82nd Midwest PDE Seminar

October 27-28, 2018, Purdue University

Schedule / Titles and Abstracts
(Talks will be held at LWSN (Lawson building) 1142)

Saturday, October 27, 2018

Morning Session:

- 8:00 - 8:45 am: Onsite Registration and Breakfast (Coffee/Bagels), outside LWSN 1142
- 8:45 - 9:40 am: Sijue Wu, University of Michigan
  On the motion of water waves with angled crests
- 9:45 - 10:40 am: Sasha Kiselev, Duke University
  Small scales and singularity formation in fluid dynamics
- 10:45 - 10:15 am: Coffee break (outside LWSN 1142)
- 11:15 am - 12:10 pm: Nam Le, Indiana University
  Hölder regularity of the 2D dual semigeostrophic equations
- 12:10 - 2:00 pm: Lunch break

Afternoon Session

- 2:00 - 2:55 pm: Ovidiu Savin, Columbia University
  Sharp regularity results for the optimal transport map between planar convex domains
- 3:00 - 3:25 pm: Max Engelstein, MIT
  Regularized distances and harmonic measure in co-dimension greater than one
- 3:30 - 3:55 pm: Robin Neumayer, IAS, Princeton
  The Cheeger constant of a Jordan domain without necks
- 4:00 - 4:30 pm: Coffee break (outside LWSN 1142)
- 4:30 - 5:25 pm: Scott Armstrong, CIMS, New York University
  Energy methods for hypoelliptic equations
- 5:30 - 5:55 pm: Guanying Peng, University of Arizona
  Null Lagrangian measures in subspaces with applications to a system of conservation laws
- 6:00 -7:30 pm: Social gather (TBA)

Sunday, October 28, 2018

Morning Session:

- 8:00 - 8:30 am: Breakfast (Coffee/Bagels), outside LWSN 1142
- 8:30 - 9:25 am: Nicola Garofalo, University of Padova
  The structure of the singular set in free boundary problems for nonlocal parabolic equations
- 9:30 - 9:55 am: Cornelia Mihaila, University of Chicago
  Bubbling with L^2 almost constant mean curvature and an Alexandrov type theorem for crystals
- 10:00 - 10:25 am: Onur Alper, Purdue University
  On the singular set of free interface in segregated critical configurations in various elliptic systems
- 10:25 -10:45 am: Coffee break (outside LWSN 1142)
- 10:45 - 11:10 am: Brian Krummel, Purdue University
  Branch sets of Dirichlet energy minimizing functions
- 11:15 - 12:10 pm: Tiziana Giorgi, New Mexico State University
  Field effects in Landau-de Gennes energies for smectic phases

Plenary Speakers

- Scott Armstrong, CIMS, New York University

  Title: Energy methods for hypoelliptic equations

  Abstract: I will present a new PDE approach to the kinetic Fokker-Planck equation. We will propose a weak formulation of the equation and present well-posedness results. We will obtain regularity estimates by iterating the basic energy estimate (essentially, a version of Caccioppoli's inequality) and combining it with a new Sobolev-type functional inequality. The proof of this Sobolev-type inequality is where the Hormander bracket condition is used. In particular, we will show that our weak solutions are smooth. We will also present a simple proof of exponential decay to equilibrium, by combining some new functional (Poincare-type) inequalities with energy estimates. This is joint work with Jean-Christophe Mourrat.

- Nicola Garofalo, University of Padova, Italy

  Title: The structure of the singular set in free boundary problems for nonlocal parabolic equations

  Abstract: I will present some recent results on the structure of the singular free boundary in the thin obstacle problem associated with the fractional heat equation. The relevant nonlocal problem model phenomena governed by the continuous time random walks (CTRW) introduced by Montroll and Weiss in 1965. In my talk the focus will be on the analysis of the singular free boundary for the relevant degenerate parabolic equation. This is joint work with A. Banerjee, D. Danielli and A. Petrosyan.

