Seminars and Advanced Graduate Courses
offered by the
Mathematics Department
Fall, 2005

Courses

MA 542: Theory Of Distributions And Applications
Instructor: Prof. Bauman, office: Math 718, phone: 49–41945, e-mail: bauman@math.purdue.edu
Time: MWF 11:30
Prerequisite: MA 544 and a knowledge of basic linear algebra
Description: Definition and basic properties of distributions; convolution and Fourier transforms; applications to partial differential equations; Sobolev spaces.
Text: Friedlander and Joshi, *Introduction to the Theory of Distributions*, Cambridge University Press, 2nd edition.

MA 543: Intrduction to the Theory of Differential Equations
Instructor: Prof. Eremenko, office: Math 450, phone: 49–41975, e-mail: eremenko@math.purdue.edu
Time: TTh 10:30-11:45
Description: Topics include: Existence and uniqueness, Sturm-Liouville theory, Dynmanical systems.

MA 546: Introduction to Functional Analysis
Instructor: Prof. Dadarlat, office: Math 708, phone: 49–41940, e-mail: mdd@math.purdue.edu
Time: MWF 1:30
Prerequisite: MA 544
Description: Banach spaces and Hilbert spaces; weak topologies; Hahn-Banach theorem; principle of uniform boundedness; open mapping theorem; Krein-Milman theorem and applications (including Stone-Weierstrass theorem). Operators on Hilbert spaces; spectral theorem for hermitian operators; Compact operators; Peter-Weyl theorem. Depending on the interest of the students, I plan to discuss additional topics related to group representations, operator algebras and/or PDEs.

The grade will be based on homework (70%) and a take home final exam (30%).
References: Most topics are covered by J. B. Conway's book, *A Course in Functional Analysis*, which is recommended.

MA 557: Abstract Algebra I
Instructor: Prof. Ulrich, office: Math 618, phone: 49–41972, e-mail: ulrich@math.purdue.edu
Time: MWF 2:30 **NOTE NEW TIME**
Prerequisite: Basic knowledge of algebra (such as the material of MA 503)
Description: The topics of the course will be commutative algebra and introductory homological algebra. We will study basic properties of commutative rings and their modules, with some emphasis on homological methods. The course should be particularly useful to students interested in commutative algebra, algebraic geometry, number theory or algebraic topology. There will be a continuation in the spring.
Text: No particular book is required, but typical texts are: M. Atiyah and I. Macdonald, *Introduction to commutative algebra*, Addison-Wesley, and J. Rotman, *An introduction to homological algebra*, Academic Press.

MA 584: Algebraic Number Theory
Instructor: Prof. Shahidi, office: Math 650, phone: 49–41917, e-mail: shahidi@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 553 and 554
Description: This is the first of a two semester course in number theory approved by COS which will eventually become an official course numbered MA 584. It covers most of the basic and fundamental facts about algebraic number fields. The second semester will cover class field theory (to be numbered MA 684). Here is the syllabus: Dedekind domains, norm, discriminant, different, finiteness of class number, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertia groups, completions and local fields.
Text: Gerald J. Janusz, *Algebraic Number Fields*, **GSM**, vol. 7, American Mathematical Society

MA 598B: Computational Introduction to Commutative Algebra
Instructor: Prof. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: MA 553 or equivalent
Text: G.-M Greuel and G. Pfister *A Singular Introduction to Commutative Algebra*
Description: From the text (paraphrased): "A rigorous introduction to Commutative Algebra, flavored with algorithms and computational practice. Abstract concepts are presented in company with concrete examples and general procedures to show how these concepts are handled by computer, thereby enriching understanding of both theory and applications."

An essential part of the text—and the course—is the computer program *Singular,* a sophisticated system for polynomial computations. Skills developed with this program are readily transferrable to other such programs, like Macaulay or CoCoA.

The text is on reserve in the library, so can (should) be perused by anyone thinking about taking the course. We will cover as much of the first four chapters as is reasonably feasible. A second semester covering the rest of the book is a possibility.

The book includes an introduction to *Singular,* as well as a CD from which you can copy the program if you want to play with it. Or, you can freely download the most recent version to your own computer from

< `http://www.singular.uni-kl.de/` >.

