RIEMANN SURFACES

Instructor: Prof. Donu Arapura (dvb@math.purdue.edu, 4-1983)
Course Number 59800 CRN:
Credits:
Time: 3:00p-4:15p TTh

Description

Riemann introduced his surfaces in the middle of the 19th century in order to "geometrize" complex analysis. In doing so, he paved the way for a great deal of modern mathematics such as algebraic geometry, manifold theory, and topology. So this would certainly be of interest to students in these areas, as well as in complex analysis or number theory. In simple terms, a Riemann surface is a surface which locally looks like the complex plane. We've all seen simple examples: open subsets of the plane, the sphere, and perhaps the domain of the "multivalued function" $\sqrt{z}$. More exotic examples include elliptic curves, which includes what you get by identifying the sides of a square with corners $0, 1, i$ and $1 + i$.

My plan is to cover the theory, assuming only the standard algebra (M553) and complex analysis (M530) courses, and to reach the major theorems such as the Riemann-Roch theorem, which is an existence theorem for meromorphic functions with prescribed singularities, and the uniformization theorem which gives the classification of simply connected Riemann surfaces. Also since my goal is to get students up to speed on topics relevent to further studies in algebraic or complex geometry or topology, I will give self contained introductions on auxillary topics such as bundles, sheaves, de Rham cohomology and so on. I was thinking of using Donaldson's recent book or Riemann Surfaces, which looks quite nice.

INTRODUCTION TO DESSINS D'ENFANTS

Instructor: Prof. Edray Goins (egoins@math.purdue.edu, 4-1936
Course Number: MA 59800 CRN: 63571
Credits: Three
Time: 3:30 p.m.–4:20 p.m. MWF

Description

Given a collection of m homogeneous polynomials F_k, the set

$$V(\mathbb{C}) = \left\{ P \in \mathbb{P}^n(\mathbb{C}) \;\middle|\; F_k(P) = 0 \text{ for } k = 1,\, 2,\, \ldots,\, m \right\}$$

is called an algebraic variety because it is defined by algebraic equations $V\colon F_1 = F_2 = \cdots = F_m = 0$. If instead of using polynomials, we use analytic functions, we have an analytic variety. Indeed, Riemann surfaces are examples of analytic varieties. There are cases there these two concepts coincide: a compact, connected Riemann surface X is actually an algebraic variety. To be more precise, $X = V(\mathbb{C})$ is a smooth, irreducible, projective variety of dimension 1 corresponding to a single equation $V : \sum_{i,j} a_{ij}\, z^i\, w^j = 0$. The French mathematician André Weil proved in 1956 that if there exists rational function $\beta : X \to \mathbb{P}^1(\mathbb{C})$ which has at most three critical values, then X can be defined by a a polynomial equation where the coefficients a_{ij} are not transcendental. Conversely – and surprisingly – the Russian mathematician Gennadiĭ Vladimirovich Belyĭ showed in 1979 that if X can be defined by a polynomial equation $\sum_{i,j} a_{ij}\, z^i\, w^j = 0$ where the coefficients a_{ij} are not transcendental, then there exists a rational function $\beta : X \to \mathbb{P}^1(\mathbb{C})$ which has at most three critical values.

Motivated by Belyĭ's discovery, the French mathematician Alexander Grothendieck wrote a letter in 1984 outlining several new directions for his research. "This discovery," he wrote, "which is technically so simple, made a very strong impression on me, and it represents a decisive turning point in the course of my reflections, a shift in particular of my centre of interest in mathematics, which suddenly found itself strongly focused. [. . .] This is surely because of the very familiar, non-technical nature of the objects considered, of which any child's drawing scrawled on a bit of paper (at least if the drawing is made without lifting the pencil) gives a perfectly explicit example. To such a 'dessin' we find associated subtle arithmetic invariants, which are completely turned topsy-turvy as soon as we add one more stroke." He realized that maps $\beta : X \to \mathbb{P}^1(\mathbb{C})$ which have at most three critical values give graphs – called "Dessins d'Enfants" !

