Seminars and Advanced Graduate Courses
offered by the
Mathematics Department
Fall, 2004

Courses

MA 531: Functions of a Complex Variable II
Instructor: Prof. Weitsman, office: Math 836, phone: 49–41909, e-mail: weits@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 530
Description: This course is intended for any student in analysis. It will cover most of chapters 11–20 of the text with some supplemented material. There will be homework sets and no exams.
Text: Rudin, *Real and Complex Analysis*

MA 542: Theory of Distributions and Applications
Instructor: Prof. SaBarreto, office: Math 604, phone: 49–41965, e-mail: sabarre@math.purdue.edu
Time: MWF 2:30
Prerequisite: For some reason the catalog says MA 510 and MA 525. I think this is inadequate. One should know what uniform convergence is and basic measure theory, at least what L^2 is.
Description: The theory of distributions is a fundamental concept in the modern theory of partial differential equations. This is an introductory course where we will study distribution solutions to differential equations, fundamental solutions, the Fourier transform, Sobolev spaces, Paley-Wiener theory and the rudiments of the theory of pseudodifferential operators.
Text: F.G. Friedlander and M.S.Joshi, *Introduction to the Theory of Distributions*, Cambridge Univ. Press, second edition, 1998.

MA 546: Introduction to Functional Analysis
Instructor: Prof. L. Brown, office: Math 704, phone: 49–41938, e-mail: lgb@math.purdue.edu
Time: MWF 12:30
Prerequisite: MA 544
Description: The course covers basic functional analysis with emphasis on bounded linear operators on Banach and Hilbert spaces. The topics listed below will be covered, and brief treatments of other topics fitting the interests of the students can be included as time permits.

Banach spaces and Hilbert spaces; Hahn-Banach theorem; closed graph and open mapping theorems; uniform boundedness principle; theory of spectrum for operators on Banach spaces and for compact operators; weak and weak* topologies; reflexivity; Hahn-Banach separation theorem and double polar theorem; operators on Hilbert spaces; spectral theorem for bounded self-adjoint operators on Hilbert spaces.
Text: M. Schechter, *Principles of Functional Analysis.*

MA 557: Abstract Algebra I
Instructor: Prof. Heinzer, office: Math 636, phone: 49–41980, e-mail: heinzer@math.purdue.edu
Time: MWF 3:30
Description: I plan to cover material from the text *Introduction to Commutative Algebra* by M. F. Atiyah and I. G. Macdonald. In particular the course will cover: rings and ideals, modules, rings and modules of fractions, primary decomposition, integral dependence and valuations, chain conditions, Noetherian rings, Artin rings, discrete valuation rings and Dedekind domains, completions and dimension theory.
Text: M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra*

MA 562: Introduction to Differential Geometry and Topology
Instructor: Prof. Catlin, office: Math 744, phone: 49–41958, e-mail: catlin@math.purdue.edu
Time: MWF 8:30
Prerequisite: MA 351 and MA 442
Description: Topics – Smooth manifolds, tangent vectors, inverse and implicit function theorems, submanifolds, vector fields, integral curves, differential forms, the exterior derivative, partitions of unity, integration on manifolds, fundamentals of Riemannian geometry, Levi–Civita connection, parallel transport, geodesics, curvature tensor.
Text: William Boothby, *Introduction to Differentiable Manifolds and Riemannian Geometry*

MA 571: Elementary Topology I
Instructor: Prof. McClure, office: Math 714, phone: 49–42719, e-mail: mcclure@math.purdue.edu
Time: MWF 1:30
Prerequisite: basic knowledge of metric spaces including compactness
Description: MA 571 will be redesigned for this fall. There will be less emphasis on point-set topology and more emphasis on the fundamental group and covering spaces. There will also be a careful treatment of the topology of surfaces, leading up to a proof of the classification theorem for compact surfaces.
Text: Munkres, *Topology* (the course will cover Chapters 2, 3, 9, 12 and 13, with possible additional topics)

MA 598B: Manifolds and Characteristic Classes
Instructor: Prof. Wilkerson, office: Math 450, phone: 49–41955, e-mail: wilker@math.purdue.edu
Time: MWF 2:30 NOTE NEW TIME
Description: This course is intended as an introduction to the algebraic topology of manifolds and vector bundles. Topics will include

1. Definitions and examples of differential manifolds.
2. Poincare duality for manifolds
3. Tangent and normal bundles of manifolds
5. The Thom class class, Euler class, and Steifel-Whitney classes
6. Universal bundles and classifying spaces
7. Characteristic classes as cohomology of classifying spaces
8. Characteristic classes and higher structures (almost complex, spin, etc).
9. Cobordism of real and complex manifolds
10. Introduction to K-theory of vector bundles

Students will be expected to do at least one presentation during the semester and participate in class discussions.
Text: Milnor and Stasheff, *Characteristic classes*, Princeton University press.
The text will be supplemented by other readings.