The program is also on the Math. Dept. machines. (Put the following line

```
set path = (/pkgs/singular-2.0.0/bin $path .  )
```

into your .cshrc file, then call the program with the command "Singular").

MA 598C: Numerical Methods for Partial Differential Equations
Instructor: Prof. Cai, office: Math 810, phone: 49–41921, e-mail: zcai@math.purdue.edu
Time: TTh 12:00-1:15
Prerequisite: MA 362, 351 and 523 or equivalent or consent of the instructor
Description: Mathematical aspects of the finite element method applied to elliptic, parabolic and hyperbolic partial differential equations. Topics in approximation theory in two dimensions and the numerical solution of sparse linear systems. Other topics at the discretion of the instructor.
Text: L. Ridgway Scott and Susanne C. Brenner, *Mathematical Theory of Finite Element Methods*, 2nd edition, ISBN: 0387954511, Springer Verlag.

MA 598D: Representation Theory of Non–Compact Lie Groups
Instructor: Prof. Yu, office: Math 738, phone: 49–41946, e-mail: jyu@math.purdue.edu
Time: TTh 3:00-4:15 **NOTE NEW TIME**
Prerequisite: MA 553, 554; experiences with Lie groups or algebraic groups recommended.
Description: Representation theory of non-compact groups is considerably more sophiscated than that of compact ones, and is a developing field of search. In this introductory course, we will cover selected aspects of representations of reductive real and complex Lie groups, including

- Admissible representations and $(\mathfrak{g}, K)$–modules.
- Infinitesimal characters.
- Spherical representations.
- Principal series representations.
- Unitary representations.
- Examples.

Text: No official textbook will be used. Some notes will be handed out. Recommended reference: A. W. Knapp, *Representation theory of semisimple groups, an overview based on examples*, Princeton University Press (1986).

MA 598E: Introduction to Knot Theory
Instructor: Prof. Lee, office: Math 734, phone: 49–47919, e-mail: yjlee@math.purdue.edu
Time: TTh 12:00–1:15
Description: Knot theory is one of the most concrete branches of topology. Traditionally, it has been important for the study of low-dimensional topology; more recently, it enjoys further attentions and has become the focus of intensive research due to new ideas arisen from physics. In spite of its long history, many basic questions remain unanswered. Techniques employed for the investigation in knot theory range from combinatorics, algebraic topology, and geometric analysis.

This course is intended to be a first course on knot theory, covering roughly the materials in the books of Lickorish and Rolfsen. Knowledge of basic algebraic topology and differential topology will be helpful, but perhaps not absolutely necessary.

Time permitting, we hope to end in some brief discussions of topics of current research, such as the knot homologies of Khovanov and Ozsvath-Szabo, finite-type invariants and Kontsevich integral.

Level of students: 2nd-year (or higher) graduate students, though well-prepared 1st-years or even undergraduates are welcome.

MA 598G: Basic Algebraic Geometry
Instructor: Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail: ram@math.purdue.edu
Time: TTh 1:30-2:45
Description: This will be an introduction to algebraic geometry. There are no prerequisites and all interested students are welcome. Here are descriptions of possible topics to be covered.

• ANALYSIS AND RESOLUTION OF SINGULARITIES OF PLANE CURVES: A plane curve C of degree n is given by a polynomial equation $F(X, Y) = 0$ of degree n. By translation of coordinates, any point P of C can be brought to the origin $(0, 0)$. Now $F = F_d + F_{d+1} + \cdots + F_n$ where F_i is homogeneous of degree i with $F_d \neq 0 \neq F_n$. P is a simple point of C means $d = 1$; otherwise it is a multiple point of multiplicity d. The distinct factors of F_d, say h of them, are the tangents to C at P. Applying a QDT = Quadratic Transformation centered at P amounts to substituting $X = X'$ and $Y = X'Y'$ to get $F(X', X'Y') = X'^d F'(X', Y')$. This explodes P into points $P'_1, \ldots, P'_h$ of the proper transform $C' : F' = 0$ of multiplicities $d'_1, \ldots . d'_h$ with $d'_1 + \cdots + d'_h \leq d$. These are points in the first neighborhood of P. Iterating this we get points in the second neighborhood, and so on. Collectively they are point infinitely near to P. Let $\delta(P) = \sum \frac{\mu(Q)(\mu(Q)-1)}{2}$ where $\mu(Q)$ is the multiplicity at Q and the summation over all points Q infinitely near to P. Assuming C to be devoid of multiple components, Max Noether (1875) proved $\delta(P) < \infty$. Dedekind (1882) proved $\delta(P)$ to be length of the conductor of the local ring of P on C. Assuming C to be irreducible he showed $g(C) = \frac{n(n-1)}{2} - \sum \delta(P)$ where the sum is over all singular (= nonsimple) points P of C and $g(C)$ is the genus of C defined by Jacobi (1830) to be the number of independent regular differentials on C.

