Advanced Graduate Courses
offered by the
Mathematics Department
Fall, 2001

MA 546: Introduction to Functional Analysis
Instructor: Prof. L. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
Time: MWF 10:30
Description: Mathematics is often divided into three disciplines: algebra, geometry, and analysis. The division is confusing in that analysis is an approach to mathematics characterized by logical thinking rather than a part of mathematics. In that sense analysis touches on all mathematics. It is therefore surprising that a fundamental course in analysis should not be a prerequisite for admission of doctoral candidates in mathematics. The reasonable explanation is that analysis is considered so difficult that such a prerequisite might reduce the number of doctoral candidates. The use of the adjective "functional" in connection with analysis adds to the confusion. Whatever logical meaning the word may have is subordinated to the aura of mystery which it connotes. The present course is offered in the belief that the Hahn–Banach theorem is the most fundamental mathematical discovery of the twentieth century. The theorem underlies all effective applications of real and complex vector spaces, even in finite dimensions. The proof of the theorem is nonconstructive since the axiom of choice is used. Analysis is applied in an axiomatic context requiring a high level of abstraction. The theorem is to modern mathematics what the Pythagorean theorem was to the mathematics of ancient Greece. This stumbling block to a mathematical career was known as a *pons assinorum*, a bridge over which asses could not pass. Modern teaching of mathematics make teachers, not students, responsible for failures in learning. The aim of the present course is careful teaching of the Hahn–Banach theorem which makes it available to all earnest students. The aim of the course is also to prepare applications which will be continued in a second semester. The Hahn–Banach theorem is often applied in conjunction with the Brouwer fixed–point theorem, whose formulation in spaces of infinite dimension is due to Ky Fan. An important application is an existence theorem for invariant subspaces which generalizes the Burnside theorem in spaces of finite dimension. Another major application is made to the structure problem for measures on Hausdorff spaces which are not locally compact. These applications are not only fundamental to mathematics but also to physics, engineering and economics. All serious students of analysis are welcome.

MA 585: Mathematical Logic I
Instructor: Prof. O. De la Cruz, office: Math 446, phone: 49–47912, e-mail: odlc@math.purdue.edu
Time: TTh 1:30-2:45
Description: This course is an introduction to several of the central fields of Mathematical Logic.

Mathematical Logic has a dual character: On one hand, it has great philosophical value, being a framework for the study of the foundations of Mathematics and Computer Science. On the other hand, it is a very lively area of research in Mathematics, and many of its leading scientists sometimes prefer to sidestep the purely philosophical concerns. The most amazing fact is that the answer to questions that could initially be considered pure philosophical turn out to have great repercussions in the mathematical development of the theory, and viceversa. In this course we will concentrate on the mathematical issues concerning contemporary logic, but both the philosophical motivations and the applications to Theoretical Computer Science will be strongly represented.

We will start with a brief introduction to "naive" Set Theory, in order to uniformize our notation and to provide a motivation for some of the formal theories to be used later as examples. Also, we will quickly introduce the theory of Boolean algebras.

After these introductory matters, we will get into the central themes. First, we will study Propositional (or Sentential) Logic, which in its simplicity provides a a good picture of the methods and goals of Logic. Then we will introduce the First Order Logic, which is the main subject of the course. First Order Logic is both fairly simple and very powerful, and we will study its formal (or syntactic) aspects, and its interpretations, or semantics (that is, its model theory). The main result that we will obtain in this direction is Gödel's Completeness Theorem.

After that, in order to study the limitations of First Order Logic, we will develop a Theory of Computability, stated in terms of Turing machines and in other syntactical versions. We will study the interplay between computability and some theories of arithmetics, and the main result of this will be Gödel's Incompleteness theorems.

If there is extra time, we will consider some extra topics, to be chosen according to the interests of the students (as well as mine). Some possibilities are: Many Valued Logics, Non-standard Analysis, Finite Models and Computational Complexity. Other topics can be considered also.
Texts: 1. Enderton, /it A Mathematical Introduction to Logic, Harcourt, 2nd Ed., 2001
2. Epstein/Carnielli, /it Computability, Wadsworth & Brooks/Cole, 2nd Ed., 1999.
The cost of these two books together is still lower than the cost of another usual textbook (Mendelson's). And these two books complement each other very nicely, providing both the "hard math" point of view and a more philosophical approach which connects with Theoretical Computer Science.