– which contain useful arithmetic information.

In this course, we discuss the budding theory behind Dessins d'Enfants. We will cover the text "Introduction to Compact Riemann Surfaces and Dessins d'Enfants" (London Mathematical Society Student Texts) by Ernesto Girondo and Gabino González-Diez. We will discuss the Riemann surfaces, algebraic curves, the Riemann-Roch Theorem, Fuchsian groups, monodromy, Galois groups, algebraic varieties, elliptic curves, and modular functions.

Prerequisite: MA 52500 (Introduction to Complex Analysis) and MA 55300 (Introduction to Abstract Algebra).

STOCHASTIC CALCULUS

Instructor: Prof. Fabrice Baudoin (fbaudoin@math.purdue.edu, 4-1406
Course Number: MA 59800 CRN: 63572
Credits: Three
Time: 10:30 a.m.–11:20 a.m. MWF

Description

This course will cover the theory of stochastic integration and its applications . We will focus on the following topics:

1. Martingales in continuous time;
2. Brownian motion;

3. Stochastic integration;
4. Stochastic di?erential equations;
5. Malliavin calculus.

Prerequisite: Basic probability theory: Random variables, Central limit theorem, Law of large numbers, Conditional expectations.

Lecture notes are posted on my blog http://fabricebaudoin.wordpress.com/

MIXED AND LEAST-SQUARES FINITE ELEMENT METHODS FOR SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

Instructor: Prof. Zhiqiang Cai (zcai@math.purdue.edu, 4-1921
Course Number: MA 59800 CRN: 63573
Credits: Three
Time: 9:00 a.m.–10:15 a.m. TTh

Description

The original physical equations for mechanics of continua are systems of partial differential equations of first-order. There are many advantages to simulate these first-order systems directly. This can be done through either mixed or least-squares finite element methods. This course is an introduction to both techniques, with applications to Darcy's flow in porous media, elastic equations for solids, incompressible Newtonian fluid flow, and Maxwell's equations in electromagnetic. We shall focus on fundamental issues such as (mixed and least-squares) variational formulations and construction of finite element spaces of H^1, H(div), and H(curl). A review of fast iterative solvers such as multigrid and domain decomposition for algebraic systems resulting from discretization will also be presented.

A tentative list of contents:

1. Mathematical Models of Continuum Mechanics
2. Construction of Finite Element Spaces in H^1, H(div), or H(curl)
3. Mixed Variational Formulations
4. Least-Squares Variational Formulations
5. Finite Element Approximations
6. Iterative Solvers
7. FOSPACK a computer package based on least-squares methods

Prerequisite: MA/CS 615 or equivalent or consent of instructor.

References

1. F. Brezzi and M. Fortin, Mixed and Hybrid Finite Element Methods, Springer-Verlag, New York, 1991.
2. V. Girault and P. Raviart, Finite Element Methods for Navier-Stokes Equations: Theory and Algorithms, Springer-Verlag, New York, 1986.
3. P. Monk, Finite Element Methods for Maxwell's Equations, Oxford University Press, Oxford, 2003.
4. W. Briggs, V. Henson, S. McCormick, A Multigrid Tutorial, Second Edition, SIAM, Philadelphia, 2000.
5. research articles and my lecture notes.

MATHEMATICAL THEORY OF OPTIMAL CONTROL

Instructor: Prof. Donatella Danielli (danielli@math.purdue.edu, 4-1920)
Course Number: MA 62000 CRN: 63083
Credits: Three
Time: 10:30 a.m.–11:20a.m. MWF

Description

This course is an introduction to the mathematical theory of optimal control of processes governed by ordinary differential equations. In recent years, control problems have arisen in very diverse areas, such as production planning, chemical and electrical engineering, and flight mechanics. The course will focus on the mathematical formulation of such problems and the existence of optimal controls both with and without convexity assumptions. One of the crucial tools for the characterization of optimal controls, namely the maximum principle, will be illustrated and (if time permits) proved. Finally, the relationship with the Calculus of Variations and applications will be discussed.