• HIGHER DIMENSIONAL DESINGULARIZATION. Extending QDTs to spaces of higher dimension we get MDTs = Monoidal Transformations. Zariski (1939-1944) in characteristic 0 and Abhyankar (1954-1965) in characteristic $p \neq 0$ showed that by using MDTs, the Noether procedure can be generalized to varieties of dimension 2 and 3. Hironaka (1964) extended this to characteristic 0 and any dimension. Abhyankar (1963) did it in the "arithmetic case" for dimension 2. We shall explore the possibilities of generalizing all this to higher dimension for nonzero characteristic as well as for the arithmetic case.

• RATIONAL AND POLYNOMIAL PARAMETRIZATION. Curve genus formulas can be used to decide when a curve can be rationally parametrized or even polynomially parametrized. Corresponding surface genus formulas can be used in a similar, but much more complicated, manner.

• CALCULATION OF FUNDAMENTAL GROUPS. Genus formulas can be used for calculating fundamental groups. In case of nonzero characteristic, they have to be supplemented with the theory of finite simple groups.
Texts: (1) *Algebraic Geometry for Scientists and Engineers,* by S. Abhyankar, Amer Math Soc.
(2) *Ramification Theoretic Methods in Algebraic Geometry,* by S. Abhyankar, Princeton U. Press.

MA 598K: Algebraic Coding Theory
Instructor: Prof. Moh, office: Math 638, phone: 49–41930, e-mail: ttm@math.purdue.edu
Time: MWF 10:30
Prerequisite: None. MA 554 will help.
Description: Code means "self-correcting code" in the literature. It is widely used in industry for telephone, e-mails, CD, photo from Mars etc. It is one of the greatest discoveries in 20th century. It is a useful subject in Quantum Computing, CS, Engineering, Mathematics etc.. The course will consist of three parts:
(1) Hamming code based on Linear Algebra (two weeks).
(2) BCH, Reed-Solomon, Classical Goppa codes based on Polynomial Algebra (four weeks).
(3) Geometric Goppa code based on Algebraic Geometry (five weeks) over a finite field (nine weeks totally).

MA 598W Cohomology of Groups
Instructor: Prof. Wilkerson, office: Math 700, phone: 49–41955, e-mail: wilker@math.purdue.edu
Time: MWF 1:30
Prerequisite: MA 553, 554.
Description: This course should be of interest to students in number theory, commutative algebra, topology, and algebraic geometry.

Topics: the group ring, projective and injective modules, invariants, resolutions, derived functors, low dimensional homology and cohomology groups, extensions, cohomology algebras, Lyndon-Hochschild-Serre spectral sequence, the spectrum of the cohomology algebra, Carlson's work.

Expectations: Each student will do at least one presentation during the term.
Text: Kenneth Brown *Cohomology of Groups*, Springer.
References: 1. Adem and Milgram *Cohomology of Groups*, Springer
2. Evens, *Cohomology of Groups*
3. Dummit and Foote, *Abstract Algebra*, Chapters 18 and 19.

MA 631: Several Complex Variables
Instructor: Prof. Catlin, office: Math 744, phone: 49–41958, e-mail: catlin@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 530
Description: Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. Local theory of analytic sets (Weierstrass preparation theorem and consequences). Functions and sets in the projective space P^n (theorems of Weierstrass and Chow and their extensions).