MA 598A Introduction to Algebraic Number Theory
Instructor: Prof. J. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA 553
Description: This will be an introduction to the basic theory of algebraic number fields (= finite extensions of the rational field $\mathbb{Q}$) and their rings of integers. Topics include Dedekind rings, unique factorization into primes, ramification theory, quadratic and cyclotomic fields, finiteness of class number, Dirichlet's unit theorem, valuations, completions and the product formula. Time permitting, we may also do decompostion groups and the Artin map (Chapter III of text).
Text: G. J. Janusz, *Algebraic Number Fields*, 2nd edition, Amer. Math. Soc., 1996.

MA 598B Spectral Sequences and Fibrations
Instructor: Prof. C. Wilkerson, office: Math 450, phone: 49–41955, e-mail: wilker@math.purdue.edu
Time: MWF 9:30
Prerequisite:
Description: Introduction to Fibrations and Spectral Sequences. Topics: Kunneth theorems in cohomology, cup products, Tor and Ext, fibrations on generalized products, the Serre Sequence, Steenrod operations, elementary group cohomology, Eilenbert–MacLane spaces.

MA 598C: Numerical Methods for Partial Differential Equations
Instructors: Prof. J. Douglas, office: Math 822, phone: 49–41927, e-mail: douglas@math.purdue.edu
Time: TTh 1:30-2:45
Prerequisite: MA 523 or consent of instructor
Description: This course is designed for two semesters to replace the original one semester course on finite element methods (Math 524). The goal of this course is to teach the basic methodology for developing accurate, robust, and efficient algorithms for the numerical solution of partial differential equations in applied mathematics, science and engineering. The course will provide the mathematical foundation of numerical methods together with important numerical aspects. Applications to some basic problems in mechanics and physics will also be considered.
Fall Semester 2001. The course will begin with finite difference and finite element methods for two-point boundary value problems and direct and iterative methods for the resulting algebraic equations. Finite difference and finite element methods will be developed and analyzed for elliptic and parabolic partial differential equations. Iterative solvers including preconditioned conjugate gradient, domain decomposition, and multigrid methods will be introduced for the resulting system of linear and nonlinear equations from the discretizations of elliptic and parabolic problems. Finally, we will discuss numerical methods for hyperbolic partial differential equations. Some implementational aspects will be considered.
Spring Semester 2002. The second semester will begin with polynomial approximation theory in Sobolev spaces. We will then develop and analyze mixed finite element methods for both elliptic and parabolic equations. As a special case of the mixed method, we will introduce the finite volume method. Domain decomposition and/or multigrid methods will be further studied. Topics on advanced methods such as methods of characteristics, least squares, and adaptive mesh refinement and on applications such as incompressible Stokes and Navier-Stokes, elasticity, Maxwell, porous media, and pseudo-differential equations are at the discretion of the instructor. These are current topics of very active research in computational mathematics.

MA 598D: Computational Financial Mathematics
Instructor: Prof. S. Stojanovic, e-mail: srdjan@math.uc.edu
Time: TTh 12:00-1:15
Description: Syllabus: Ordinary differential equations on Mathematica; modeling stock price dynamics: stochastic differential equations; European options - symbolic solutions of the Black-Scholes partial differential equation and its extensions; stock market statistics - stock market data manipulation and statistical analysis; implied volatility for European options - option market data and calibration of the Black-Scholes model - implied volatility via optimal control problem for the Dupire partial differential equation; American options - fast numerical solutions and optimal control of obstacle problems; optimal portfolio rules - symbolic solutions of stochastic control problems in portfolio management.
Teaching Style: Sophisticated theories are presented in practical style, which with the help of the programming capabilities of Mathematica, help the students develop good intuition about the real trading of stocks and options, as well as about the wide variety of the mathematics involved.
Text: S. Stojanovic, *Computational Financial Mathematics*, Birkhauser, Boston, May, 2001

MA 598G Introduction to Combinatorics
Instructor: Prof. V. Gasharov, e-mail: vesko@math.cornell.edu
Time: MWF 2:30
Prerequisite: Linear Algebra
Description: The goal of this course is to present a broad selection of topics in combinatorics, including partitions, compositions, permutation statistics, generating functions, the principle of inclusion-exclusion, symmetric functions, Littlewood-Richardson rule, Robinson-Schensted-Knuth correspondence, simplicial complexes, Kruskal-Katona theorem.
Text: 1. T. Hibi, *Algebraic combinatorics on convex polytopes*
2. J. H. van Lindt and R. M. Wilson, *A course in combinatorics*
3. R. Stanley, *Enumerative combinatorics,* volume 2