Prerequisite: MA 54400 or instructor's consent.

References

1. L. D. Berkoviz, Optimal Control Theory, Springer.

METHODS OF LINEAR AND NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS I

Instructor: Prof. Patricia Bauman (bauman@math.purdue.edu, 4-1945)
Course Number: MA 64200 CRN: 63568
Credits: Three
Time: 9:30 A.M.-10:30 A.M. MWF

Description

This is the first semester of a one-year course in the theory of second order elliptic and parabolic PDEs. The aim of the course is to study the solvability of boundary value problems and regularity properties of solutions. The first semester will focus on linear elliptic equations, both in divergence and nondivergence form. The starting point for the study of classical solutions will be the theory of Laplace's and Poisson's equations. The emphasis here will be on: (1) existence of solutions to the Dirichlet problem for harmonic functions via the Perron method (based on the maximum principle) and (2) Holder estimates for solutions of Poisson's equation derived from the analysis of the Newtonian potential. The crowning achievement of the theory of classical solutions is Schauder's theory, which extends the results of potential theory to a general class of non-divergence form equations with Holder-continuous coefficients. In the second part of the semester we will consider a more general and modern approach to linear problems, based not on potential theory, but on Hilbert space methods for so-called weak solutions. Our main goal will be to proave the celebrated De Giorgi-Nash-Moser theorem on the regularity of weak solutions. The relevant tools from the theory of Sobolev spaces will be developed concurrently.

Prerequisites: MA 54400 and MA 61100, or instructor's approval.

RIEMANN HYPOTHESIS

Instructor: Prof. Louis deBranges (branges@math.purdue.edu, 4-6057)
Course Number: MA 69000 CRN: XXXXX
Credits: Three
Time: 9:30-10:30 A.M. MWF

Description

A proof of the Riemann hypothesis is given in Fourier analysis on skew–fields. The course is primarily concerned with the use of Laplace transformation for the computation of Fourier transforms and the resulting use of the Mellin transformation to produce Jacobian zeta functions multiplied by gamma function factors. A verification is made of a conjecture made in 1986 which implies the conjectured line of zeros of these zeta functions. The classical zeta functions of Euler and Dirichlet are factors of the simplest Jacobi–zeta functions. The classical Riemann hypothesis is a consequence of its generalization to Jacobi zeta function (which are essentially the same as the zeta functions constructing from modular forms. Students should have passed qualifying examinations before registering.

INVERSE SCATTERING PROBLEMS FOR WAVE PROPAGATION

Instructor: Prof. Peijun Li (lipeijun@math.purdue.edu, 4-0846)
Course Number: MA 69200 CRN: 63578
Credits: Three
Time: 12:00p.m–1:15 p.m. TTh

Description

Scattering problems are concerned with the effect that an inhomogeneous medium has on an incident field. In particular, if the total field is viewed as the sum of an incident field and a scattered field, the direct scattering problem is to determine the scattered field from the incident field and the differential equation governing the wave motion; the inverse scattering problem is to determine the nature of the inhomogeneity, such as location, geometry, or material property, from a knowledge of the scattered field. These problems have played a fundamental role in diverse scientific areas such as radar and sonar (e.g., stealth aircraft design and submarine detection), geophysical exploration (e.g., oil and gas exploration), medical imaging (e.g., breast cancer detection), near-field optical microscopy (e.g., imaging of small scale biological samples), and nano-optics. This course introduces mathematical models and computational methods for four classes of inverse problems that arise from the acoustic and electromagnetic wave propagation in complex and random media, which include the inverse surface scattering problem, inverse obstacle scattering problem, inverse medium scattering problem, and inverse source scattering problem.

