Advanced Graduate Courses
offered by the
Mathematics Department
Spring, 2001

Courses

MA 598A: Continuum Mechanics in the Geosciences

Instructor: Prof. Gabrielov, office: Math 648, phone: 49-47911, e-mail: agabriel@math.purdue.edu
Time: MWF 9:30
Prerequisites: Prerequisites include standard undergraduate calculus sequence: linear algebra, ordinary differential equations and multivariate calculus.
Description: This course will provide an introduction to the fundamental ideas and methods of continuum mechanics, with a view towards geophysical applications. The goal is to provide beginning graduate students with the physics background and mathematical tools necessary for more advanced, special topics courses in continuum mechanics.

The topics to be covered include: Essential Mathematics, Stress Principles, Kinematics of Deformation and Motion, Fundamental Laws and Equations, Linear Elasticity, Friction and Fracture, Classical Fluids, Geophysical Fluid Dynamics, Numerical Methods.

The course will be based on the book *Continuum Mechanics in the Geosciences* by Prof. W.I. Newman (UCLA), to be published by Oxford University Press. Copies of the text will be distributed in class.

MA 598D: Numerical Partial Differential Equations

Instructor: Prof. Santos, office: Math 808, phone: 49-41925, e-mail: santos@math.purdue.edu
Time: TTh 3:00-4:15
Prerequisite: MA 523 or consent of instructor
Description: This course is designed for two semesters to replace the original one semester course on finite element methods (Math 524). The goal of this course is to teach the basic methodology for developing accurate, robust, and efficient algorithms for the numerical solution of partial differential equations in applied mathematics, science and engineering. The course will provide the mathematical foundation of numerical methods together with important numerical aspects. Applications to some basic problems in mechanics and physics will also be considered.
Fall Semester 2000. The course will begin with finite difference and finite element methods for two-point boundary value problems and direct and iterative methods for the resulting algebraic equations. Finite difference and finite element methods will be developed and analyzed for elliptic and parabolic partial differential equations. Iterative solvers including preconditioned conjugate gradient, domain decomposition, and multigrid methods will be introduced for the resulting system of linear and nonlinear equations from the discretizations of elliptic and parabolic problems. Finally, we will discuss numerical methods for hyperbolic partial differential equations. Some implementational aspects will be considered.
Spring Semester 2001. The second semester will begin with polynomial approximation theory in Sobolev spaces. We will then develop and analyze mixed finite element methods for both elliptic and parabolic equations. As a special case of the mixed method, we will introduce the finite volume method. Domain decomposition and/or multigrid methods will be further studied. Topics on advanced methods such as methods of characteristics, least squares, and adaptive mesh refinement and on applications such as incompressible Stokes and Navier-Stokes, elasticity, Maxwell, porous media, and pseudo-differential equations are at the discretion of the instructor. These are current topics of very active research in computational mathematics.

MA/STAT 598G: Advanced Probability and Options, with Numerical Methods

Instructor: Prof. Viens, office: Math 504, phone: 49-46035, e-mail: viens@stat.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: MA 598F or consent of the instructor.
Description: The second course in a two–course sequence on the mathematics of finance, and especially on option pricing. Topics covered will include: Interest rate models, American options and stochastic optimal stopping, Binomial, Monte–Carlo, and finite difference methods for solving the partial differential equations of option pricing.
Text:1. T. Bjork, *Arbitrage Pricing in Continuous Time*, Oxford, 1998.
2. D. Lambertson and B. Lapeyre, *Stochastic Calculus applied to Finance*, Chapman and Hall/CRC, 1996, reprinted 2000 by CRC Press.

MA 598W: Lie Groups
Instructor: Prof. Wilkerson, office: Math 450, phone: 49-41955, e-mail: wilker@math.purdue.edu
Time: MWF 10:30
Description: This will be an introduction to the basic facts about Lie groups and Lie algebras, with concentration on the compact connected case. Although metric space topology and differential geometry could be helpful, algebraic topology will not be assumed.

Rough outline: First half: manifolds, topological groups, Lie groups, exponential map, tori, maximal tori, Weyl groups, classification of simple Lie groups/Lie algebras. Second half: representation theory for the compact case.

Homework can be customized to student interests.

