

Seminars and Advanced Graduate Courses  
offered by the  
Mathematics Department  
Fall, 2007

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Courses

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**MA 542: Theory of Distributions and Applications**

**Instructor:** Prof. A. SaBarreto, office: Math 410, phone: 49-41965, e-mail: [sabarre@math.purdue.edu](mailto:sabarre@math.purdue.edu)

**Time:** MWF 1:30

**Prerequisite:** Good knowledge of real analysis, (MA 544 is desirable. You need to know at least what  $L^2$  is). Some knowledge of complex analysis, and a good motivation to study PDEs (you need to know what this means). If you want to take the course but are not sure if you have the necessary prerequisites, please stop by my office, MA 410.

**Description:** The theory of distributions is a very important tool in the study of linear and nonlinear partial differential equations. We will first cover the basics, and then study homogeneous distributions, fundamental solutions of the Laplace, wave and heat operators. We will also study the Fourier transform of distributions and the Paley-Wiener theory.

**References:** L. Hormander, *The Analysis of Linear Partial Differential Operators*, vol. 1, Springer Verlag.

**Text:** F.G. Friedlander, *Introduction to the Theory of Distributions, with an appendix by M. Joshi*, Cambridge University Press, 1998. (about \$30 on amazon.com)

**MA 562: Introduction to Differential Geometry and Topology**

**Instructor:** Prof. M. Leok, office: Math 430, phone: 49-64578, e-mail: [mleok@math.purdue.edu](mailto:mleok@math.purdue.edu)

**Time:** TTh 12:00-1:15

**Prerequisite:** MA 351 and MA 442

**Description:** This course is an introduction to the theory of differentiable manifolds, as well as vector and tensor analysis and integration on manifolds. The course is particularly useful for students interested in differential geometry, Lie groups, and global analysis, and serves as a foundation course for work in geometric mechanics and geometric control theory. In addition, it is the basis of the modern approach to applied fields such as fluid mechanics, elasticity, and general relativity.

Topics will include smooth manifolds, tangent vectors, inverse and implicit function theorems, submanifolds, vector fields, integral curves, differential forms, the exterior derivative, partitions of unity, integration on manifolds, and if time permits, fundamentals of Riemannian geometry, Levi-Civita connection, parallel transport, geodesics, curvature tensor.

**Text:** William Boothby *Introduction to Differentiable Manifolds and Riemannian Geometry*

**MA 546: Introduction to Functional Analysis**

**Instructor:** Prof. L. Brown, office: Math 704, phone: 49-41938, e-mail: [lgb@math.purdue.edu](mailto:lgb@math.purdue.edu)

**Time:** MWF 10:30

**Prerequisite:** MA 544

**Description:** The course covers basic functional analysis with emphasis on bounded linear operators on Banach and Hilbert spaces. The topics listed below will be covered, and brief treatments of other topics fitting the interests of the students can be included as time permits.

Banach spaces and Hilbert spaces; Hahn-Banach theorem; closed graph and open mapping theorems; uniform boundedness principle; theory of spectrum for operators on Banach spaces and for compact operators; weak and weak\* topologies; reflexivity; Hahn-Banach separation theorem and double polar theorem; operators on Hilbert spaces; spectral theorem for bounded self-adjoint operators on Hilbert spaces.

**Text:** M. Schechter, *Principles of Functional Analysis*, AMS.

**MA 557: Abstract Algebra I**

**Instructor:** Prof. B. Ulrich, office: Math 618, phone: 49-41972, e-mail: ulrich@math.purdue.edu

**Time:** MWF 1:30

**Prerequisite:** Basic knowledge of algebra (such as the material of MA 503).

**Description:** The topics of the course will be commutative algebra and introductory homological algebra. We will study basic properties of commutative rings and their modules, with some emphasis on homological methods. The course should be particularly useful to students interested in commutative algebra, algebraic geometry, number theory or algebraic topology. There will be a continuation in the spring.

**Texts:** No particular book is required, but typical texts are:

- M. Atiyah and I. Macdonald, *Introduction to commutative algebra*, Addison-Wesley.

- J. Rotman, *An introduction to homological algebra*, Academic Press.

**MA 598C: Mixture and Polar Continuum Field Theories**

**Instructor:** Prof. J. Cushman, office: Math 416, phone: 49-48040, e-mail: jcushman@math.purdue.edu

**Time:** TTh 10:30-11:45

**Description:** Classical continuum mechanics approaches and principles are reviewed and then extended to multiphase and multi-constituent mixtures with overlaying continua. As in classical mechanics, these theories have three degrees of freedom per continua, corresponding to the spatial coordinates or velocities of material particles. Applications of these theories to drug delivery substrates, contaminant transport in the environment and reservoir engineering will be presented.

