Introduction to Differential Geometry and Topology
Instructor: Prof. Harold Donnelly
Course Number: MA 56200
Credits: Three
Time: MWF 1:30 P.M.

Description

Smooth manifolds; tangent vectors; inverse and implicit function theorems; submanifolds; vector fields; integral curves; differential forms; the exterior derivative; DeRham cohomology groups; surfaces in E3., Gaussian curvature; two dimensional Riemannian geometry; Gauss-Bonnet and Poincare theorems on vector fields. Typically offered Fall.

Required Textbook: Introduction to Differentiable Manifolds and Riemannian Geometry, by William Boothby, Revised second edition, 2003.

Algebraic Geometry and Sheaf Theory
Instructor: Professor Donu Arapura
Course Number: MA 59800
Credits: Three
Time: T Th 10:30 AM

Description

This will be a course on algebraic geometry with emphasis on sheaf theoretic and homological methods. Such methods are ubiquitous in the subject, so any serious student should understand them. Although this is more like a second course in algebraic geometry, I will try to structure it so that a beginning student can, with enough effort, follow everything. I will start by telling you what a sheaf is, and then by defining an algebraic variety, 'a la Serre, as a topological space with sheaf of functions which looks locally like an affine variety i.e. the set of zeros of a bunch of polynomials. This is a lot easier than defining a scheme, but it is in the same spirit. Once the basics are out of the way, I?ll discuss sheaf cohomology and some variants, and then give geometric applications.

I probably won't use a book as such, but here are some standard references:

Hartshorne, Algebraic geometry

Griffiths, Harris, Principles of algebraic geometry

Mumford, The "Red book" of varieties and schemes

Wells, Differential analysis on complex manifolds.

An introduction to homological algebra

Machine Learning and Uncertainty Quantification for Data Science

Instructor: Professor Guang Lin

Course Number: MA 59800

Credits: Three

Time: T Th 3:00–4:15 PM

Description

This introductory course will cover many concepts, models, algorithms and Python codes in machine learning and uncertainty quantification in data science. Topics include classical supervised learning (e.g., regression and classification), unsupervised learning (e.g., principle component analysis and K-means), uncertainty quantification algorithms (e.g., importance sampling, Markov Chain Monte Carlo) and recent development in the machine learning and uncertainty quantification field such as deep machine learning, variational Bayes, and Gaussian processes. While this course will give students the basic ideas, intuition and hands on practice behind modern machine learning and uncertainty quantification methods, the underlying theme in the course is probabilistic inference for data science.

Commutative Algebra

Instructor: Professor Bernd Ulrich

Course Number: MA 650

Credits: Three

Time: MWF 4:30 PM

Description

This is an intermediate course in commutative algebra. The course is a continuation of MA 557/558, but should be accessible to anybody with basic knowledge in commutative algebra (localization, Noetherian and Artinian modules, associated primes, dimension theory, Ext and Tor).

The topics of this semester will include: Depth and Cohen-Macaulayness, structure of finite free resolutions, regular rings and normal rings, canonical modules and Gorenstein rings, local cohomology and local duality.

Prerequisites: Some basic knowledge of commutative algebra.

Texts: No specific text will be used, but possible references are:

- H. Matsumura, Commutative ring theory, Cambridge University Press
- W. Bruns and J. Herzog, Cohen-Macaulay rings, Cambridge University Press
- D. Eisenbud, Commutative algebra with a view toward algebraic geometry,Springer.

Eisenstein Series, Automorphic Forms and L–functions
Instructor: Professor Freydoon Shahidi
Course Number: MA 69000
Credits: Three
Time: MWF 9:30 AM

Description

I am planning to teach a course on Eisenstein Series, Automorphic Forms and L-functions. I will discuss the basic facts about these objects and develop the theory of intertwining operators and sketch the theory of Eisenstein series including proofs of their meromorphic continuation and functional equations in the rank one cases which is what is needed in proving the analytic properties of all the L-functions appearing in their constant terms which we will explicitly compute. We then discuss automorphic L-functions and the computation of the non-constant terms in the generic case which allow us to prove many results about these L-functions including their functional equations and non-vanishing on Re(s)=1. Time permitting, we will discuss their connections to representation theory, functoriality and other aspects of the theory. We will complement the theory through examples.