Text: Frank Adams, *Lectures on Lie Groups*, Midway Press, about $20.
References:1. F. Warner,*Foundations of Differential Manifolds and Lie Groups*,
2. J.–P. Serre*Algebres de Lie semi-simples complexes*
3. Broecker–tom Dieck, *Representations of Compact Lie Groups*
4. Fulton–Harris, *Representation Theory*

MA 611: Methods of Applied Mathematics
Instructor: Prof. Phillips, office: Math 706, phone: 49-41939, e-mail: phillips@math.purdue.edu
Time: MWF 1:30
Prerequisite: MA 511 and MA 544
Description: This course develops the functional analysis needed to study differential equations and applied math. Examples and applications will be given using Sobolev spaces and Sturm-Liouville theory.Topics include Banach and Hilbert spaces; convex analysis; weak topologies; linear oper ators; Lax-Milgram theorem; compact operators; Riesz-Fredholm theory; spectral theory of compact operators.
Text: Avner Friedman, *Foundations of Modern Analysis*, Dover
Reference: Haim Brezis, *Analyse Fonctionelle Theorie et applications*. Students are encouraged to order the reference directly from the publisher at http://www.dunod.com/cgi-bin/booke.pl?179:1:'au=(Brezis%2CH*)'

MA 631: Several Complex Variables
Instructor: Prof. Catlin, office: Math 744, phone: 49-41958, e-mail: catlin@math.purdue.edu
Time: MWF 12:30
Prerequisite: MA 530
Description: Holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains, L^2–theory in pseudoconvex domains, Levi problem, Cousin rpoblems, Weierstrass preparation theorem and consequences.

MA 643: Methods of Linear and Nonlinear Partial Differential Equations II
Instructor: Prof. Phillips, office: Math 706, phone: 49-41939, e-mail: phillips@math.purdue.edu
Time: MWF 2:30
Prerequisites: MA 642
Description: Continuation of MA 642. Topics to be covered are L^p theory for solutions of elliptic equations, including Moser's estimates, Aleksandrov maximum principle, and the Calderon–Zygmund theory. Introduction to evolution problems for parabolic and hyperbolic equations, including Galerkin approximation and semigroup methods. Applications to nonlinear problems.
Text: Lawrence C. Evans, *Partial Differential Equations*, AMS, Graduate Studies in Mathematics, vol. 19, 1998.

MA 646: Banach Algebras and C^* Algebras
Instructor: Prof. L. Brown, office: Math 704, phone: 49-41938, e-mail: lgb@math.purdue.edu
Time: MWF 11:30 **Note New Time**
Prerequisite: MA 546 or equivalent
Description: The subject of operator algebras originated from models for quantum physics and has since been related to many other fields. Some aspects of the theory can be described as "non–commutative topology", "non–commutative geometry", "non–commutative probability theory", etc. For more on this, see:
http://msri.org/activities/programs/0001/opalg/ Although all of these topics are beyond the scope of the course, the intent is to provide the prerequisites for such advanced topics.

Topics will include Banach algebras, Gelfand theory, the commutative Gelfand–Naimark theorem and applications to normal operators, C*–algebras and representations, the non–commutative Gelfand–Naimark theorem, von Neumann algebras, and Murray–von Neumann equivalence. Some operator theory or other topics may be included as time permits.

References:

1. W. Arveson, *An Invitation to C^*–algebras*
2. J. Conway, *A Course in Functionsl Analysis*, chaps VII, VIII, and IX
3. R. Kadison and J. Ringrose, *Fundamentals of Operator Algebras*
4. J. von Neumann, *Collected Works*, vol. III, chaps 2,3,4,5
5. G. Pedersen, *C^*–algebras and their Automorphism Groups*
6. G. Simmons, *Introduction to Topology and Modern Analysis*
7. M. Takesaki, *Theory of Operator Algebras*

MA 664: Algebraic Curves and Functions II
Instructor: Prof. Abhyankar, office: Math 432, phone: 49-41933, e-mail: ram@math.purdue.edu
Time: TTh 3:00-4:15
Description: Althought a continuation of MA 663 offered in fall, 2000 semester, it will be quite possible to take the Spring course without having taken the Fall course. So all interested students are welcome. The first lecture of the course will be on January 30 (and not on January 9)

MA 665: Algebraic Geometry
Instructor: Prof. Archava, office: Math 748, phone: 49-43173, e-mail: archava@math.purdue.edu
Time: TTh 10:30-11:45 **Note New Time**
Description: This will be a continuation of the Riemann surfaces course in the fall, 2000 semester, covering more advanced topics: cohomology, Riemann-Roch formula, Serre's duality, Picard and Jacobi varieties, Abel's theorem, Jacobi inversion and Torelli's theorem.
Text: 1. R. Narasimhan, *Compact Riemann surfaces*
2. O. Forster *Lectures on Riemann surfaces*