When dealing with bodies or processes involving micro-structure such as blood flowing in a micro-capillary bed, granular materials or colloid filtration, mixture theories often fail and a more sophisticated approach is required. One such approach is the theory of polar continua where, in addition to the three degrees of freedom associated with the spatial coordinates, there are nine degrees of freedom representing micro-stretch and micro-shear. This theory will be developed and applied to blood flow in micro-capillaries, i.e. capillaries with diameters similar to the effective diameter of a red blood cell

**MA 598T: Bridge to Research Seminar**

**Instructor:** Prof. F. Milner, office: Math 628, phone: 49-41967, e-mail: milner@math.purdue.edu

**Time:** M 4:30

**Description:** The seminar has two main goals, both aimed at helping students early in their graduate career find their place in the department. The first is to help students discover what area of mathematics they might be interested in researching, as well as who they might like to work with. The second is to provide students with an opportunity to interact with faculty in a casual setting. This is achieved by having professors from the department give brief talks about their research area at a level that is accessible to those in their first and second year of graduate study.

**MA 598W: Algebraic Topology of Lie Groups**

**Instructor:** Prof. C. Wilkerson, office: Math 700, phone: 49-41955, e-mail: wilker@math.purdue.edu

**Time:** TTh 12:00-1:15

**Description:** This will be an introduction to Lie Groups, stressing tools that arise from algebraic topology, e.g. Hopf algebras, fixed point theory, and classifying spaces.

The lectures will be largely from papers of Dwyer-Wilkerson over the last decade or so. However, the following monographs will be useful:

1). J. F. Adams, *Lie Groups*, Midway Press.

2) Milnor-Stasheff, *Characteristic Classes*, Study 76, Princeton Press.

I'm looking for some class participation, i.e., some lectures by students.

**MA 631: Several Complex Variables**

**Instructor:** Prof. L. Lempert, office: Math 728, phone: 49-41952, e-mail: lempert@math.purdue.edu

**Time:** MWF 12:30

**Prerequisite:** MA 530

**Description:** Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. Local theory of analytic sets (Weierstrass preparation theorem and consequences). Functions and sets in the projective space  $P^n$  (theorems of Weierstrass and Chow and their extensions).

**MA/STAT 638: Stochastic Processes I**

**Instructor:** Prof. A. Yip, office: Math 432, phone: 49-41941, e-mail: yip@math.purdue.edu

**Time:** MWF 10:30

**Prerequisite:** MA/STAT 539 or equivalent.

**Description:** This course studies Ito's Stochastic Calculus and the solutions of Stochastic Differential Equations. This will be used to provide examples of stochastic processes for the investigation of the concept of Markov and Diffusion Processes, Gaussian and Stationary Processes, Ergodic Theory and others. The emphasis will be on the applications of probability theory in mathematics, engineering and physical sciences.

**Text:** Ioannis Karatzas, Steven E. Shreve, *Brownian motion and stochastic calculus*.

**MA 642: Methods of Linear and Nonlinear Partial Differential Equations I**

**Instructor:** Prof. D. Phillips, office: Math 706, phone: 49-41939, e-mail: phillips@math.purdue.edu

**Time:** MWF 9:30

**Prerequisite:** MA 544 and MA 611 or equivalent.

**Description:** This is the first semester of a one-year course in pde theory with applications to nonlinear equations. The first semester focuses on second order elliptic equations (in divergence and nondivergence form). Topics to be developed include maximum principles, Harnack inequalities, Schauder theory for classical solutions, and Sobolev estimates for weak solutions.

**Text:** D. Gilbarg and N. Trudinger, *Elliptic Partial Differential Equations of Second Order*

**MA 650: Commutative Algebra**

**Instructor:** Prof. B. Ulrich, office: Math 618, phone: 49-41972, e-mail: ulrich@math.purdue.edu

**Time:** MWF 3:30

**Prerequisite:** Basic knowledge of commutative algebra (such as the material of MA 557/558).

**Description:** This is an intermediate course in commutative algebra. The course is a continuation of MA 557/558, but should be accessible to any student with basic knowledge in commutative algebra (localization, Noetherian and Artinian modules, associated primes and primary decomposition, integral extensions, dimension theory, completion, Ext and Tor).

