Functions Of A Complex Variable I
Instructor: Professor Alexandre Eremenko
Course Number: MA 53000
Credits: Three
Time: 11:30–12:20 PM MWF

Catalog Description

Complex numbers and complex-valued functions of one complex variable; differentiation and contour integration; Cauchy's theorem; Taylor and Laurent series; residues; conformal mapping; special topics. More mathematically rigorous than MA 52500.

Elements Of Stochastic Processes
Instructor: Professor Samy Tindel
Course Number: MA 53200
Credits: Three
Time: 3:30–4:20 PM MWF

Catalog Description

A basic course in stochastic models, including discrete and continuous time Markov chains and Brownian motion, as well as an introduction to topics such as Gaussian processes, queues, epidemic models, branching processes, renewal processes, replacement, and reliability problems.

Probability Theory I
Instructor: Professor Christopher Janjigian
Course Number: MA 53800
Credits: Three
Time: 10:30–11:4 AM TTh

Catalog Description

A basic course in stochastic models, including discrete and continuous time Markov chains and Brownian motion, as well as an introduction to topics such as Gaussian processes, queues, epidemic models, branching processes, renewal processes, replacement, and reliability problems.

Ordinary Differential Equations and Dynamical Systems
Instructor: Professor Yuan Gao
Course Number: MA 54300
Credits: Three
Time: 4:30–5:45 PM TTh

Description

This is a graduate–level course on Ordinary differential equations and Dynamical systems. The course will start with an introduction to the basic properties of differential equations, including solving linear systems, existence and uniqueness theory, flows and linearization, and linearized stabilities. With these preparations, I will introduce advanced concepts related to global existence, invariant/stable/unstable manifold, periodic orbits, limit sets, averaging, chaotic and bifurcation theory. Preliminary knowledges on basic concepts in linear algebra, calculus, analysis are required. The goal of the course is to introduce the fundamental mathematical ideas in dynamical systems. Each student will give a final presentation on related topics.

Real Analysis And Measure Theory
Instructor: Professor Rodrigo Banuelos
Course Number: MA 54400
Credits: Three
Time: 12:30–1:20 PM MWF

Catalog Description

Metric space topology; continuity, convergence; equicontinuity; compactness; bounded variation, Helly selection theorem; Riemann-Stieltjes integral; Lebesgue measure; abstract measure spaces; LP-spaces; Holder and Minkowski inequalities; Riesz-Fischer theorem.

Introduction to Functional Analysis

Instructor: Professor Marius Dadarlat
Course Number: MA 54600
Credits: Three
Time: 1:30–2:20 PM MWF

Description

1. Banach spaces
2. Hilbert spaces
3. Linear Operators and functionals
4. The Hahn-Banach Theorem
5. Duality
6. The Open Mapping Theorem
7. The Uniform Boundedness Principle
8. Weak Topologies
9. Spectra of operators
10. Compact operators
11. Banach algebras and C*-algebras
12. Riesz calculus
13. Fredholm index
14. Gelfand transform

15. Spectral theorem for normal operators

If time allows:

16. Unbounded Operators

17. Applications: Differential operators, Peter-Weyl theorem

Prerequisites: Familiarity with basic measure theory

Grading:

Attendance 35%,

HW 40%,

Final Exam 25% (a take-home 36 hours no collaboration exam).

No specific textbook is required. These topics are covered by most books on functional analysis. A good reference is: Gert Pedersen, *Analysis Now*, (Graduate Texts in Mathematics) *Corrected Edition*!

Introduction To Abstract Algebra
Instructor: Professor Freydoon Shahidi
Course Number: MA 55300
Credits: Three
Time: 10:30–11:20 AM MWF

Catalog Description

Group theory: Sylow theorems, Jordan-Holder theorem, solvable groups. Ring theory: unique factorization in polynomial rings and principal ideal domains. Field theory: ruler and compass constructions, roots of unity, finite fields, Galois theory, solvability of equations by radicals.