MA 598E: Singularities of Differentiable Mappings
Instructor: Prof. Gabrielov, office: Math 648, phone: 49–47911, e-mail: agabriel@math.purdue.edu
Time: MWF 12:30
Prerequisite: Standard calculus sequence.
Description: The theory of singularities is concerned with local properties of differentiable maps $\mathbb{R}^n \to \mathbb{R}^m$ (or, as an important special case, differentiable functions $\mathbb{R}^n \to \mathbb{R}$). It allows one to identify and classify "generic" and "stable" singularities, often reducing their study to a finite list of simple algebraic "normal forms".

This theory has numerous applications in mathematics, mechanics, physics, and many other areas. In particular, it provides theoretical background for the "catastrophe theory" that studies loss of stability of physical, biological, economic, and social systems under gradual changes of control parameters.

The course, aimed at beginning graduate students, will introduce basic ideas of the singularity theory: transversality, stability, genericity, elements of classification of singularities.
Text: M. Golubitsky, V. Guillemin. *Stable mappings and their singularities.*
Reference: V. Arnold. Selected chapters from: *Singularities of differentiable maps.*

MA 598K: Algebraic Coding Theory
Instructor: Prof. Moh, office: Math 638, phone: 49–41930, e-mail: ttm@math.purdue.edu
Time: MWF 10:30
Prerequisite: None. MA 554 will help.
Description: There will be three parts: a) elementary coding theory. b) linear coding and polynomial coding (BCH, RS and Goppa codes) c) algebraic geometric codings.
Text: Undecided. (maybe J. H. van Lint or my own notes).

MA 598M: Introduction to Algebraic Geometry (with a view toward commutative algebra)
Instructor: Prof. Matsuki, office: Math 614, phone: 49–41970, e-mail: kmatsuki@math.purdue.edu
Time: MWF 11:30 NOTE NEW TIME
Prerequisite: MA 553, 554, and some basic knowledge of complex analysis will help but not necessary.
Description: This is an introductory course in Algebraic Geometry.

We would like to put an emphasis on how commutative algeba plays an indispensable role in analyzing the geometry of algebraic varieties.

I would like to design the course so that it will cater to the group of students in commutative algebra, and provide them with some geometric aspects.

For example, I will spend some time discussing the geometric meaning of the material covered in the popular textbook by Eisenbud, *Commutative Algebra with a View toward Algebric Geometry.*
Text: 1. Hartshorne, *Algebraic Geometry*
2. Eisenbud, *Commutative Algebra with a View toward Algebraic Geometry*

MA 598U: Introduction to Mixed and Least-Squared Finite Element Methods
Instructor: Prof. Cai, office: Math 810, phone: 49–41921, e-mail: zcai@math.purdue.edu
Time: TTh 12:00–1:15
Prerequisite: MA 598C or CS 615 or equivalent or consent of instructor
Description: The original physical equations for mechanics of continua are first-order partial differential systems. There are many advantages to simulate these first-order systems directly. This can be done through either mixed or least-squares finite element methods. This course is an introduction to both techniques, with applications to Darcy's flow in porous media, elastic equations for solids, incompressible Newtonian fluid flow, and Maxwell's equations in electromagnetic. We shall focus on fundamental issues such as (mixed and least-squares) variational formulations and finite element approximations of important function spaces H1, H(div), and H(curl). A review of fast iterative solvers such as multigrid and domain decomposition for algebraic systems resulting from discretization will also be presented.
A tentative list of contents:

1. Mathematical Models of Continuum Mechanics
2. Function Spaces H1, H(div), and H(curl)
3. Mixed Variational Formulations
4. Least-Squares Variational Formulations
5. Finite Element Approximations
6. Iterative Solvers
7. FOSPACK a computer package based on least-squares methods

References:

[1] F. Brezzi and M. Fortin, *Mixed and Hybrid Finite Element Methods*, Springer-Verlag, New York, 1991.
[2] P. Monk, *Finite Element Methods for Maxwell's Equations*, Oxford University Press, Oxford, 2003.
[3] V. Girault and P. Raviart, *Finite Element Methods for Navier-Stokes Equations: Theory and Algorithms*, Springer-Verlag, New York, 1986.
[4] W. Briggs, V. Henson, S. McCormick, *A Multigrid Tutorial*, Second Edition, SIAM, Philadelphia, 2000.
[5] research articles and my lecture notes.