MA 690B: Topics in Commutative Algebra
Instructor: Prof. Heinzer, office: Math 636, phone: 49-41980, e-mail: heinzer@math.purdue.edu
Time: MWF 3:30
Prerequisite: MA 557
Description: The course will cover material concerning the following topics: (1) the number of equations needed to describe an algebraic variety, (2) the graded ring and conormal module of an ideal, (3) regular and singular points of algebraic varieties, (4) projective resolutions and Hilbert's syzygy theorem.
Text:, *Introduction to Commutative Algebra and Algebraic Geometry* by Ernst Kunz, Birkhauser, Boston 1985

MA 690D: Automorphic L–functions
Instructor: Prof. Shahidi, office: Math 650, phone: 49-41917, e-mail: shahidi@math.purdue.edu
Time: MWF 9:30 **NOTE NEW TIME**
Description: I will try to cover Langlands' *Euler Products* manuscript. This book is the origin of the method that eventually led to recent suprising progress in Langlands functoriality conjecture which is one of the central conjectures in modern number theory. Some knowledge of structure theory of algebraic groups (roots, weights, ...) and number theory (local and global fields, adeles, ideles, ...) will be helpful, although, I am planing to review some. Otherwise I will try to keep it as self–contained as I can.

MA 692C/AGRY 598C/BME 695: Mathematical Modeling of Controlled Drug and Pesticide Release
Instructor: Prof. Cushman, office: Math 816, phone: 49-48040, e-mail: jcushman@math.purdue.edu
Time: TTh 1:30-2:45
Prerequisite:
Description: Controlled release of chemicals plays an ever increasing role in such diverse fields as medicine, biotechnology and agriculture. The use of biologically active ingredients in an uncontrolled fashion is environmentally hazardous, dangerous to human health, and not cost effective. Because of these concerns there has been a proliferation of research on simulating controlled release and using these simulation tools to optimally design new controlled release substrates. The purpose of this course is to provide the mathematical background necessary to model controlled release. We introduce the student to mathematical homogenization, hybrid mixture theory, and the requisite stochastic tools.
Outline:

- Introduction to controlled release systems: Diffusion in homogeneous and heterogeneous substrates, Surface dissolution and chemical reactions, Swelling and dissolution, Diffusion in swelling systems, Osmotic pumps.
- Diffusion Stochastic differential equations (lagrangian perspective), Brownian processes, Levy flights, Fokker Planck equations (eulerian perspective), Classical parabolic equations, Fractional equations, Other anomalous diffusions.
- Mathematical Homogenization Formal asymptotic expansions, Effective properties, Distributed microstructures.
- Mathematical Homogenization (con't) Cell problems, Multiple scales, Dual porosity and Greens functions, Multiple phases.
- Mixture Theory Classical continuum physics, Mixtures of species, Porous mixtures, Swelling porous mixtures, Multiple-scale hybrid mixture theory.
- Uncertainty in heterogeneous substrates and boundaries Stochastic perturbation, Stochastic Greens functions, Stochastic initial and boundary data, Stochastic PDE's.

MA 692F: Biomathematical Modeling
Instructor: Prof. Feng, office: Math 848, phone: 49-41915, e-mail: zfeng@math.purdue.edu
Time: MWF 9:30
Prerequisite: Calculus. Some knowledge of linear algebra would be helpful.
Description: This course is an introduction to the application of mathematical methods and concepts to the description and analysis of biological processes. The mathematical contents consist of difference and differential equations and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, epidemiology, coevolutionary systems, structured population models, nonlinear dynamics and chaos, stochastic processes, and introduction to *Mathematica* (computer package). Bio-mathematical research projects (in small group) may be carried out.
Text: Selected chapters from various texts, Class notes, Handouts, and Articles on Reserve.

MA 693B: Riemann Theta Functions
Instructor: Prof. de Branges, office: Math 800, phone: 49-46057, e-mail: branges@math.purdue.edu
Time: MWF 10:30
Description: When the Riemann hypothesis is accepted as a fundamental issue, the question arises for what functions the conjecture made by Riemann for the classical zeta function applies. An incomplete answer is given when the Euler product and functional identity are offered as the properties essential to the Riemann hypothesis. Unanswered is the question of how these properties arise. An answer lies in an adelic generalization of Fourier analysis. A Riemann zeta function is the Mellin transform of a Riemann theta function, which is used to define an adelic generalization of the Laplace transformation. Quadratic extensions of the p–adic line are combined with the complex plane to construct the adelic plane for the adelic Laplace transformation. Although the real line has a unique quadratic extension, a p–adic line has several quadratic extensions. A large class of Riemann theta functions result from the choices of p–adic planes in an adelic plane. The corresponding Fourier analysis not only motivates the Riemann hypothesis but also supplies a background for applications. A knowledge of classical Fourier analysis and of p–adic number fields is presumed. There are no other prerequisites.