Linear Algebra
Instructor: Professor Bill Heinzer
Course Number: MA 55400
Credits: Three
Time: 3:30–4:20 PM MWF

Catalog Description

Review of basics: vector spaces, dimension, linear maps, matrices determinants, linear equations. Bilinear forms; inner product spaces; spectral theory; eigenvalues. Modules over a principal ideal domain; finitely generated abelian groups; Jordan and rational canonical forms for a linear transformation.

Abstract Algebra II
Instructor: Professor Vaibhav Pandey
Course Number: MA 55800
Credits: Three
Time: 12:00–1:15 PM TTh

Description

This course is a continuation of MATH 557. The course will cover topics in dimension theory, regular sequences, Koszul and local cohomology, Cohen Macaulay rings, and Gorenstein rings. Following this, we will give an introduction to the theory of tight closure and relook at Cohen-Macaulay and Gorenstein rings through the lens of tight closure techniques.

Emphasis will be laid on calculating examples and possibly presentation of particular topics by students towards the end of the course. Weekly homeworks will be assigned.

The course should be accessible to anyone who has done MATH 557 or has a working knowledge of the text in Atiyah–McDonald.

No particular book will be followed, through broadly we will follow Bruns–Herzog for the general theory.

Introduction To Differential Geometry And Topology
Instructor: Professor Harold Donnelly
Course Number: MA 56200
Credits: Three
Time: 9:30–10:20 AM MWF

Description

Smooth manifolds; tangent vectors; inverse and implicit function theorems; submanifolds; vector fields; integral curves; differential forms; the exterior derivative; DeRham cohomology groups; surfaces in E3., Gaussian curvature; two dimensional Riemannian geometry; Gauss-Bonnet and Poincare theorems on vector fields.

Textbook: William Boothby, An Introduction to Differential Manifolds and Riemannian Geometry, Second Edition, 2003, first six chapters

Prerequisites: several variable calculus, linear algebra, basics of general topology

Introduction To Algebraic Topology
Instructor: Professor Xingshan Cui
Course Number: MA 57200
Credits: Three
Time: 1:30–2:45 PM TTh

Description

The course will be an introduction to algebraic topology, which is not only a basic tool in topology, but is also important in many other fields such as differential geometry, algebraic geometry, number theory, mathematical physics, data science, quantum information, etc. The main focus is on homology and cohomology. Along the way, we will cover CW-complexes, Universal Coefficient Theorem, Kunneth Theorem, Poincare duality, basics of category theory, basics of homological algebra, and (if time permits) an introduction to 3-/4-manifolds.

Introduction To Additive Combinatorics
Instructor: Professor Ilya Shkredov
Course Number: MA 59500AC
Credits: Three
Time: 4:30–5:45 PM TTh

Description

Additive combinatorics is a rapidly developing new field of modern mathematics, lying between number theory and combinatorics. A variety of tools are used such as (besides number theory and combinatorics) dynamical systems, computer science, probability, geometry, algebra and so on. Roughly speaking, additive combinatorics is the field that studies combinatorial problems that can be expressed through the group operation.

To get an idea of additive combinatorics, you can refer to the first result in this area, namely the famous Cauchy theorem (1813) concerning addition in Z/pZ, which says that the power of the sum $A + B := a + b : a \in A, b \in B$ of two sets A, B from Z/pZ is either p or at minimum $|A| + |B| - 1$. Thus, we have a general combinatorial statement for arbitrary sets, but this combinatorics includes the group operation +. Other results of additive combinatorics are those of van der Waerden theorem on arithmetic progressions (which Khintchin called "a pearl of number theory"), Freiman's structural result on sumsets, the amazing Green-Tao theorem on arithmetic progressions in the prime numbers, Bourgain-Glibichuk-Konyagin theorem on the uniform distribution of multiplicative subgroups and many others.

In this course we plan to introduce you to the fundamental results of the area and describe some relationships and connections of additive combinatorics with number theory, combinatorics, ergodic theory, graph theory, Fourier analysis, geometry and other branches of mathematics.