MA 598Z: Applied Functional Analysis
Instructor: Prof. Lucier, office: Math 634, phone: 49–41979, e-mail: lucier@math.purdue.edu
Time: MWF 8:30
Prerequisite: A knowledge of metric spaces (MA 504) and linear algebra (MA 511).
Description: The course will cover the following topics: (1) Quick review of metric spaces; contraction mapping theorem. (2) Banach spaces—linear spaces and norms; linear functionals and the dual space; weak convergence; Hahn-Banach Theorem, Open Mapping Theorem, Uniform Boundedness Theorem. (3) Hilbert spaces—inner products; Lax-Milgram Lemma. (4) Other topics—compact operators; the spectral theorem; nonlinear operators.
Text: E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley

MA/STAT 638: Stochastic Processes I
Instructor: Prof. Viens, office: Math 504, phone: 49–46035, e-mail: viens@stat.purdue.edu
Time: TTh 3:00–4:15
Prerequisite: MA/STAT 539
Description: Advanced topics in probability theory which will include Gaussian proceses; martingales, Markov processes; stochastic calculus and stochastic differential equations; fractional Brownian motion, local time.

MA 642: Methods of Linear and Nonlinear Partial Differential Equations
Instructor: Prof. N. Garofalo, office: Math 616, phone: 49–41971, e-mail: garofalo@math.purdue.edu
Time: TTh 10:30–11:45
Prerequisite: MA 523 and MA 611.
Description: Second order elliptic equations including maximum principles, Harnack inequality, Schauder estimates and Sobolev estimates. Applications of linear theory to nonlinear equations.

MA 650: Commutative Algebra
Instructor: Prof. Ulrich, office: Math 618, phone: 49–41972, e-mail: ulrich@math.purdue.edu
Time: MWF 1:30
Prerequisite: MA 557/558 or basic knowledge in commutative algebra.
Description: This is an intermediate course in commutative algebra. The course is a continuation of MA 557/558, but should be accessible to any student with basic knowledge in commutative algebra (localization, Noetherian and Artinian modules, associated primes and primary decomposition, integral extensions, dimension theory, completion, Ext and Tor).

The topics of this semester will include: Graded rings and modules, depth and Cohen-Macaulayness, structure of finite free resolutions, regular rings and normal rings, canonical modules and Gorenstein rings, local duality.
Text: There is no specific text, but possible references are:

1. H. Matsumura, *Commutative ring theory*, Cambridge University Press
2. W. Bruns and J. Herzog, *Cohen-Macaulay rings*, Cambridge University Press
3. D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, Springer.

MA 665: Algebraic Geometry
Instructor: Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail: ram@math.purdue.edu
Time: TTh 1:30–2:45
Description: Algebraic geometry is concerned with solutions of systems of polynomial equations, and their graphical representations. This is aided by field theory, ideal theory, valuation theory, and local algebra. In the complex domain, analysis and topology also play a significant role. This course is intended as an introduction to various topics in algebraic geometry such as:

- Analysis and resolution of singularities
- Rational and polynomial parametrization
- Intersections of curves and surfaces
- Polynomial maps
- Fundamental Groups and Galois groups

The lectures will be expository in nature and so will be accessible to everyone. Thus there are no formal prerequisites and all interested students are welcome. In particular the required algebra, analysis, and topology will be developed simultaneously. The course will continue with its second part in the Spring.
Texts: 1. Shreeram S. Abhyankar, *Algebraic Geometry for Scientists and Engineers*, Amer Math Soc.
2. Shreeram S. Abhyankar, *Resolution of Singularities of Embedded Algebraic Surfaces*, Springer Verlag.

MA 690A: Transcendental Algebraic Geometry
Instructor: Prof. Arapura, office: Math 642, phone: 49–41983, e-mail: dvb@math.purdue.edu
Time: TTh 12:00–1:15
Description: The title of this course might appear to be an oxymoron, but it isn't. It refers to the study of complex algebraic varieties – sets of solutions of complex polynomials – by a combination of algebraic, analytic, differential geometric, and topological methods. If you have taken a course on elliptic curves, you will have seen this sort of thing already. Elliptic curves can be approached from both an algebraic and analytic viewpoint, and much of the beauty of the subjects stems from this interplay .