References: Borel's Corvallis notes:Automorphic L-functions, AMS Proceedings in Pure Mathematics, No. 33, Volume 2, F. Shahidi, Eisenstein Series and L-functions, AMS Colloquium Publications, Vol. 58, 2010. F. Shahidi, An Overview of Eisenstein Series, in "p-adic Representations, Theta-correspondence and the Langlands-Shahidi Theory", Lectire Series of Modern Number Theory, Science Press, Beijing, 2013. C. Moeglin and J-L Waldspurger, Spectral Decomposition and Eisenstein Series, Cambridge Tracts in Mathematics 113, Cambridge University Press, 1995.

Topics on Theories and Methods for Structured Matrix Computations
Instructor: Professor Jianlin Xia
Course Number: MA 692
Credits: Three
Time: T Th 4:30-5:45 PM

Description

In this topic course, we will dicuss the analysis of a series of imporatant matrix computations topics, as well as the efficient and reliable numerical methods involving matrix structures. In particular, we will discuss fast structured direct solvers, effective preconditioners, and fast eigenvalue solvers. The underlying matrix structures often leads to the development of new methods that are both fast and stabble. Related stability and accuracy will also be studied. Applications to PDE solutions, image processing, and some engineering problems will also be discussed.

Complex Analysis
Instructor: Professor Louis de Branges
Course Number: MA 693
Credits: Three
Time: MWF 9:30 AM

Description

This second course in complex analysis presumes a knowledge of the text by Lars Ahlfors which is commonly used as an introductory course at the graduate level. He gives a proof of the Riemann mapping theorem which depends on the Bieberbach conjecure for the second coefficient, a theorem which Bieberbach proved in 1916. He conjectured an estimate of every coefficient which was proved in 1984 by the present analyst. The proof is interesting because of its use of special functions related to the special functions applied in a still incomplete proof of the Riemann hypothesis. This relationship makes special function a topic of current interest in complex analysis. Students having the necessary background in the text by Alhfors are welcome.

An Introduction to Fourier Integral Operators
Instructor: Antônio Sá Barreto
Course Number: MA 693
Credits: Three
Time:

Description

Fourier integral operators (Fios) were originally created for studying singularities of solutions of hyperbolic differential equations. They were intoduced by Hörmander in 1971 following the work of several people including Egorov, Hörmander, Lax, Ludwig and Maslov. Hörmander's work and a subsequent paper by Duistermaat and Hörmander have had a major impact in many areas of mathematics including spectral theory, geosphysics, scattering theory, etc. Fios operators can also be viewed as a quantization of classical objects such as canonical transformations. The goal of this course is to cover the global calculus of Fios on manifolds and use it to study some problems in spectral and scattering theory. We will cover basic topics in symplectic geometry that are necessary to develop the calculus of Fios. We will also do a short introduction to semiclassical microlocal analysis and introduce the theory of semiclassical Fios.

This course can be viewed as a continuation of MA693–Introduction to Microlocal Analysis taught by Prof. Stefanov in spring 2016. I will assume students are familiar with the calculus of pseudodifferential operators (Ψdos) (which are the simplest type of Fios), but I would not discourage *very* motivated students from taking this course without having taken MA693 in spring 2016. After all, the calculus of Ψdos will be a consequence of the results we will prove.

Outline of the syllabus

1. The stationary phase theorem, non-degenerate phase functions, Morse lemma and oscillatory integrals

2. A short review of differentiable manifolds, transversality, vector fields, differential forms, the Lie derivative
3. Distributions on manifolds, half-densities, the wave front set of a distribution
4. Symplectic linear algebra, Lagrangian subspaces
5. Symplectic manifolds, the cotangent bundle of a C^∞ manifold (T^*X), Darboux's theorem, Moser's method
6. Lagrangian submanifolds and conic Lagrangian submanifolds
7. Local parametrization of Lagrangian submanifolds by phase functions
8. Equivalence of phase functions
9. Lagrangian distributions, Melrose's definition
10. The principal symbol of a Lagrangian distribution
11. The Maslov bundle
12. Composition theorems, special cases, Egorov's theorem
13. L^2-boundedness
14. Semiclassical microlocal analysis, the semiclassical wave front set
15. Semiclassical Fios
16. Applications will de done throughout the course. Topics include Weyl's law, singularities of the trace of the wave group, propagation of singularities, global parametrices for the Cauchy problem of hyperbolic equations.

