Vector Calculus
Instructor: Prof. Harold Donnelly
Course Number: MA 51000
Credits: Three
Time: MWF 2:30 P.M.

Description

Calculus of functions of several variables and of vector fields in orthogonal coordinate systems. Optimization problems, implicit function theorem, Green's theorem, Stokes' theorem, divergence theorems. Applications to engineering and the physical sciences. Not open to students with credit in MA 362 or 410. Typically offered Fall Spring Summer.

Required Textbook: Thomas' Calculus, Early Transcendentals, by Thomas, Weir, and Hass, Twelfth Edition, 2010.

Introduction to Partial Differential Equations
Instructor: Changyou Wang
Course Number: MA52300
Credits: Three
Time: TR 12:00 P.M.

Description

First order quasi-linear equations and their applications to physical and social sciences; the Cauchy-Kovalevsky theorem; characteristics, classification and canonical forms of linear equations; equations of mathematical physics; study of Laplace, wave and heat equations; methods of solution.

Complex Analysis
Instructor: Prof. Louis de Branges
Course Number: MA 53100
Credits: Three
Time: MWF 9:30 A.M.

Description

The course pursues the classical aims of mathematics as formulated in 1900 by Whittaker and Watson "Modern Analysis" and as implimented in the twentieth century in quantum mechanics and number theory. Complex analysis treats numbers having two real coordinates for a plane or four real coordinates for the skew–plane of quaternions. These numbers have an associative multiplication which in essential cases is also commutative. Special functions in these spaces are studied by harmonic analysis. The eventual goal of this research is the quantum mechanics of electrons in atoms and the relationship to number theory of the Riemann hypothesis.

The text for the course is an unpublished set of lecture notes on Complex Analysis available at the author's webpage. The course is open to all students who accept its aims.

Abstract Algebra I
Instructor: Prof. Bernd Ulrich
Course Number: MA55700
Credits: Three
Time: MWF 4:30 P.M.

Description

Description: The topics of the course will be commutative algebra and introductory homological algebra. We will study basic properties of commutative rings and their modules, with some emphasis on homological methods. The course should be particularly useful to students interested in commutative algebra, algebraic geometry, number theory, or algebraic topology. There will be a continuation in the spring.

Prerequisites: Basic knowledge of algebra (such as the material of MA 50300).

Texts: We will not follow any particular book, but typical texts are:

- M. Atiyah and I. Macdonald, Introduction to commutative algebra, Addison-Wesley.
- J. Rotman, An introduction to homological algebra, Academic Press.

Introduction to Differential Geometry and Topology
Instructor: Prof. Harold Donnelly
Course Number: MA 56200
Credits: Three
Time: MWF 10:30 A.M.

Description

Smooth manifolds; tangent vectors; inverse and implicit function theorems; submanifolds; vector fields; integral curves; differential forms; the exterior derivative; DeRham cohomology groups; surfaces in E3., Gaussian curvature; two dimensional Riemannian geometry; Gauss-Bonnet and Poincare theorems on vector fields. Typically offered Fall.

Required Textbook: Introduction to Differentiable Manifolds and Riemannian Geometry, by William Boothby, Revised second edition, 2003.

Representation Theory
Instructor: Prof. David Goldberg
Course Number: MA59800
Credits: Three
Time: T-TH 3:00 P.M.

Description

The theory of group representations has played (and continues to play) a role in several branches of mathematics, including algebra, number theory, geometry, and harmonic analysis. Simply put a representation of a group is a homomorphism from the group to some group of matrices. Such objects are extremely useful in understanding the structure of the group itself — for example representations of finite groups play a crucial role in the classification of finite groups. If the group has a topology in which the group operations are continuous, then one can ask that the representations have certain analytic properties, and this is how representation theory plays a role in harmonic analysis. Maybe most surprising, is that representation theory is extremely useful, even if the group you start with is already a matrix group (e.g., $SL_2(\mathbb{R})$). In this course we will discuss the representation theory of finite groups, compact groups, and then move towards representations of p-adic groups (that is "nice" subgroups of $GL_n(F)$ with F a p-adic field). Students should have taken MATH 553 and 544, and MATH 554 is useful as well. At some point we may introduce some ideas from algebraic geometry, but we will cover the basic notions from this subject that we will need. Time allowing we will discuss how p-adic representation theory plays a key role in number theory.

While there won't be a fixed textbook for the course, we will work from several references, all of which will be made available either online or on reserve in the MATH Library.

Introduction to the Hodge Theory

Instructor: Prof. Kenji Matsuki

Course Number: MA59800

Credits: Three

Time: MWF 11:30 A.M.