MA 642: Methods of Linear and Nonlinear Partial Differential Equations I
Instructor: Prof. Phillips, office: Math 706, phone: 49–41939, e-mail: phillips@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 523 or consent of instructor
Description: Second order elliptic equations including maximum principles, Harnack inequality, Schauder estimates, and Sobolev estimates. Applications of linear theory to nonlinear equations.
Text: Gilbarg and Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer

MA 650: Commutative Algebra
Instructor: Prof. Heinzer, office: Math 636, phone: 49–41980, e-mail: hienzer@math.purdue.edu
Time: MWF 3:30 **NOTE NEW TIME**
Description: I plan to cover material from the text *Commutative Ring Theory* by H. Matsumura. In particular, the course will cover: properties of extension rings, integral extensions, valuation rings, dimension theory of graded rings, the Hilbert function and Hilbert polynomial, systems of parameters and multiplicity, the dimension of extension rings, regular sequences and the Koszul complex, Cohen-Macaulay rings, Gorenstein rings, regular rings and UFDs.

MA 692D: Applied Inverse Problems
Instructor: Prof. de Hoop, e-mail: mdhoop@math.purdue.edu
Time: TTh 10:30-11:45
Description: This seminar/course addresses topics in imaging, inverse scattering, tomography, and interferometry. The techniques covered are derived from the analysis of hyperbolic partial differential equations, Fourier analysis, microlocal analysis, and symplectic geometry. The main applications to be touched upon this semester are in geophysics and (medical) elastography.

Additional Information: This course will connect quite well with MA 542. The first part of the course will be introductory, starting from Fourier transform and Radon transform techniques. References for the introduction include: Bleistein, Cohen and Stockwell, *Mathematics of multidimensional imaging, migration and inversion*, Springer 2001, but the course will provide more material. There will be a couple of guest speakers as well.

MA 693E: KK-Theory
Instructor: Prof. L. Brown, office: Math 704, phone: 49–41938, e-mail: lgb@math.purdue.edu
Time: MWF 10:30
Prerequisite: Some knowledge of C^*–algebras is essential. Some knowledge of K–theory of C^*–algebras is desirable.
Description: KK–theory is a two–variable generalization of K–theory. K–theory produces two groups, K_0 and K_1, for a C^*–algebra A, and K_0 and K_1 are both covariant functions. When A is commutative, A is isomorphic to $C(X)$ in the unital case, and these K–groups are the same as the topological K–groups $K_i(C(X)) \simeq K^i(X)$. For two C^*–algebras A and B, there are groups $KK_i(A, B), i = 0$ or 1, and the KK–functor is contravariant in A and covariant in B. Also $KK_0(\mathbb{C}, B) \simeq K_0(B), KK_0(C_0(\mathbb{R}), B) \simeq K_1(B), KK_1(A, \mathbb{C}) \simeq Ext(A)$ (a group formed from extensions of C^*–algebras), for example. The Kasparov product, $KK(A, B) \times KK(B, C) \to KK(A, C)$, makes it possible to regard elements of $KK(A, B)$ as generalized maps from A to B. KK–theory has many applications.

MA 694E: Elliptic and Parabolic PDEs
Instructor: Prof. Garofalo, office: Math 616, phone: 49–41971, e-mail: garofalo@math.purdue.edu
Time: TTh 10:30-11:45
Description: This is intended as the first part of a one year long course which will develop some recent, and less recent, trends in elliptic and parabolic PDE's related to variational inequalities and free boundary problems. We will begin with a detailed discussion of those results on the boundary behavior of nonnegative solutions of elliptic and parabolic PDE's in non-smooth domains whch constitute the backbone of the theory. We will then move on to discuss the classical obstacle problem, and then develop the relevant regularity theory which culminated with the groundbreaking work of Caffarelli. We will also discuss the Stefan problem and those more recent developments concerning two-phase free boundary problems, such as monotonicity formulas due to Alt-Caffarelli-Friedman and to Caffarelli, and their applications. The course will have an entirely self-contained character.

