Abstract Algebra I
Instructor: Professor Donu Arapura
Course Number: MA 55700
Credits: Three
Time: 3:00–4:15 PM TTh

Description

My plan is do commutative algebra, mostly following Atiyah-Macdonald. Although, since their style is bit compressed, I may supplement it with other sources from time to time. This is a basic course, so I will assign homework.

Algebraic Number Theory
Instructor: Professor Freydoon Shahidi
Course Number: MA 58400
Credits: Three
Time: 9:30–10:20 AM MWF

Description

Syllabus: Dedekind domains, norm, discriminant, different, finiteness of class number, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertia groups, completions and local fields.

Prerequisite: MA553

References:

1. My own lecture notes,
2. G. Janusz, Algebraic Number Fields, Graduate Studies in Mathematics, Vol. 7, AMS (optional).
3. E. Artin, Theory of Algebraic Numbers, Gottingen Notes (optional).

Alg Groups Class Field Theory
Instructor: Professor Kenji Matsuki
Course Number: MA 59800AAG
Credits: Three
Time: 10:30–11:20 AM MWF

Description

The purpose of this course is to give a concise introduction to the theory of algebraic curves, and then to proceed onto the analysis of the Abelian extensions of their function fields (Class Field Theory).

When one tries to study the subject of Algebraic Geometry using Hartshorne's textbook as a beginner, he is already exhausted by the time he reaches Chapter 3, buried in the myriads of technicalities and abstract notions of schemes, sheaves, functors etc. But actually the fun begins when he starts reading Chapter 4, dealing with the concrete geometry of curves. Also he would realize that all the technicalities and abstract notions start making more sense.

So the 1st part of this course is to present a concise introduction to the theory of algebraic curves at the level of Chapter 4 of Hartshorne's textbook, hoping that this would serve as a motivational introduction to the subject of Algebraic Geometry as a whole. We will then go more in depth into the study of algebraic curves, analyzing the singular curves and their (generalized) Jacobians, following Chapters IV and V of Serre's book.

It has been observed and well-known that there is a strong analogy between the behavior of the number fields and that of the function fields of algebraic curves. Knowing that the classical class field theory dictates the Abelian extensions of a given number field, one is naturally led to seek the corresponding story for the analysis of the Abelian extensions of the function field of a given algebraic curve. This is exactly what we do in the 2nd part of this course. We prove that any Abelian extension (covering) of an algebraic curve is obtained as the pull back of an isogeny of the (generalized) Jacobian of the curve. We prove the main theorem of Class Field Theory, study the reciprocity map and some cohomological aspects.

Contents of the course

Part I: Introduction to the theory of algebraic curves

1. Basics of algebraic curves following Chapter 4 of Hartshorne
2. More in depth study of the theory of algebraic curves following Chapters IV and V of Serre's book
    - Riemann–Roch Theorem
    - (Generalized) Jacobians
    - Maps to a commutative groups
    - Singular curves

Part II: Class Field Theory

- Covering as the pull back of an isogeny
- Main Theorem of Class Field Theory
- Reciprocity Map
- Cohomological Aspects

(i) Who should consider taking the course?

- a student who wants to start studying Algebraic Geometry and be expsoed to the basics of the theory of algebraic curves
- a student whose major interest leans toward the number theory, and yet wants to know some geometrical aspects and insights of the theory

- a student whose major interest leans toward the geometry of algebraic varieties, and yet wants to learn some arithmetical aspects and insights bridging the algebra and geometry

(ii) Textbooks

- **Algebraic Geometry by R. Hartshorne, GTM 52, Springer**
- **lgebraic Groups and Class Fields by J.P. Serre, GTM 117, Springer**

(iii) Classes:

- Lectures on Mondays, Wednesdays, and Fridays 10:30 - 11:20 in MATH 215

(iv) Grading scheme:

- Attendance is required for the student to get A.
- An end-of-the-semester report may be required, where a student discusses and solves a couple of problems mentioned during the lectures.

Mathematics of Quantum Information Theory

Instructor: Professor Thomas Sinclair

Course Number: MA 59800AMQIT

Credits: Three

Time: 3:30–4:20 PM MWF

Description

The purpose of this course is to survey some of the mathematical foundations of quantum information theory. This will include an introduction to operator systems and quantum channels, discussion of Bell's inequality, and applications of the geometry of finite–dimensional Banach spaces.

Though not a required text, we will be following the book, "Alice and Bob Meet Banach" by Aubrun and Szarek. I will supplement the lectures with my own course notes and material from other sources. No knowledge of quantum mechanics or quantum information theory is expected, but a basic working knowledge of functional analysis will be assumed.

Methods of Linear & Nonlinear Partial Differential Equations

Instructor: Professor Emanuel Indrei

Course Number: MA 64200

Credits: Three

Time: 9:00–10:15 AM TTh

Commutative Algebra
Instructor: Professor Giulio Caviglia
Course Number: MA 65000
Credits: Three
Time: 12:30–1:20 PM MWF

Algebraic Geometry III
Instructor: Professor Jaroslaw Wlodarczyk
Course Number: MA 69000AG
Credits: Three
Time: 4:30–5:45 PM TTh

Description

We shall give a continuation of the basic course in Algebraic Geometry based upon Hartshorne's *Algebraic Geometry*: Chapter 3 with emphasis on cohomology of sheaves and a remaining section 2.9 of Chapter 2 on formal schemes. We also review some concepts and Hartshorne problems from Section 2.8. The important part of the class will be solving problems from Hartshorne's book and others.The emphasis of this course is on the proofs and algebraic tools used in Algebraic Geometry.