The topics of this semester will include: Graded rings and modules, depth and Cohen-Macaulayness, structure of finite free resolutions, regular rings and normal rings, canonical modules and Gorenstein rings, local cohomology and local duality.

**Texts:** No specific text will be used, but possible references are:

- H. Matsumura, *Commutative ring theory*, Cambridge University Press
- W. Bruns and J. Herzog, *Cohen-Macaulay rings*, Cambridge University Press
- D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, Springer

**MA 690A: Topics in Algebra and Algebraic Geometry**

**Instructor:** Prof. S. Abhyankar, office: Math 600, phone: 49-41933, e-mail: ram@math.purdue.edu

**Time:** TTh 3:00-4:15

**Description:** This course will prepare students to write a thesis in algebra or algebraic geometry. There are no prerequisites. All interested students are welcome. Students are encouraged to attend the Thursday Seminar in the same room at 4:30.

**Texts:** Shreeram S. Abhyankar, *Algebraic Geometry for Scientists and Engineers*, Americal Mathematical Society  
2. Shreeram S. Abhyankar, *Lectures on Algebra I*, World Scientific.

**MA 690B: Topics in Commutative Algebra**

**Instructor:** Prof. W. Heinzer, office: Math 636, phone: 49-41980, e-mail: heinzer@math.purdue.edu

**Time:** MWF 12:30

**Prerequisite:** MA 557

**Description:** The course will cover topics in commutative algebra. The topics to be covered will depend on the interests of the students enrolled in the course and the competency of the instructor.

- References:** 1. W. Bruns and J. Herzog, *Cohen-Macaulay rings*  
 2. I. Swanson and C. Huneke, *Integral closure of ideals, rings and modules*  
 3. M. Nagata, *Local rings*  
 4. H. Matsumura, *Commutative ring theory*

**MA 690M: Hodge Theory and Complex Algebraic Geometry**

**Instructor:** Prof. K. Matsuki, office: Math 614, phone: 49-41970, e-mail: kmatsuki@math.purdue.edu

**Time:** MWF 11:30

**Prerequisite:** Basics of complex analysis and algebra

**Description:** We will follow and present the material in the beautiful book by C. Voisin of the same title as the course. For the content we quote the ad. by the publisher: "This is a modern introduction to Kaehlerian geometry and Hodge structure. It starts with basic material on complex variables, complex manifolds, holomorphic vector bundles, sheaves and cohomology theory. The book culminates with the Hodge decomposition theorem, while in route proving the hard Lefschetz theorem and the Hodge index theorem."

**Text:** C. Voisin *Hodge Theory and Complex Algebraic Geometry*

**MA 690S: Automorphic  $L$ -functions and Applications**

**Instructor:** Prof. F. Shahidi, office: Math 650, phone: 49-41917, e-mail: shahidi@math.purdue.edu

**Time:** MWF 9:30

**Prerequisite:** MA 684 and some background in algebraic groups and basic representation theory. A course such as MA 690D presently taught by Professor Yu is more than enough.

**Description:** The bulk of the course will discuss the analytic properties of automorphic  $L$ -functions and deduce them from the theory of Eisenstein series in a large number of cases. After discussing what Langlands Functoriality Conjecture is, we will apply converse theorems (to be explained) to these results to prove certain cases of the conjecture some of which will not be possible using any other methods presently available such as trace formulas and theta liftings. We will then discuss several applications of these results in number theory and group representations.

**References:** 1. *Automorphic forms and Applications*, Proceedings of 2002 Graduate Summer School at Park City, IAS/Park City Mathematics Series, vol. 12, AMS, 2007. (Lectures by Borel, Cogdell and myself)

2. My notes: *Eisenstein Series and Automorphic  $L$ -functions*

3. *Automorphic  $L$ -functions*, Fields institute Monographs, AMS, 2004.

**MA 690Y: Galois Cohomology**

**Instructor:** Prof. J.-K. Yu, office: Math 738, phone: 49-41946, e-mail: jyu@math.purdue.edu

**Time:** TTh 10:40-11:45

**Prerequisite:** Algebraic number theory, basic knowledge of algebraic groups

**Description:** This course is devoted to the cohomology of the Galois group of a field, and the applications to number theory and representation theory, and cohomological invariants.