Text: No textbook is required. Lecture notes will be made available to students.

Course grade: No exams. Students are required to present course-related material in class.

Prerequisite: Basic knowledge of functional and numerical analysis, and partial differential equations.

References

1. D. Colton and R. Kress, Inverse Acoustic and Electromagnetic Scattering Theory
2. H. Engle, M. Hanke, and A. Neubauer, Regularization of Inverse Problems
3. J.-C. Nédélec, Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems

AN INTRODUCTION TO COMPRESSED SENSING

Instructor: Prof. Ben Adcock (adcock@math.purdue.edu, 4-1981)

Course Number: MA 69200 CRN: 63576
Credits: Three
Time: TTh 12:00pm-1:15pm

Description

This is a course in compressed sensing and its applications in signal and image processing. In the past decade, compressed sensing has emerged as a powerful new theory that overcomes traditional barriers in sampling. Under appropriate conditions, it states that signals and images to be recovered from seemingly highly incomplete data sets. Moreover, not only is this possible in theory, reconstruction can be carried out efficiently in practice by standard numerical algorithms. This has important implications for many real-world applications, not least medical imaging, radar, analog-to-digital conversion, and sensor networks. The goal of this course is to provide a comprehensive introduction to this new field. Although the course will be primarily mathematical, applications will also be emphasized.

Graduate students in sciences and engineering are encouraged participate.

Prerequisites: A good knowledge of linear algebra, analysis, introductory probability and basic programming skills are essential. Some knowledge of functional analysis is useful but not necessary.

Textbook: No textbook is required. Lecture notes will be made available.

References:

1. E. J. Cand'es, An Introduction to compressive sensing. IEEE Signal Process. Mag. 25(2):21-30, 2008.
2. E. J. Cand'es, J. Romberg, and T. Tao. Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information. IEEE Trans. Inform. Theory, 52(2):489509, 2006.
3. D. L. Donoho. Compressed sensing. IEEE Trans. Inform. Theory, 52(4):12891306, 2006.
4. Y. Eldar and G. Kutyniok, Compressed Sensing. Cambridge, 2012.
5. M. Fornasier and H. Rauhut. Compressive sensing. Handbook of Mathematical Methods in Imaging, p.187-228. Springer, 2011.

SPARSE AND STRUCTURED MATRIX ANALYSIS (MA 69200)

Instructor: Jianlin Xia (xiaj@math.purdue.edu, 4-1922
Course Numer: MA 69200
Credits:
Time: 1:30-2:45PM TTh

Description

This course includes advanced sparse and structured matrix analysis and computations. Classical structured solvers for large matrices are reviewed. Topics related to the error and stability analysis, randomization, fast discretized PDE solutions, etc. are discussed in detail. The generalizations and applications to imaging and engineering areas are also included.

Text: Lecture notes, handouts, slides, and research papers.

References

1. Steffen Boerm, Efficient Numerical Methods for Non-Local operators, H^2-matrix compression, algorithms and analysis, European Mathematical Society, 2010.
2. Iain Duff, Direct Methods for Sparse Matrices, Clarendon Press
3. Gene Golub and Charles Van Loan, Matrix Computations, 4th edition, John Hopkins.

Prerequisite: Numerical linear algebra or similar, MA 514, CS 515, or consent of instructor.

INTRODUCTION TO THE GINZBURG-LANDAU EQUATION

Instructor: Prof. Dan Phillips (phillips@math.purdue.edu, 4-1939)
Course Number: MA 69400 CRN: 63579
Credits: Three
Time: 9:30 a.m.–10:20a.m. MWF

Description

This course gives an introductory development for the study in two dimensions of stationary solutions to the complex valued Ginzburg-Landau equation. The course prerequisite is MATH 642, however the course will be self contained beyond this. We will study the limiting structure for a family of solutions depending on a small parameter consisting of quantized vortices or defects where we will be able to determine their location and characterize their nature. The theory has a number of important applications in physics and illustrates an interesting interplay between analysis, geometry, and topology.