Extended Program:

1. Introduction, coloring problems.

2. Combinatorial ergodic theory and the regularity lemma.

3. Sumsets and difference sets.

4. Applications of Fourier analysis to additive combinatorics.

5. Sets having no arithmetic progressions of length three.
6. Bohr sets and the spectrum.
7. Almost periodicity.
8. Freiman's theorem on sets with small doubling.
9. The sum-product phenomenon: the real case.
10. The sum-product phenomenon: the finite fields case.
11. Gowers norms.
12. Multiplicative combinatorics.

Book: Terence Tao and Van H. Vu, Additive combinatorics

Prerequisites: 16*** (first year calculus).

All levels, undergraduate/graduate.

Elliptic Curves
Instructor: Professor Daniel Le
Course Number: MA 59500EC
Credits: Three
Time: 12:30–1:20 PM MWF

Description

Elliptic curves are the simplest "non-trivial" objects across a wide swath of subjects including complex geometry, algebraic geometry, algebraic topology, and number theory. They correspondingly played a large role in the development of all of these. In the course, elliptic curves will serve as examples of or as an introduction to general phenomena in these areas. The course will start with analytic aspects before moving to algebraic and arithmetic aspects. Our focus will be on (co)homology of elliptic curves in various guises. Prerequisites are complex analysis and field theory. Some knowledge of commutative algebra, algebraic number theory, and algebraic varieties is recommended.

Geometry Of Characteristic Classes
Instructor: Professor Sam Nariman
Course Number: MA 59500GCC
Credits: Three
Time: 12:00–01:15 PM TTh

Description

Characteristic classes are central to the modern study of the topology and geometry of manifolds. They were first introduced in topology, where, for instance, they could be used to define obstructions to the existence of certain fiber bundles. Characteristic classes were later defined using connections on vector bundles, thus revealing their geometric side.

This course aims to introduce the students to the three theories of characteristic classes. They include characteristic classes of flat bundles, characteristic classes of foliations, and characteristic classes of surface bundles. We will mainly follow a book and a lecture note by Shigeyuki Morita. For the characteristic classes of surface bundle, if time permits we will discuss the Madsen-Weiss theorem.

We assume some basic knowledge in algebraic topology, differential topology in particular de Rham forms, manifold, fiber bundle, homotopy groups.

Geometric Measure Theory
Instructor: Professor Monica Torres
Course Number: MA 59500GMT
Credits: Three
Time: 12:30–1:20 PM MWF

Description

Geometric Measure Theory is widely applied to many areas of Analysis and Partial Differential Equations. This class is an introduction to Geometric Measure Theory and is composed of two parts:

In Part I we will study Hausdorff measures, Besicovitch's covering theorem, differentiation of measures, Lipschitz functions and Rademacher's theorem, Rectifiable sets and blow-ups of Radon measures, the area formula, sets of finite perimeter, compactness of sets of finite perimeter, reduced boundary and De Giorgi's structure theorem, coarea formula, isoperimetric inequality. With all these techniques at hand, we can show the existence of minimizers of important geometric variational problems (i.e. minimal surface).

In Part II of the class we will introduce methods of geometric measure theory to study regularity of minimizers, including the monotonicity formula. We will obtain Lipschitz continuity and $C^{1,\alpha}$ regularity of local perimeter minimizers. Analysis of singularities and the Federer's dimension reduction argument will also be discussed. If time permits, we will include other important topics in geometric measure theory such as functions of bounded variation, and their corresponding traces and Gauss-Green formulas on rough domains (i.e; sets of finite perimeter).

Prerequisites: Basic measure theory (as in MA544 or equivalent).