**Text:** J-P Serre *Galois Cohomology*

**MA 692D: High Performance Computing and Grid Computing**

**Instructor:** Prof. S. Dong, office: Math 436, phone: 49-63875, e-mail: sdong@math.purdue.edu

**Time:** TTh 1:30-2:45

**Prerequisite:** Ability to program in any of the high-level languages: FORTRAN, C, or C++.

**Description:** High performance computing involves the use of the most efficient algorithms on parallel computers capable of the highest performance to solve the most demanding scientific problems. It is an enabling technology for many scientific disciplines such as weather forecasting, genome sequencing, turbulent drag reductions, drug design, and cryptography. It is an indispensable skill for students in applied mathematics, science and engineering.

This course targets graduate students and senior undergraduate students. It is designed to enable students to efficiently exploit high performance parallel computers to support their own research activities. The course will expose students to the fundamental concepts and principles in high performance computing, and to enable them to efficiently program a spectrum of parallel computers ranging from workstation clusters with a few CPUs to supercomputers with thousands of CPUs. The course emphasizes on practical applications, and will cover several important aspects of high performance computing and the emerging area of Grid computing.

**MA 693B: Riemann Hypothesis****Instructor:** Prof. L. de Branges, office: Math 800, phone: 49-46057, e-mail: branges@math.purdue.edu**Time:** MWF 9:30

**Description:** The Riemann hypothesis is a unifying concept of mathematical analysis whose proof applies Fourier analysis on the space of quaternions with real coordinates and on related spaces of quaternions whose coordinates are  $p$ -adic numbers for all primes  $p$ . Zeta functions are constructed using the space of quaternions with rational coordinates. The functional identity for a zeta function is an application of the Poisson summation formula to functions which originate in the relationship between Fourier analysis on a space of quaternions and Fourier analysis on a plane. The Radon transformation which relates the two kinds of Fourier analysis has a maximal dissipative property which implies the Riemann hypothesis for the constructed zeta functions. These functions are the Hecke zeta functions associated with the congruence subgroup of the modular group modulo two. The course is based on a preprint, *The Riemann hypothesis for Hecke functions*, available at the instructor's webpage.

**MA 696A: Heat Kernels and Dirac Operators****Instructor:** Prof. Y.-J. Lee, office: Math 734, phone: 49-47919, e-mail: yjlee@math.purdue.edu**Time:** TTh 12:00-1:15**Prerequisite:** Familiarity with basic differential geometry (e.g. MA 562).

**Description:** The heat kernel approach to index theorems was proposed and pursued in the 70's in the works of Atiyah, Bott, Patodi, Singer, Gilkey, et al. Inspired by supersymmetric quantum mechanics from physics, this subject was revived in the 80's via the works of Bismut, Getzler, Quillen, and many other authors. This new approach is more direct than the previous proofs; it not only simplifies but made generalizations possible. The goal of this course is to develop the tools leading to these results, such as asymptotic expansion of the heat kernel, Clifford modules, and the superspace formalism. These tools have become basic to many aspects of modern differential geometry and topology. Time permitting, we'll also discuss more advanced topics such as equivariant and family versions of the local index theorems, secondary invariants.

**Text:** Berline, Getzler, Vergne *Heat Kernels and Dirac operators*


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**Seminars**


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**Algebra and Algebraic Geometry Seminar**, Prof. Abhyankar**Time:** Thursday 4:30-6:00**Automorphic Forms and Representation Theory Seminar**, Prof. Goldberg**Time:** Thursdays, 1:30**Cauchy-Riemann Equations Seminar**, Prof. Catlin**Time:** Thursdays, 2:30**Commutative Algebra Seminar**, Profs. Heinzer and Ulrich**Time:** Wednesdays 4:30**Computational and Applied Math Seminar**, Prof. Shen**Time:** Fridays 3:30**Function Theory Seminar**, Prof. Eremenko**Time:** Tuesdays, time flexible**Foundations of Analysis Seminar**, Prof. de Branges**Time:** Thursday 9:30**Geometric Analysis Seminar**, Prof. Lempert**Time:** Monday 3:30**Number Theory Seminar**, Prof. Goins**Time:** Thursday 3:30**Joint Geometry/Topology Seminar**, Prof. Y. J. Lee**Time:** Saturdays, once a month**Operator Algebras Seminar**, Prof. Dadarlat**Time:** Tuesdays, 2:30

**PDE Seminar**, Prof. Phillips

**Time: Thursday, 3:30**

**Probability Seminar**, Prof. Banuelos

**Time: Wednesday, 3:30**

**Spectral and Scattering Theory Seminar**, Prof. SaBarreto

**Time: Thursday 4:30**

**Topology Seminar**, Prof. McClure

**Time: Thursdays 3:30**

**Working Algebraic Geometry Seminar**, Prof. Matsuki

**Time: Wednesday 3:30rm**