MA 598F: Mathematics of Finance
Instructor: Prof. F. Viens, office: Math 504, phone: 49–46035, e-mail: viens@stat.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: MA 519 (or equivalent) + MA 261 (or equivalent) + MA 440 (or equivalent); or consent of the instructor.
Description: We will provide an introduction to the mathematical tools and techniques of modern finance theory, in the context of Black-Scholes-style option pricing. The typical (pricing) question is: how much should you charge someone for allowing them the right to purchase a certain stock from you at a given price and given time in the future? The typical (Black-Scholes) assumption is that the differential of the (log of the) stock price is the sum of a constant term (r.dt, constant interest rate) and a random noise term (dW(t), a Brownian increment). Under this assumption, to answer the pricing question, the main mathematical tool is stochastic calculus and its connection to partial differential equations. These mathematics will be the object of a thorough introduction at an elementary level, without measure theory. This toolbag will enable us to derive the main pricing and hedging results in complete and incomplete markets, and to treat many examples of exotic and path-dependent options, as well as an introduction to stochastic optimal control and portfolio optimization.
Text: T. Bjork, *Arbitrage Pricing in Continuous Time*, Oxford, 1998.
Suggested additional reading: D. Lambertson and B. Lapeyre, *Stochastic Calculus applied to Finance*, Chapman and Hall/CRC, 1996, reprinted 2000 by CRC Press.; P. Wilmott, S. Howison, J. Dewynne, *The mathematics of financial derivatives. A student introduction.* Cambridge U.P. 1995. Chapters 11 to 16.

MA 620: Mathematical Theory of Optimal Control
Instructor: Prof. L. Berkovitz, office: Math 700, phone: 41936, e-mail: brkld@math.purdue.edu
Time: MWF 8:30
Prerequisite: MA 544
Description: The course will be concerned with the mathematical theory of optimal control problems for systems governed by ordinary differential equations. The topic outline for the course is as follows: Examples of contral problems from applied areas. Existence theorems. The Pontryagin Maximum Principle. Linear systems, Linear quadratic problems, Time optimal problems. Relationship to the calculus of variations. Hamilton–Jacobi theory and optimal feedback The "text" will be notes for a projected revised edition of my 1974 book *Optimal Control Theory*, Applied Mathematical Sciences, Vol. 12, Springer Verlag. An overview of the course content can also be gotten from L. Cesari, *Optimization – Theory and Applications. Problems with Ordinary Differential Equations*, Springer Verlag, 1983.

MA/STAT 638: Stochastic Processes I
Instructor: Prof. J. Ma, office: Math 620, phone: 49–41973, e-mail: majin@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA/STAT 538/539, or consent of the instructor.
Description: MA/STAT 638-639 is a two semester coaurse I plan to cover various topics ranging from measure theoretic probability to stochastic differential equations, with an eye on the applications to stochastic control and mathematical finance.

Detailed topics include:

A. Some complements to measure theory: analytic sets, capacities, measurability of debuts, ...
B. Some general properties of stochastic processes: regularity of paths, optional and predictable times, quasi-left- continuous filtrations, cross section theorem(s)...
C. Martingale theory and stochastic integrations: optional sampling, Doob-Meyer decomposition, stochastic integration (for general martingales), Ito's formula(s), Martingale representation theorem, Girsanov transformations,...
D. Stochastic differential equations: general theory on stochastic differential equations, strong solutions, weak solutions, backward stochastic differential equations,...
E. (optional) Applications of stochastic differential equations: basics in stochastic control, linear and nonlinear filtering, finance,...

This course will provide a solid foundation and many necessary tools for further study in the directions of stochastic (ordinary and partial) differential equations, stochastic control, stochastic finance theory, and any subject involving stochastic analysis.

MA 642: Methods of Linear and Nonlinear Partial Differential Equations
Instructor: Prof. N. Garofalo, e-mail: garofalo@math.purdue.edu
Time: TTh 12:00-1:15
Prerequisite: MA 523 & MA 611
Description: Second order elliptic equations including maximum principles, Harnack inequality, Schauder estimates, and Sobolev estimates. Applications of linear theory to nonlinear problems.

MA 665: Algebraic Geometry
Instructor: Prof. S. Abhyankar, office: Math 432, phone: 49–41933, e-mail: ram@math.purdue.edu
Time: TTh 3:00-4:15
Description: Various topics of current interest will be discussed such as resolution of singularities, Jacobian problem, and computation of Galois groups. There are no prerequisites. All students are welcome.