Description

The purpose of this course is to study the Hodge theory of the cohomology groups of complex manifolds, discussing such celebrated subjects as the Hodge decomposition of the cohomology of compact Kähler manifolds through harmonic analysis and the Hodge structures, the Hodge Index theorem, the hard Lefshetz theorem, Kodaira's vanishing theorem, Kodaira's projective embedding theorem, etc. The main sources of the lectures will be PART II of the textbook by Claire Voisin titled "Hodge Theory and Complex Algebraic Geometry", and Chapters IV - VI of the textbook by R.O. Wells titled Differential Analysis on Complex Manifolds". The prerequisites are the basics of holomorphic functions of several variables and sheaf theory, to the level covered in PART I of Voisin's book, or Chapters I - III of Well's book. However, I will try to make the prerequistes to a minimum, often reviewing the basic materials and/or at least mentioning what I am using as a black-box with some explicit reference. My goal is not to cover many topics at a high speed, but rather to learn and sit on these celebrated therems until they will be absorbed into our minds with a good understanding of the essential points. I will give several report problems along the way, and the

nal grade will be determined by the report submitted at the end of the semester.

Textbook:

- "Hodge Theory and Complex Algebraic Geometry" by Clair Voisin
- "Differential analysis on Complex Manifolds" by R.O. Wells

Prerequisites:

- basic knowledge of complex analysis of several variables,
- basic knowledge of sheaf theory and homological algebra, and
- willingness to work hard :) (the most important prerequisite)

Geometric Invariant Theory and Applications to Constructing Moduli Spaces
Instructor: Prof. Deepam Patel
Course Number: MA59800
Credits: Three
Time: T Th 10:30 A.M.

Description

This course will be an introduction to Mumford?s geometric invariant theory and it?s application to the construction of various modulii spaces in algebraic geometry. We will begin by discussing the construction of Hilbert and Picard schemes (following FGA) followed by an introduction to the basics of geometric invariant theory. In the second half of the course we will apply GIT methods to construct and study moduli spaces of stable bundles on curves. If there is enough time, we will also discuss the construction of moduli spaces of stable sheaves on surfaces and higher dimensional varieties. Prerequisites: The basic prerequisite for this course is a working knowledge of basic algebraic geometry. For example, the first 3 chapters of Hartshorne or Mumford?s ?Red Book? would be sufficient. However, I will try to keep the pre-requisites for this course as minimal as possible and briefly review some concepts (such as cohomology, smoothness, flatness etc.) as needed.

Methods of Linear & Nonlinear Partial Differential Equations
Instructor: Prof. Daniel Phillips
Course Number: MA64200
Credits: Three
Time: MWF 1:30 P.M.

Description

This is the first semester in a one-year course on the theory of PDEs. The Fall semester focuses on linear second order elliptic equations. Topics to be covered include Laplace's equation, the maximum principle, Poisson's equation and the Newtonian potential, Schauder estimates for classical solutions, Sobolev spaces, weak solutions and their regularity.

Prerequisite; MA 523.

Required Text: D. Gilbarg, N. S. Trudinger, Elliptic partial differential equations of second order. Second edition.

Introduction to the Hodge Theory
Instructor: Prof. William Heinzer
Course Number: MA65000
Credits: Three
Time: MWF 3:30 P.M.

Description

I plan to cover material from the text "Commutative ring theory" by H. Matsumura. In particular, the course will cover: properties of extension rings, integral extensions, valuation rings, dimension theory of graded rings, the Hilbert function and Hilbert polynomial, systems of parameters and multiplicity, the dimension of extension rings, regular sequences and the Koszul complex, Cohen-Macaulay rings, Gorenstein rings, regular rings and UFDs.

Schemes and Galois Theory
Instructor: Prof. Donu Arapura
Course Number: MA66500
Credits: Three
Time: TTh 3:00 P.M.

Description

Algebraic geometry went through a state of flux in the 1950's as new tools from commutative and homological algebra, algebraic topology and sheaf theory became available. By the end of that decade Grothendieck succeeded in rewriting the foundations of the subject using his theory of *schemes*, and then applied it in several interesting ways. Unfortunately, scheme theory by itself tends to be a bit dry and rather technical. As an experiment, I would like to try to teach the basic theory in concert with one of its applications, namely Grothendieck's generalization of Galois theory [2]. This is also related to the fundamental group in topology. The key idea is to define a notion which generalizes separable field extensions and plays the same role as covering spaces do in topology. These are the so called *étale* covers. Of course, we won't get to this for a while since we have basic material to cover first. I may use Hartshorne [3] or Vakil [4] (which I've heard good things about) for the basic stuff, and then I will follow the original source [2] for the more advanced material.

As for prerequites, let me suggest that at a minimum, everyone should know algebra at the level of say Atiyah-Macdonald [1] and some point set topology. Knowing more than this would be helpful, but I won't insist on it. While this class is obviously of interest to students who plan to go into the subject, it should also be interesting to people in algebra, number theory or topology.

