed in recent years. Issues pertaining to the analysis and implementation of such schemes will also be addressed.

Text: Leimkuhler, Reich, *Simulating Hamiltonian Dynamics*, Cambridge University Press, 2005. ISBN: 0521772907

Reference: Hairer, Lubich, Wanner, *Geometric Numerical Integration*, 2nd Edition, Springer-Verlag 2006. ISBN: 3540306633.

MA 692B: Pseudodifferential and Paradifferential Operators

Instructor: Prof. Sa Barreto, office: Math 410, phone: 49-41965, e-mail: sabarre@math.purdue.edu

Time: MWF 12:30

Prerequisites: MA 542, MA 545 or equivalent, or permission of the instructor.

Description: Topics:

- 1: The calculus of pseudodifferential operators, Garding and sharp Garding estimates.
- 2: Boundedness of operators with symbols of low regularity including the Caldern-Vaillancourt theorem.
- 3: Paradifferential operators. Bony's theorem on propagation of singularities.
- 4: Operators with symbols in $S_{1,1,0}$. Hormander's theorem.

Reference: 1) A. Grigis and J. Sjostrand, *Microlocal analysis for differential operators*. Cambridge Univ. Press

2) L. Hormander, *The theory of linear differential operators*. Springer-Verlag, Vol III.

3) M. Taylor, *Tools for PDE. AMS Math surveys and monographs*, Vol 81

4) M. Taylor, *Pseudodifferential operators and non-linear PDE*, Birkhauser, Boston, 1991

MA 693B: Riemann Hypothesis

Instructor: Prof. de Branges, office: Math 800, phone: 49-46057, e-mail: branges@math.purdue.edu

Time: MWF 9:30

Description: The Riemann hypothesis is a unifying concept of mathematical analysis which applies the noncommutative algebra of quaternions with rational coefficients with completions to which Fourier analysis is applied. Zeta functions are constructed in the right of integral quaternions, which resembles the ring of integers despite noncommutativity. The structure of left and right ideals are determined by an Euclidean algorithm. The functional identity of the zeta function is an application of the Poisson summation formula as in the classical theory of Dirichlet zeta functions. The zeta functions constructed are the Hecke zeta functions. A formulation of the Riemann hypothesis in the theory of Hilbert spaces of entire functions applies to these zeta functions. The maximal dissipative property of the Radon transformation in Fourier analysis on quaternions verifies this strengthening of the classical Riemann hypothesis.

MA 696B: Topics in Complex Geometry

Instructor: Prof. Yeung, office: Math 712, phone: 49-41942, e-mail: yeung@math.purdue.edu

Time: MWF 11:30

Prerequisite: Some knowledge in several complex variables and differential geometry.

Description: The tentative topics include multiplier ideals and multiplier ideal sheaves, their use in algebraic and complex geometry, and some techniques in Ricci flows.

Seminars

Algebraic Geometry Seminar, Prof. Abhyankar

Time: Thursday 4:30–6:00

Automorphic Forms and Representation Theory Seminar, Prof. Yu

Time: Thursdays, 1:30

Bridge to Research Seminar, Prof. Milner

Time: Mondays 4:30

Commutative Algebra Seminar, Profs. Heinzer and Ulrich

Time: Wednesdays 4:30

Computational and Applied Math Seminar, Prof. Shen

Time: Fridays 3:30

Computational Finance Seminar, Prof. Figueroa-Lopez

Time: Fridays 2:30

Function Theory Seminar, Prof. Eremenko

Time: flexible.

Geometric Analysis Seminar

Time: Monday 4:30

Foundations of Analysis Seminar, Prof. de Branges

Time: Thursday 9:30

Joint Geometry/Topology Seminar, Prof. Lee

Time: To Be Announced

Number Theory, Prof. Goins

Time: Thursday, 3:30

Operator Algebras Seminar, Prof. Dadarlat

Time: Tuesdays, 2:30

PDE Seminar, Prof. Phillips

Time: Thursdays, 3:30

Probability Seminar, Prof. Sellke

Time: Mondays 3:30

Spectral and Scattering Theory Seminar, Prof. Sá Barreto

Time: Wednesday 4:30

Topology Seminar, Prof. McClure

Time: Thursday 3:30

Working Algebraic Geometry Seminar, Profs. Arapura and Matsuki

Time: Wednesday 3:30-5:00