MA 690A: Introduction to Algebraic Geometry
Instructor: Prof. D. Arapura, office: Math 642, phone: 49–41983, e-mail: dvb@math.purdue.edu
Time: TTh 1:30-2:15
Description: This is a basic course in algebraic geometry which seems long overdue. I'll try and follow what could be called a semiclassical approach, in that I'll emphasize the study of the basic examples of curves, surfaces, algebraic groups, Grassmanians … rather than general scheme theory (but of course, we will develop some technical machinery along the way). I most likely will not have a chance to do any cohomology theory.

As for prequistes, I would like students to have had some prior exposure to commutative algebra at the level of Atiyah-Macdonald, and algebraic curves or Riemann surfaces or manifolds.
Text: Joe Harris *Algebraic geometry*

MA 690B: Topics in Commutative Algebra
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49–41980, e-mail: heinzer@math.purdue.edu
Time: MWF 12:30
Prerequisite: MA 557
Description: The course will cover selected topics in commutative algebra concerned with the generation of modules and ideals and the number of equations needed to describe an algebraic variety. It will also cover:

(1) properties of extensions rings such as flatness,
(2) valuation rings,
(3) dimension and multiplicity theory.

Text: H. Matsumura *Commutative Ring Theory*, Cambridge University Press.

MA 690C: Introduction to Elliptic Curves
Instructor: Prof. K. Matsuki, office: Math 614, phone: 49–41970, e-mail: kmatsuki@math.purdue.edu
Time: MWF 11:30
Description: From the early attempt to express elliptic integrals in terms of elementary functions and understanding of its inevitable failure in terms of homology, the subject of elliptic curves has been the central merging point of analysis, topology and number theory, whose origins may be traced back to the subject to begin with.

The main purpose of this course is to give a beginner's guide to this subject from an Algebraic Geometry point of view, supplementing a course in basic Algebraic Geometry offered by Prof. Arapura.

We will start with discussing the classical theory, such as uniformization by Weierstrass p-functions and the group structure given by Chord–Tangent Law, progressing toward the end (hopefully) to discuss more advanced topics such as modular forms.

Prerequisites are a good understanding of algebra and complex analysis at the level of MA 553 and MA 530.

I hope this course will serve as an introduction to Algebraic Geometry even to the students whose main interests are not Algebraic Geometry.

MA 690D: Automorphic L–Functions
Instructor: Prof. F. Shahidi, office: Math 650, phone: 49–41917, e-mail: shahidi@math.purdue.edu
Time: MWF 10:30
Description: The course will include a proof of the functional equation from all the L–functions obtained from the method of Langlands and myself, by developing the theory of local coefficients and computing them in general over real and complex numbers, as well as the notion of multiplicativity for them and so on. These are fundamental recent results proving a large number of cases of functionality using this method which are not available from any other method at present.

MA 696A: Mathematical Foundation of Classical Mechanics
Instructor: Prof. L. Lempert, office: Math 734, phone: 49–41952, e–mail: lempert@math.purdue.edu
Time: MWF 9:30
Prerequisite: Some exposure to analysis in $\mathbb{R}^n$ and to ordinary differential equations (MA 303 or 304); in the latter half of the course some differential geometry such as manifolds and tangent bundles.
Description:

All through its history mathematics has been greatly influenced by problems in mechanics, celestial or otherwise. Mechanics has been instrumental in the development of Riemannian and symplectic geometry, ordinary and partial differential equations, dynamical systems, etc. While it is now accepted that the physical world is described by quantum (and more advanced) physics rather than classical mechanics, modern physical theories are still built on mechanics.

The course will introduce the audience to fundamental notions of the subject and to some highlights, including rather recent ones.

Contents: Newtonian description of mechanics, conservative systems, motion in central fields, Kepler's problem. Lagrangian mechanics, principle of least action, Legendre transformation, Hamiltonian, Noether's theorem, preservation of phase volume, Poincare's recurrence theorem. Constraints, mechanics on manifolds, geodesics on surfaces of revolution. Small oscillations. Hamiltonian mechanics, symplectic manifolds, Gromov's uncertainty principle. Hamilton–Jacobi theory, generating functions, complete integrability, Kolmogorov–Arnold–Moser theory.
Reference: Arnold, *Foundation of Classical Mechanics*, Springer.