References:

[1] M. Atiyah, I. Macdonald, Commutative algebra

[2] A. Grothendieck, Revêtement étale et groupe fondamental (SGA1)

[3] R. Hartshorne, Algebraic geometry

[4] R. Vakil, Foundations of algebraic geometry

Topics in Real Algebraic Geometry
Instructor: Prof. Saugata Basu
Course Number: MA69000
Credits: Three
Time: TR 1:30 P.M.

Description

Real algebraic geometry has been a very active area recently with connections to different areas of mathematics such as discrete geometry, harmonic analysis and theoretical computer science. The course will cover the aspects of effective and quantitative real algebraic geometry that underlie these connections – including recently obtained quantitative bounds on the Positivstellensatz, on the topological complexity of semi-algebraic sets, topology of semi-algebraic sets admitting group actions, and connections with complexity theory.

The course will be self-contained and only familiarity with basic abstract algebra will be assumed.

Special Topics on Mathematical Biology
Instructor: Prof. Julie Feng
Course Number: MA69200
Credits: Three
Time: TR 10:30 A.M.

Description

This course is an introduction to the application of mathematical methods and concepts to the description and analysis of biological processes. The mathematical contents consist of difference and differential equations, basic probability theory and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population dynamics, life history evolution, epidemiology, ecology, structured population models, and stochastic processes. Bio-mathematical research projects (in small group) may be carried out.

Modeling and Computation in Optics and Electromagnetics
Instructor: Prof. Peijun Li
Course Number: MA69200
Credits: Three
Time: TR 12:00 P.M.

Description

Prerequisite: Basic knowledge of functional and numerical analysis, and partial differential equations.

This course addresses some recent developments on the mathematical modeling and the numerical computation of problems in optics and electromagnetics. The fundamental importance of the fields is clear, since they are related to technology with significant industrial and military applications. The recent explosion of applications from optical and electromagnetic scattering technology has driven the need for modeling the relevant physical phenomena and developments of fast, efficient numerical algo- rithms. As the applied mathematics community has begun to address a few of these challenging problems, there has been a rapid development of the theory, analysis, and computational techniques in these areas. The course will provide introductory material to the areas in optics and electromagnetics that offer rich and challenging mathematical problems. It is also intended to convey some up-to-date results to students in applied and computational mathematics, and engineering disciplines as well. Particular emphasis of this course is on the formulation of the mathematical models and the design and analysis of computational approaches. Topics are organized to present model problems, physical principles, mathematical and computational approaches, and engineering applications corresponding to each of these problems.

Text: No textbook is required. Lecture notes will be made available to students

Course grade: No exams. Students are required to present course-related material in class.

References

1. G. Bao, L. Cowsar, and W. Master, Mathematical Modeling in Optical Science
2. D. Colton and R. Kress, Inverse Acoustic and Electromagnetic Scattering Theory
3. J. Jin, The Finite Element Method in Electromagnetics

4. P. Monk, Finite Element Methods for Maxwell's Equations
5. J.-C. Nédélec, Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems

Wavelets and Approximation Theory
Instructor: Prof. Bradley J. Lucier
Course Number: MA69200
Credits: Three
Time: MWF 11:30 A.M.

Description

This course will cover now-classical results on approximation theory using wavelets, together with applications to image compression, noise removal, and image reconstruction.

Emphasis will be placed on nonlinear methods in each of these areas, corresponding to coefficient quantization, wavelet shrinkage, calculations in fixed-point arithmetic, etc.

Specific topics will include: Piecewise polynomial approximation, moduli of smoothness, characterizing Besov smoothness spaces by the decay in wavelet coefficients, embeddings of Besov spaces, linear and nonlinear approximation using wavelets, linear and nonlinear noise removal, inversion of the Radon transform with noisy data. Other possible topics depending on the interests of the audience: High order wavelets, matching quantization strategies with applications, bi-orthogonal wavelets, etc.

The material will be presented from notes by the lecturer and some papers and book chapters

Homotopy Theory
Instructor: Prof. David Gepner
Course Number: MA69700
Credits: Three
Time: TR 1:30pm-2:45 P.M.

Description

This course will constitute an introduction to homotopy theory, beginning with the notion of homotopy between maps of topological spaces, (higher) homotopy groups of topological spaces, weak homotopy equivalence, and certain elementary but crucial results such as Whitehead's theorem, cellular approximation, homotopy excision, the Freudenthal suspension theorem, and the Hurewicz theorem. We will then proceed towards fibration and bundle theory, with applications to K-theory, cohomology, Chern classes, Postnikov towers, and obstruction theory. Time permitting, we will cover more specialized topics such as Brown representability, the Steenrod algebra, and stable homotopy theory.