A more precise description is as follows. The first part will consist of basic sheaf theory and applications to algebraic geometry and manifold theory (De Rham and Poincaré duality). Next we do Hodge theory and its consequences (the Hodge and Lefschetz decompositions). In the third part, we will study coherent algebraic sheaves and their relation to analytic sheaves (GAGA theorems). My notes for the course "Complex algebraic varieties and their cohomology" can be downloaded off my web page:

http://www.purdue.edu/ dvb

They would provide a more precise idea of what to expect.

This is a second semester course in algebraic geometry, so I will assume some background beyond the basic courses. But it is simpler for me to discuss it with you individually than to try to list all the formal prerequisites.

MA 690G: Multiplicities and Chern Classes
Instructor: Prof. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: MWF 10:30
Prerequisite: One course in commutative algebra.
Description: This course will be about some conjectures which have driven many developments in commutative algebra over the past few decades–Serre's multiplicity conjectures, and the related homological conjectures developed by Hochster and others following the work of Peskine and Szpiro. Some essential ideas come from topology (Chern classes, intersection numbers), via intersection theory in algebraic geometry, as (re)formulated in the 1980s by Fulton and MacPherson. The associated global methods can actually be carried over to the context of local algebra, where they have enabled decisive progress.

We will work our way through the text, which is on reserve in the library, and which should be perused by anyone interested in the subject.

The course will be run like a seminar: there will be no homework or exams, but students will be expected to share in the presentation of the basic parts of the material.
Text: Paul Roberts, *Multiplicities and Chern classes in Local Algebra*

MA 690K: Class Field Theory
Instructor: Prof. Shahidi, office: Math 650, phone: 49–41917, e-mail: shahidi@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 598Y, Algebraic Number Theory, Spring, 2004 semester
Description: This course will be a continuation of Spring, 2004 course MA 598Y, taught by Prof. J.-K. Yu. Contents: Ideles, adeles, L-functions, Artin symbol, reciprocity, local and global class fields, Kronecker-Weber theorem.
Text: Serge Lang, *Algebraic Number Theory*, Addison Wesley, 1970.

MA 692B: Special Topics on Approximations of Navier-Stokes equations: Numerical Analysis and Implementation
Instructor: Prof. Shen, office: Math 806, phone: 49–41923, e-mail: shen@math.purdue.edu
Time: TTh 1:30–2:45 NOTE NEW TIME
Prerequisite: MA611 and CS514. The courses CS614 and CS615 would be helpful but not necessary.
Description: Topics:

1. Basic existence and uniqueness theory for the Stokes and Navier-Stokes equations
2. Numerical Analysis:
    - Approximations of Stokes equations — inf-sup conditions
    - Approximations of time dependent Navier-Stokes equations
        - penalty methods, artificial compressibility methods, projection methods
3. Implementation:
    - A basic spectral-Galerkin code will be provided for the Navier-Stokes equations. The students will be asked to perform simulations of some model problems such as driven cavity and spin-up.
4. Other related equations:
    - Phase-field equations: Allen-Cahn and Cahn-Hillard Coupling of Navier-Stokes equations with phase-field equations

Text and reference books: 1. R. Temam, *Navier-Stokes equations. Theory and numerical analysis.*
2. V. Girault and P.A. Raviart, *Finite element methods for Navier-Stokes equations.*

MA 693B: Riemann Hypothesis
Instructor: Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
Time: MWF 10:30
Description: A proof of the Riemann hypothesis is the underlying aim of the course. An axiomatic treatment is given of Hilbert spaces, whose elements are entire functions, which appear in the spectral theory of ordinary differential operators of first and second order. The Fourier transformation for the real line, the Laplace transformation, and the Mellin transformation are applied in the construction of Hilbert spaces of entire functions. The Radon transformation in the plane is shown to be a maximal dissipative transformation whose properties permit a proof of the Riemann hypothesis. The course will expand on lectures given at the Mathematical Institute in Paris in May 2002. The lectures are published as an appendix in the book by Karl Sabbagh on *The Riemann Hypothesis*, on reserve in the Mathematics Library. Students require a level of mathematical maturity measured by qualifying examinations in analysis.