The tentative list of the topics includes but is not limited to:

- Smoothness and formal smoothness (over a field)
- Cohen Macauley rings
- Formal schemes
- Derived functors and cohomology of sheaves.
- Cohomology of Noetheriaan affine space
- Cohomology of Projective space
- Cech cohomology
- Ext groups and sheaves
- Serre duality

Textbook and course notes:

The textbooks is Algebraic geometry(AG) by Hartshorne.

Additional reading

- Vakil's notes (V)
- Matsumura's Commutative ring theory
- Introduction to commutative algebra (AM) by Atiyah and Macdonald
- Commutative algebra with a view towards algebraic geometry by Eisenbud (E)

Homework:

There is no grader for this course. I encourage you to solve as many homework problems as you can. The solutions of homework problems will be presented by volunteers in the class or by me. The homeworks will be assigned at least a week before the presentation. In principle we are trying to solve all the Hartshorne problems with possibly some exceptions.

Introduction to Lie Groups, Lie Algebras and Their Representations
Instructor: Professor Saugata Basu
Course Number: MA 69000B
Credits: Three
Time: 1:30–2:20 PM MWF

Description

I will cover the basic theory of Lie groups and algebras, including solvability, semi-simplicity, and the Cartan–Killing classification of semi-simple Lie algebras via their Dynkin diagrams. We will then study the representation theory of Lie groups and algebras, weight spaces, highest weight theory, Schur-Weyl duality, Weyl character formula, Verma modules and BGG resolutions. If time permits I will discuss some applications of the classical theory in more modern applied areas – such as tensor decompositions and geometric complexity theory.

References:

1. Lie Groups Beyond an Introduction – A. W. Knapp.
2. Representation theory – A first course, Fulton and Harris.
3. An introduction to Lie Groups and Lie Algebras, A. Kirillov, Jr.

Fast Matrix Methods and Data Science
Instructor: Professor Jianlin Xia
Course Number: MA 69200DS
Credits: Three
Time: 12:00–1:15 PM TTh

Description

This course will cover some theories and computations related to fast matrix methods as well as the connections to various data science subjects. Selected topics include sparse linear solvers, large eigenvalue solvers, structured matrices, fast multipole methods, randomized linear algebra, fast direct solvers, data compression and clustering, neural networks, mutual interactions between matrix computations and deep learning, etc. The course materials will be based on a series of lecture notes and research papers. The purpose of this course is to survey some of the mathematical foundations of quantum information theory. This will include an introduction to operator systems and quantum channels, discussion of Bell's inequality, and applications of the geometry of finite-dimensional Banach spaces. Though not a required text, we will be following the book, "Alice and

Bob Meet Banach" by Aubrun and Szarek. I will supplement the lectures with my own course notes and material from other sources. No knowledge of quantum mechanics or quantum information theory is expected, but a basic working knowledge of functional analysis will be assumed.

Topics Mathematical Biology
Instructor: Professor Zhilan Feng
Course Number: MA 69200F
Credits: Three
Time: 9:00–10:15 AM TTh

Description

This course is an introduction to the application of mathematical methods and concepts to the description and analysis of biological processes. The mathematical contents consist of difference and differential equations and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, ecology, epidemiology, coevolutionary systems, structured population models, nonlinear dynamics, and stochastic simulations. Bio-mathematical research projects (in small group) may be carried out.

Topics on Kähler–Einstein metrics
Instructor: Professor Chi Li
Course Number: MA 69600K
Credits: Three

The purpose of this course is to survey some of the mathematical foundations of quantum information theory. This will include an introduction to operator systems and quantum channels, discussion of Bell's inequality, and applications of the geometry of finite-dimensional Banach spaces. Though not a required text, we will be following the book, "Alice and Bob Meet Banach" by Aubrun and Szarek. I will supplement the lectures with my own course notes and material from other sources. No knowledge of quantum mechanics or quantum information theory is expected, but a basic working knowledge of functional analysis will be assumed.

Time: 10:30–11:20 AM MWF

Description

I will discuss various topics related to Kähler-Einstein metrics, which may include the following (depending on the time and interest)

1. Kähler manifolds, Kähler-Einstein metrics and Chern number inequalities
2. Aubin–Yau theorem ($c_1 < 0$ case)
3. Calabi conjecture, Yau's existence theorem ($c_1 = 0$ case)
4. $c_1 > 0$ case: obstructions, continuity methods and reduction to the C^0-estimate

5. Variational characterization of Kähler-Einstein metrics, space of Kähler metrics
6. K-stability, non-Archimedean approach
7. Yau–Tian–Donaldson conjecture
8. Singular Kähler-Einstein metrics

Prerequisites: 562, 525

Reference: G. Tian. Canonical metrics in Kähler geometry. Lectures in Mathematics ETH Zurich. Birkhäuser, Basel, 2000.