- Tiziana Giorgi, New Mexico State University

  Title: Field effects in Landau-de Gennes energies for smectic phases

  Abstract: We investigate a Landau-de Gennes free energy for smectic A phases in rod-like liquid crystals to apprehend the chevron pattern formed in the presence of an applied magnetic field. We establish a small parameter ϵ, and explore via Γ-convergence the behaviors of the minimizers when the field strength is of order $\mathcal{O}(\epsilon^{-1})$. We note that the same approach can be exploited to analyze a Landau-de Gennes functional used to model the switching mechanism due to an electric field in a smectic tilted columnar phase of bent-core liquid crystals. Numerical simulations illustrating the chevron structures and the switching mechanism are featured. This is joint work with Carlos J. Garca-Cervera, Sookyung Joo and Lidia Mrad.

- Sasha Kiselev, Duke University

  Title: Small scales and singularity formation in fluid dynamics

  Abstract: We review recent advances in understanding singularity and small scales formation in solutions of fluid dynamics equations. The focus is on the Euler and surface quasi-geostrophic (SQG) equations and associated models. The Euler equation describes flow of incompressible and inviscid fluid, and is one of the fundamental equations of fluid mechanics. As is the case for most fluid mechanics equations, it is nonlinear and nonlocal, and its solutions are often unstable and spontaneously generate small scales. I will discuss a construction of an example of double exponential growth in derivatives of the solutions to two-dimensional Euler equation, the rate that is known to be sharp due to upper bounds going back to 1930s. The SQG equation comes from atmospheric science, and is perhaps the simplest-looking fluid

mechanics equation for which the question of singularity formation vs global regularity remains open. I will discuss a recent proof of singularity formation in modified SQG patch solutions on half-plane.

- Nam Le, Indiana University

  Title: Hölder regularity of the 2D dual semigeostrophic equations

  Abstract: The system of 2D dual semigeostrophic equations is a fully nonlinear version of the 2D incompressible Euler equations in vorticity-stream formulation where the Monge-Ampere operator replaces the Laplace operator. It arises in a meteorology model used to describe large scale atmospheric flows. In this talk, we discuss the Holder regularity of time derivative of solutions to the 2D dual semigeostrophic equations when the initial potential density is bounded away from zero and infinity. Our main tool is an interior Holder estimate in 2D for an inhomogeneous linearized Monge-Ampere equation when the source term is the divergence of a bounded vector field but not of high integrability.

- Ovidiu Savin, Columbia University

  Title: Sharp regularity results for the optimal transport map between planar convex domains

  Abstract: Given two domains with the same volume, the optimal transport, in its most basic form, consists in mapping one domain into the other by a measure preserving transformation which minimizes a total transport cost. For the quadratic cost, the regularity theory of the map was developed by L. Caffarelli in the early 90s, by making use of its connection with the Monge-Ampere equation. In my talk I will discuss some recent work in collaboration with Hui Yu concerning the optimal transport map between two arbitrary planar convex domains.

- Sijue Wu, University of Michigan

  Title: On the motion of water waves with angled crests

  Abstract: In this talk, I will discuss some recent progress on the understanding of the motion of water waves that allows for angled crested type singularities in the interface, with and without surface tension.

Invited Speakers

- Onur Alper, Purdue University

  Title: On the singular set of free interface in segregated critical configurations in various elliptic systems

  Abstract: Segregation phenomena arise in numerous contexts such as: the partitions of domains optimally with respect to the Dirichlet eigenvalues, the clustering problem in machine learning, the asymptotic limits of reaction-diffusion systems with strong competition interactions, the Lotka-Volterra model for the population dynamics of competitive species, and the standing waves in multi-phase Bose-Einstein condensates. In this talk I will discuss the extension of the analysis in the paper "On the singular set of free interface in an optimal partition problem" (A. 2018) to the case of segregated critical configurations under a weak reflection law, with applications to these problems in mind. The class of problems we consider include singular perturbation limits of elliptic systems of nondivergence type, as well as divergence type systems with nontrivial competitive interaction structure. Our main result is the rectifiability of the singular set of free interface and local bounds on its upper $(n-2)$-dimensional Minkowski content. This is joint work with Salvatore Stuvard (UT Austin) and Monica Torres (Purdue).