MA 693D: Topics in Geometric Measure Theory and Partial Differential Equations
Instructor: Prof. Danielli, office: Math 802, phone: 49–41920, e-mail: danielli@math.purdue.edu
Time: TTh 9:00-10:15 NOTE NEW TIME
Description: The course is planned as a continuation of MA 598G offered this semester, but new interested students can follow it with a little extra work. Topics presented in the class will include: Theory of BV spaces (compactness, approximation with smooth functions, traces); Sets of locally finite perimeter; Regularity almost everywhere of minimal hypersurfaces; The Dirichlet problem for the minimal surface equation; The Bernstein problem; Harmonic measure and Green's function for the Laplacian; Potential theory in Lipschitz domains.
Texts: 1. E. Giusti, *Minimal Surfaces and Functions of Bounded Variation*, Birehauser, 1984.
2. C. E. Kenig, *Harmonic Analysis Techniques for Second Order Elliptic Boundary Value Problems*, AMS, 1994

MA 694C: Topics on Backward Stochastic Differential Equations
Instructor: Prof. Ma, office: Math 620, phone: 49–41973, e-mail: majin@math.purdue.edu
Time: TTh 1:30–2:45
Prerequisite: MA538, 539 or equivalent. Some exposure to partial differential equations will be helpful.
Description: This course is a continuation of the MA694A, "Introduction to Backward Stochastic Differential Equations" offered in Fall 2003, but it will be delivered in a self-contained manner. The preliminaries of BSDEs will be briefly reviewed, and the topics are then expanded to the non-standard cases including forward-backward differential equations, and those with reflections or functional terminals. A thorough study of path-regularity of the solutions and its relation with numerical methods for BSDEs/FBSDEs will be another main topic. If time permits, the applications of BSDEs and FBSDEs to mathematical finance theory will be revisited and further explored.
Text: Jin Ma and Jiongmin Yong, *Forward-Backward Stochastic Differential Equations and Their Applications*, Lecture notes in Mathematics, **1702** (1999), Springer.
Suggested reading:

1. N. Ikeda and S. Watanabe, *Stochastic Differential Equations and Diffusion Processes*, North Holland, 1981.
2. I. Karatzas and S.E. Shreve, *Brownian Motion and Stochastic Calculus*, Springer, 1987.
3. P. Protter, *Stochastic Integration and Stochastic Differential Equations*, Springer, 1990.

MA 694D: Topics on Differential Equations
Instructor: Prof. Garofalo, office: Math 616, phone: 49–41971, e-mail: garofalo@math.purdue.edu
Time: TTh 12:00–1:15
Description: This is intended as the first part of a one year long course which will develop some recent trends in PDE's arising in diverse areas, such as sub-Riemannian, or CR geometry, several complex variables, dynamical systems and mathematical finance. The main focus will be on sub-Laplacians and various nonlinear and fully nonlinear PDE's related to sub-Laplacians which are the object of intense recent attention. We will also develop the necessary metric tools, and thus in particular prove the existence of Carnot-Caratheodory metrics, and establish their main properties. Various important explicit examples will be discussed in detail, and many open problems will be presented. The course will have a completely self-contained character. Some familiarity with PDE's beyond the level of MA 523 will be helpful, but not strictly necessary.

MA 696C: Topics in Complex Geometry
Instructor: Prof. Yeung, office: Math 712, phone: 49–41942, e-mail: yeung@math.purdue.edu
Time: MWF 11:30
Prerequisite: MA 530, MA 562. Some basic understandings in algebraic geometry and several complex variables will be helpful as well.
Description: In this course, some basic techniques in complex manifolds will be studied. Tentatively, the following topics will be covered.

1. L^2-estimates and its applications and generalizations
2. Multiplier ideal sheaf and its use in complex geometry.
3. Kähler-Einstein metrics, existence, uniqueness and applications.
4. Kähler-Ricci flow in geometry.

Probably only parts of the materials would be covered, depending on the progress of the course.
The references would be given as we proceed. Here are some relevant materials.

1. Hörmander, *Introduction to complex analysis in several variables.*
2. Aubin, T., *Some non-linear problems in Riemannian geometry.*

Seminars

Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Automorphic Forms and Group Representations Seminar, Prof. Yu
Time: Thursdays, 1:30-2:30

Commutative Algebra Seminar, Prof. Ulrich
Time: Wednesdays 4:30-5:20

Computational and Applied Math Seminar, Prof. Shen
Time: Fridays 3:30

Function Theory Seminar, Prof. Weitsman
Time: Tuesdays, 3:00

Geometric Analysis Seminar, Prof. Lempert
Time: Monday 3:30

Linear and Complex Analysis Seminar, Prof. de Branges
Time: Thursday 10:30-11:20

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30-3:20

PDE Seminar, Prof. Phillips
Time: Tuesdays, 9:30-10:20

Probability Seminar, Prof. Banuelos
Time: Mondays 3:30

Spectral and Scattering Theory Seminar, Prof. SaBarreto
Time: To be determined

Topology Seminar, Prof. McClure
Time: Tuesday 1:30-2:20

Working Algebraic Geometry Seminar, Prof. Arapura
Time: Wednesday 3:30-4:30