Prerequisite: MA 54400 or instructor's consent.

References

1 F. Bethuel, H. Brezis, and F. Helein, *Ginzburg-Landau Vortices*, Birkhauser, 1994.

2 E. Sandier and S. Serfaty, *Vortices In The Magnetic Ginzburg-Landau Model*, Birkhauser, 2007.

GROUP ACTIONS, TORIC VARIETIES AND BIRATIONAL GEOMETRY

Instructor: Prof. Jaroslaw Wlodarczyk (wlodar@math.purdue.edu, 6-7414)
Course Number 69600 CRN: 63581
Credits: 3
Time: TTh 12:00pm-1:15pm

Description

The purpose of this course is to give a survey on various techniques used in birational geometry and its interactions with invariant theory and toric geometry.

-We will introduce C^*-actions and show their verious applications. We will prove a fundamental Bialynicki-Birula theorem on decomposition of varieties , which allows to decompose the varieties into cells.

-We will discuss Weil conjectures and virtual Poincare polynomials. We compute cohomologies of various varieties using B-B decomposition and Poincare polynomials.

-We will inroduce birational cobordisms (techniques inspired by topological cobordisms and Morse theory),. We will discuss moment maps tor toric actions as an analogue of Morse function.

- In the course we introduce and discuss the theory of toric varieties as the illustration of the above mentioned techniques with particular emphasis Morelli cobordisms and C^* -actions, on Mori theory, Moment maps and theory of valuations.

- One of the main goals will be the the sketch of a proof of the Weak Factorization Theorem which states that any birational map between smooth projective varieties is a composition of blow-ups and blow-downs along smooth centers. The focus of this course is to give an intuition about the interplay of different areas of algebraic geometry.

Prerequisite: Basic knowledge about algebraic geometry (like R.Hartshorne 'Algebraic Geometry' Chapter I or similar).

Main texts:

Bialynicki-Birula, Some theorems on group actions.

Michel Brion, Emmanuel Peyre The virtual Poincare polynomials of homogeneous spaces

Igor Dolgachev, Lectures on Invariant Theory

W. Fulton, Introduction to Toric Varieties, Annals of Math. Studies 131,

Jerzy Konarski, The B-B decomposition via Sumihiro Theorem

Kenji Matsuki, Introduction to Mori Theory

Tadao Oda, Convex bodies and Toric Varieties

J. Wlodarczyk Algebraic Morse Theory and Factorization of Birational Maps.

J.Wisniewski Toric Mori Theory and Fano Manifolds.pdf.

Tentative contents of the course.

Final exam project information.

CURVATURE DIMENSION INEQUALITIES
IN RIEMANNIAN AND SUB-RIEMANNIAN GEOMETRY

Instructor: Prof. Fabrice Baudoin (fbaudoin@math.purdue.edu, 4-1406)
Course Number: MA 69600 CRN: 66073
Credits: Three
Time: 11:30 a.m.–12:20 p.m. MWF

Description

This course will be an introduction to the theory of curvature dimension inequalities. We will present the tools from the theory of semigroups that can be used to study the geometry of Riemannian or sub-Riemannian manifolds. We will cover the following topics

1. The Laplace-Beltrami operator on a Riemannian manifold, Bochners formula;
2. The heat semigroup on a Riemannian manifold;
3. Li-Yau type inequalities and Harnack estimates;
4. The heat kernel proof of Bonnet-Myers theorem;
5. Sub-Riemannian manifolds with transverse symmetries;
6. Sub-Riemannian Li-Yau inequalities;
7. Open problems and recent developments: Geometric analysis of contact manifolds.

Lecture notes will be posted on my blog http://fabricebaudoin.wordpress.com/

Prerequisites: Very basic Riemannian geometry: The most important tools will be reminded at the beginning of the class.