Introduction to Quantum Computing
Instructor: Professor Eric Samperton
Course Number: MA 59500IQC = CS 59300
Credits: Three
Time: 10:30–11:45 AM TTh

Description

An introduction to the theory of quantum computation focused primarily on foundations, theory, and rigor, rather than specific hardware implementations or heuristic applications. We will begin with the axioms of quantum mechanics and the most common formulation of quantum computation based on quantum circuits. We will then develop the core primitives in the quantum algorithms toolkit (such as quantum Fourier transforms, phase estimation, and Trotterization/quantum simulation) and establish some elementary complexity-theoretic results (including some oracle separations, and various lower and upper bounds), as well as work through the crown jewel of quantum algorithms to date—Shor's factoring algorithm. Along the way, we will see some of the more curious aspects of quantum information facilitated by quantum entanglement (such as Grover search, quantum teleportation, superdense coding, Bell violations). The last portion of the course will develop the basic theory of quantum error-correcting codes and the fault tolerance problem.

In particular, you may want to note that I *do not* plan to cover quantum optimization, quantum machine learning, or post-quantum cryptography in any depth (if at all).

Textbook: Quantum Computation and Quantum Information, Nielsen and Chuang.

Prerequisites: Some first-year and most second-year graduate students in CS, physics or mathematics should be comfortable in the class. Familiarity with at least one of the following at the level of a first year graduate student will be expected of all students: CS theory, quantum mechanics, abstract algebra, functional analysis, linear algebra. Please inquire with the instructor if you have any questions about whether this course is appropriate for you.

Quantum Groups and Applications

Instructor: Professor Oleksandr Tsymbaliuk
Course Number: MA 59500QGA
Credits: Three
Time: 1:30–2:20 PM MWF

Description

This is an introductory course to (finite) quantum groups, the subject ubiquitous in various branches of modern mathematics and mathematical physics. The course is expected to fit a wide variety of math/physics students: both graduate as well as advanced undergraduate.

The theory of quantum groups originates from the math-physics work of Faddeev's school in mid 80s in the study of quantum inverse scattering method.

The theory was soon generalized in the papers of Drinfeld and Jimbo through the formalism of Hopf algebras created by algebraic topologists in the middle of the 20th century. The basics of this theory were outlined in Drinfeld's 1986 ICM talk, which still remains an excellent reference on the subject.

While being of independent interest, this subject finds many astonishing applications in other areas of mathematics and physics: algebraic combinatorics, algebraic geometry, algebraic topology, category theory, differential equations, harmonic analysis, integrable systems, knot theory, representation theory, quantum field theory, quantum computations, and others.

Tentative list of topics: coalgebras, bialgebras, modules and comodules; Hopf algebras, examples of $U(\mathfrak{sl}_n)$ and $\mathbb{C}[SL(n)]$, and the pairing between them; quantum plane, quantum $SL_q(2)$, and quantum $U_q(\mathfrak{sl}_2)$; quantum groups $U_q(\mathfrak{g})$ and their Hopf algebra structure; finite dimensional representations of $U_q(\mathfrak{g})$ for $q \neq \sqrt{1}$; center of $U_q(\mathfrak{g})$ and non-degenerate pairings; R-matrices, Yang-Baxter equation, and Faddeev-Reshetikhin-Takhtajan construction; universal R-matrices and Drinfeld's quantum double; tensor categories and tensor functors; braid group action, root vectors, and PBW-type basis; two integral forms of $U_q(\mathfrak{g})$ and $q = \sqrt{1}$ case; tangle category and its representations, leading to the Jones polynomials; crystal bases.

Prerequisites: Familiarity with basic results on Lie algebras will be helpful. Familiarity with basic notions of category theory may be useful for the second part of the course.

Introduction to Type III Factors
Instructor: Professor Thomas Sinclair
Course Number: MA 59500TF
Credits: Three
Time: 3:30–4:20 PM MWF

Description

This course will serve as an introduction to the theory of von Neumann algebras, focusing on factors of type III. We will give an elementary account of

Tomita–Takesaki theory, including the theory of the modular automorphism group, the noncommutative flow of weights, and the classification of type III_λ factors $0 \leq \lambda \leq 1$. Examples and constructions from dynamical systems and mathematical physics will be discussed. Time permitting, a selection of topics such as Araki's notion of relative entropy, Cuntz algebras, the axiomatization of Algebraic Quantum Field Theory, and connections with quantum information theory will be presented.