MA 694F: Lévy processes and Stochastic Analysis
Instructor: Prof. Ma, office: Math 620, phone: 49–41973, e-mail: majin@math.purdue.edu
Time: TTh 1:30-2:45
Prerequisite: MA 538, 539, or consent of the instructor
Description: The aim of this course is to introduce the stochastic analysis based on Lévy processes, a random motion that contains the properties of both Brownian motion and compound Poisson process. Such a process has recently found novel applications in diverse areas such as mathematical finance and quantum field theory. The course will start from reviewing some basics in probability theory such as infinite divisibility, convolution semigroups of probability measures, etc., and end up with stochastic differential equations driven by Lèvy processes, as well as their applications in finance. The theory of semimartingales, Markov processes, and stochastic integrals with respect to Lévy processes and its calculus will be covered along the way.
Text: David Applebaum, *Lévy Processes and Stochastic Calculus*, Cambridge Studies in Advanced Mathematics **93**, Cambridge University Press, 2004.
Suggested Reading: 1) J. Bertoin, *Lévy Processes*, Cambridge University Press (1996).
2) K-I. Sato, *Lévy Processes and Infinitely Divisible Distributions*, Cambridge University Press (1999).

MA 694R: Introduction to Stochastic Partial Differential Equations
Instructor: Prof. Roeckner, e-mail: roeckner@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: theory of martingales, basic knowledge on stochastic integration
Description: This course will focus on nonlinear SPDE of evolutionary type. Examples will include e.g. stochastic reaction diffusion and porous media equations. We shall start with the theory of stochastic integration in Hilbert spaces including Ito's formula. Then we shall present one of the simplest techniques to prove existence and uniqueness of solutions based on monotonicity arguments. Time permitting also the more technical semigroup approach to construct mild solutions will be treated.

Further topics: asymptotic properties of solutions, invariant measures, corresponding infinite dimensional Kolmogorov equations. If required, the necessary basic ingredients from the theory of Hilbert spaces and stochastic differential equations on $\mathbb{R}^d$ will be recalled in compact form during the course, as soon as they are needed. No preknowledge of PDE is necessary. The deterministic case will be a special case of the theory to be presented.
Texts: (1) Boris Rozowskii *Stochastic evolution systems.* Mathematics and its Applications, 35. Kluwer Academic Publishers Group, Dordrecht, 1990.
(2) Giuseppe Da Prato *Kolmogorov Equations for Stochastic PDEs.* Advanced Courses in Mathematics - CRM Barcelona. Birkhaeuser, Basel, 2004.

MA 696C: Topices in Complex Geometry
Instructor: Prof. Yeung, office: Math 712, phone: 49–41942, e-mail: yeung@math.purdue.edu
Time: MWF 10:30
Prerequisite: 530, 562. Some basic understandings in algebraic geometry and several complex variables will be helpful as well.
Description: In this course, some basic techniques in complex manifolds will be studied. Tentatively, aspects of the following topics will be covered.
1. Kähler-Einstein metrics and their applications.
2. Ricci flow and Kähler-Ricci flow in geometry.
3. Hichin-Kobayashi correspondances and uniqueness of metrics of constant scalar curvature.
4. Metric properties of Teichmüller spaces and moduli spaces of curves.

Probably only parts of the materials would be covered, depending on the progress of the course.

Seminars

Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Working Algebraic Geometry Seminar,
Time: Wednesday 3:30

Automorphic Forms and Group Representations Seminar, Prof. Yu
Time: Thursdays, 1:30-2:30

Commutative Algebra Seminar, Prof. Ulrich
Time: Wednesdays 4:30-5:20

Computational and Applied Math Seminar, Prof. Shen
Time: Fridays 3:30

Function Theory Seminar, Prof. Weitsman
Time: Tuesdays, 3:00

Geometric Analysis Seminar, Prof. Lempert
Time: Monday 3:30

Foundations of Analysis Seminar, Prof. de Branges
Time: Thursday 10:30-11:20

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30-3:20

PDE Seminar, Prof. Phillips
Time: Thursday, 3:30

Probability Seminar, Prof. Viens
Time: Tuesday, Thursday 12:00

Spectral and Scattering Theory Seminar, Prof. SaBarreto
Time: Wednesday 4:30

Student Tea Time Seminar on Applied Analysis, Prof. Petrosyan
Time: Tuesday 3:00

Topology Seminar, Prof. McClure
Time: Tuesday 1:30-2:20

Working Algebraic Geometry Seminar, Prof. Arapura
Time: Wednesday 3:30-4:30