- Max Engelstein, MIT

  Title: Regularized distances and harmonic measure in co-dimension greater than one

  Abstract: The past thirty years have seen several advances in characterizing the geometry of a co-dimension one set by the behavior of singular operators on that set and the boundary behavior of harmonic functions in complement of that set. We will talk about recent extensions of some of these characterizations to higher co-dimension, using the behavior of a regularized distance function. However, we will also discuss a surprising situation, in which the regularized distance function is itself a solution to a degenerate elliptic operator, giving us results in sharp contrast with the co-dimension one case. This is joint work with Guy David (Paris Sud) and Svitlana Mayboroda (University of Minnesota).

- Brian Krummel, Purdue University

  Title: Branch sets of Dirichlet energy minimizing functions

  Abstract: Dirichlet energy minimizing multivalued functions were introduced by Almgren as approximations of area minimizing submanifolds. Almgren showed that the branch set of a Dirichlet energy minimizing multivalued function on a domain in $\mathbb{R}^n$ has Hausdorff dimension at most $n-2$. We show that the branch set is countably $(n-2)$-rectifiable and that there is a unique tangent function at $\mathcal{H}^{n-2}$-a.e. branch point. Our approach involves a modification of a blow-up method due to L. Simon, which was originally applied to multiplicity one classes of minimal submanifolds. We use new estimates and strategies together with techniques from prior work to adapt L. Simon?s blow-up method to the higher multiplicity setting of Dirichlet energy minimizing functions.

- Cornelia Mihaila, University of Chicago

  Title: Bubbling with L^2 almost constant mean curvature and an Alexandrov type theorem for crystals

  Abstract: I will discuss a recent result in which an Alexandrov-type theorem for L^2 almost constant anisotropic mean curvature sets is proven. In addition I will provide a description of

critical points/local minimizers for elliptic energies interacting with a confinement potential. An improvement on previous almost constant mean curvature results is our use of L^2 versus C^0 closeness, since this should have applications in mean curvature flow and is new even in the isotropic case. This talk is based on a joint work with Matias Delgadino, Francesco Maggi, and Robin Neumayer.

- Robin Neumayer, IAS, Princeton University

  Title: The Cheeger constant of a Jordan domain without necks

  Abstract: In 1970, Cheeger established lower bounds on the first eigenvalue of the Laplacian on compact Riemannian manifolds in terms of a certain isoperimetric problem. The analogous problem on domains of Euclidean space has generated much interest in recent years, due in part to its connections to capillarity theory, image processing, and landslide modeling. In this talk, based on joint work with Leonardi and Saracco, we give an explicit characterization of minimizers in this isoperimetric problem for a very general class of planar domains.

- Guanying Peng, University of Arizona

  Title: Null Lagrangian measures in subspaces with applications to a system of conservation laws

  Abstract: In this talk we will discuss the Null Lagrangian measures which are probability measures in the space of matrices $\mathbb{R}^{m\times n}$ that commute with all minors. Given a closed set $K \subset \mathbb{R}^{m\times n}$, we denote by $\mathcal{M}^{pc}(K)$ the set of Null Lagrangian measures supported on K. Such measures arise in the study of certain nonlinear PDEs that are naturally associated to a set K in the space of matrices, and triviality of $\mathcal{M}^{pc}(K)$, i.e., $\mathcal{M}^{pc}(K)$ consisting of Dirac measures, often leads to nice properties of the PDEs. A particular example comes from the method of compensated compactness, in which the Young measures can be viewed as Null Lagrangian measures. There is overall little understanding on conditions on a set K in order for $\mathcal{M}^{pc}(K)$ to be trivial. We provide a necessary and sufficient condition for triviality of $\mathcal{M}^{pc}(K)$ for subspaces $K \subset \mathbb{R}^{m\times n}$. When the dimension of K is less than 4, the condition is equivalent to the more classical condition of K having no rank-one connections. The ideas allow us to answer a question raised by Kirchheim, Müller and Šverák on the structure of $\mathcal{M}^{pc}(K)$ for some nonlinear submanifold $K \subset \mathbb{R}^{3\times 2}$ that is associated to a well known 2×2 system of conservation laws with one entropy/entropy flux pair. This is joint work with A. Lorent.