References:

- U. Haagerup, "Tomita–Takesaki Theory for Pedestrians"
- M. Rieffel and A. van Daele, "A bounded operator approach to Tomita–Takesaki theory"
- M. Takesaki, "Theory of Operator Algebras II"

Numerical Methods For PDEs I

Instructor: Professor Xiangxiong Zhang
Course Number: MA 61500
Credits: Three
Time: 1:30–2:45 PM TTh

Description

This is an introductory course of numerical solutions to partial differential equations for any graduate students interested in computational mathematics, with emphasis on breadth rather than depth. The course will cover key concepts with a balance between analysis and implementation, including accuracy, stability and convergence of finite difference methods for time-dependent problems such as wave equations, parabolic equations and conservation laws. The finite element method for elliptic equations on structured meshes will also be introduced. Linear system solvers such as the conjugate gradient method and the multigrid method, and ODE solvers such as

Runge-Kutta method will also be discussed, if time permits. Homework and the take-home final exam will consist of both analysis and coding by Matlab. Sample Matlab codes will be provided to assist beginners, thus no prior knowledge of coding is required. Recommended prerequisites include linear partial differential equations, linear algebra, and Fourier analysis, all of which will be reviewed during the lectures. Feel free to send an email to zhan1966@purdue.edu for any questions. The lecture notes in previous years are available at

http://www.math.purdue.edu/~zhan1966/teaching/615/MA615_notes.pdf

Methods of Linear and Nonlinear Partial Differential Equations II

Instructor: Professor Changyou Wang
Course Number: MA 64300
Credits: Three
Time: 3:00–4:15 PM TTh

Description

This is the continuation of MA64200. We plan to cover the Moser-Nash-De Giorgi continuity theory and the Calderon-Zygmund Lp-theory for second order uniformly elliptic equations with diver-gence structures. Introduction of the theory of viscosity solutions to elliptic equations introduced by P. Lions, M. Crandall, L. Evans, and others. If time is permitted, some nonlinear problems will be discussed.

Textbook and References

1. *Elliptic Partial Differential Equations of Second Order*, 2nd edition, Springer, Berlin, by David Gilbarg and Neil S. Trudinger.
2. *Elliptic Partial Differential Equations*, Courant Lecture Notes Math., 1, by Qing Han and Fanghua Lin.
3. *Fully Nonlinear Elliptic Equations*, American Mathematical Society Colloquium Publication, 43, by X. Cabre and L. Caffarelli.

4. *User's guide to viscosity solutions of second order partial differential equations*, Bull. Amer. Math. Soc. 27 (1992), no.1, 1-67, by M. Crandall, H. Ishii, and P. L. Lions.

Algebraic Geometry II

Instructor: Professor Donu Arapura
Course Number: MA 66500
Credits: Three
Time: 10:30–11:45 AM   TTh

Description

The one sentence definition of algebraic geometry is that it is the study of spaces defined by polynomial equations. However, this description is a bit misleading. Like a lot of mathematics, the subject underwent a paradigm shift in the mid 20th century with the introduction of abstract methods, and in particular, of methods from homological algebra. Topics such as the classical Riemann-Roch theorem were reinterpreted in this language, and then massively generalized.

This is ostensibly a second semester algebraic geometry class. So I will assume that people are reasonably comfortable with basic algebraic geometry, or things close to it, such as real or complex manifold theory. I plan to develop sheaf cohomology more or less from scratch, and then apply it to algebraic geometry and surrounding areas. For example, de Rham's theorem is fairly easy to prove using this machinery, as is the Riemann-Roch theorem mentioned above. I also plan to cover the basics of complex algebraic geometry. Some things, such as the genus of a curve or Jacobian varieties, are much easier to understand over $\mathbb{C}$.

If you are unsure whether you have the background, then just ask me. I will not follow any book very closely, but some useful references are:

1. Griffiths, Harris, Principles of Algebraic Geometry

2. Hartshorne, Algebraic Geometry

3. Voisin, Hodge theory and Complex Algebraic Geometry I, II