References

1. I will type (or at least scan) my lecture notes and make them available.
2. J.J. Duistermaat. *Fourier integral operators.* Vol. 130. Springer, (1996).
3. A. Grigis and J. Sjöstrand. *Microlocal analysis for differential operators. An introduction.* Cambridge Univ. Press, (1994).
4. L. Hörmander. *The Analysis of Linear Partial Differential Operators.* vol. I-IV. Springer Verlag, (1994)
5. M. E. Taylor. *Pseudodifferential operators.* Princeton Mathematical Series, 34. Princeton Univ. Press, (1981)

Topics in Harmonic Analysis; Applications to PDE, Functional Analysis and Number Theory

Instructor: Professor Victor Lie

Course Number: MA 69300

Credits: Three

Time: T Th 9:00 AM

Description

This is intended as a basic introductory course to the modern methods of Analysis. Topics include L^p spaces, tempered distributions, Fourier transform, the Riesz and Marcinkiewicz interpolation theorems, the Hardy-Littlewood maximal function, Calderon-Zygmund theory, the spaces H^1 and BMO, oscillatory integrals, almost orthogonality, restriction theorems, Radon transforms.

Specific applications of these methods to problems in other fields such as

Partial Differential Equations (e.g. Schrodinger equation, homogeneous/non-homogeneous); wave equation

Number theory: Counting Lattice Points; Poisson Formula; Hardy-Littlewood circle method

Materials/Books:

1) Main book: Stein and Shakarchi : Functional Analysis: Introduction to Topics in Analysis
2) Terence Tao: Fourier Analysis; Harmonic Anlaysis - Notes (Blog)
3) Stein - Harmonic Analysis
4) Thomas Wolff - Lecture Notes in Harmonic Analysis

Topics in Kahler–Ricci flow

Instructor: Professor Chi Li

Course Number: MA 696

Credits: Three

Time: T Th 4:30-5:45 PM

Description

Ricci flow has been proved to be a powerful tool in the study of geometric structures on Riemannian manifolds, mainly through the work of Hamilton and Perelman. The Kahler-Ricci flow is the Ricci flow on Kahler manifolds, in particular on projective manifolds. Perelman's work has a great impact on the study of Kahler–Ricci flow. One example is his breakthrough on the subject: the diameter and scalar curvature estimates along Kahler-Ricci flows on Fano manifolds. On the other hand, because of the Kahler structure, the Ricci flow can be reduced to a Monge-Ampere flow which is a natural flow of Kahler potential function and can be studied by PDE and pluripotential theoretic methods. This point of view is used extensively in the recent study of Kahler–Ricci flows on general projective varieties with its connection to the Minimal Model Program (MMP) in birational algebraic geometry. It is hoped that the above two points of view (metric and potential) can be combined to produce more deeper results through the Kahler–Ricci flow. This course aims to give an introduction to this line of thoughts, emphasizing the above two points of view. Planned materials:

1. Basic facts of Ricci flow: curvature evolution equations, maximal principle, Kahler–Ricci flow.
2. Perelman's W-functional, heat kernel estimates, local non-collapsing.
3. Kahler–Ricci flow on Fano manifolds, Perelman's estimates.
4. Kahler–Ricci flow on general projective manifolds and the relation to MMP.

Reference books on (Kahler-)Ricci flow:

1. Chow, B. et al.: The Ricci flow, techniques and applications. Part I, II, AMS, 2007.
2. Zhang, Q. S.: Sobolev inequalities, heat kernels under Ricci flow, and the Poincare conjecture, CRC Press, Boca Raton, FL, 2011.
3. S. Boucksom, P. Eyssidieux, V. Guedj: An introduction to the Kahler–Ricci flow. Lecture Notes in Math. 2086, Springer, Heidelberg 2013.

Reference papers/notes could be found in the folder (updating):
www.math.purdue.edu/ li2285/courses/KRflow.