Advanced Graduate Courses
offered by the
Mathematics Department
Spring, 2002

MA 545: Functions of Several Variables and Related Topics
Instructor: Prof. R. Bañuelos, office: Math 608, phone: 49-41977, e-mail: banuelos@math.purdue.edu
Time: MWF 9:30
Prerequisites: Math 544. However, we will review, depending on the need, some of the topics from the last few weeks of 544 including differentiation of monotone functions, the Radon-Nikodym theorem, duality of L^p spaces, and Fubini's theorem.
Description: This course will cover some of the basic tools of analysis which are useful in several areas of mathematics including PDE's, stochastic analysis, modern harmonic analysis and complex analysis. Topics include: The Hardy–Littlewood maximal function; convolutions; approximations to the identity and their applications to density theorems in $L^p(R^n)$ and to the basic boundary value problems in the the upper half space of R^n (the Dirichlet problem for the heat equation and the Laplacian with L^p–data); the Fourier transform and its basic properties on L^1 and L^2 (including Plancherel's theorem); interpolation of linear operators; basic singular integral theory and its applications to the Beltrami equation and to the Cauchy Riemann equations in several dimensions; Sobolev inequalities. Along the way we will develop some of the basic tools of Littlewood–Paley theory and prove the Calderón reproducing formula which is very useful in the theory of wavelets.
Text: The students enrolled in this course will receive a free copy of my book, *"Lecture Notes: A Second Semester Course in Analysis."*
Other References: 1. E. H. Lieb and M. Loss, *"Analysis."*
2. E. M. Stein, *"Singular Integrals and Differentiability Properties of Functions."*

MA 572: Introduction to Algebraic Topology
Instructor: Prof. J. McClure, office: Math 714, phone: 49-42719, e-mail: mcclure@math.purdue.edu
Time: MWF 1:30
Prerequisites: Basic point-set topology, up to compactness and connectedness. Some knowledge of the fundamental group is also desirable.
Description: This spring I will be teaching MA 572 (Introduction to Algebraic Topology) in a way which is designed to be useful for students in other fields, especially algebraic geometry, commutative algebra, and several complex variables. The first part of the course will be about line integrals in open subsets of the plane and their paths of integration.
Text: 1. Fulton, *Algebraic Toploogy.*
2. Greenberg and Harper, *Algebraic Topology: A First Course.*

MA 586: Mathematical Logic II
Instructor: Prof. O. De la Cruz, office: Math 446, phone: 49-47912, e-mail: odlc@math.purdue.edu
Time: TTh 9:00 - 10:15
Prerequisites: MA 585
Description: This course will be run as a seminar, covering several points which complement the material in MA 585. The grade will be based on class participation and the presentation of at least one topic in class.

A few of the topics that might be covered include: Some techniques in "classical" Model Theory; linear, temporal, many–valued, non-monotonic or modal logics; Chaitin's information–theoretical approach to incompleteness; and more. The final choice of topics will depend on the students' and my interests.

MA 598A: Homotopy Theory
Instructor: Prof. J. Smith, office: Math 720, phone: 49-47910, e-mail: jhs@math.purdue.edu
Time: MWF 1:30
Prerequisites: Math 572 and some basic algebra.
Description: Homotopy theory begins with a definition: a homotopy is a one parameter family of maps. Homotopy defines an equivalence relation on the set of continuous maps. Like Janus, Homotopy Theory has two faces, an inward face that studies the homotopy equivalence classes of maps between spaces and and outward face that finds applications of homotopy theory in other areas of mathematics. MA 598A will be run as a seminar. Students will be introduced to methods for understanding homotopy classes and to the many applications of Homotopy Theory.

MA 598D: Numerical Partial Differential Equations
Instructor: Prof. Santos, office: Math 808, phone: 49-41925, e-mail: santos@math.purdue.edu
Time: TTh 3:00-4:15
Prerequisite: MA 523 or consent of instructor
Description: The objective of the course is to teach the methodology for developing efficient and accurate algorithms for the numerical solution of partial differential equations in problems arising in applied mathematics, physics, geophysics and engineering.

The course will begin with polynomial approximation theory in Sobolev spaces. Then the theory and application of mixed finite element method for the solution of second order elliptic and parabolic problems will be covered, including the computer implementation of the algorithms to simulate flow in porous media.

Iterative procedures for the numerical simulation of waves in dispersive media and the approximate solution of Maxwell's equations as well as the associated inverse problems will be covered at the discretion of the instrutcor and depending on time limitations.
Reference Textbooks: 1. S. C. Brenner and L. R. Scott, *The Mathematical Theory of Finite Element Methods*, Springer, 1994.
2. P. G. Ciarlet, *The Finite Element Method for Elliptic Problems*, North-Holland, 1980.

MA/STAT 598E: Computational Financial Mathematics II
Instructor: Prof. S. Stojanovic, office: Math 427, phone: 49-41957, e-mail: stojanov@math.purdue.edu
Time: TTh 12:00-1:15 & 1:30-2:45, Weeks 1 – 7.5 only (January 7th through February 27th)
Prerequisites: MA & STAT 598D Computational Financial Mathematics I (or equivalent knowledge in Mathematica programming and financial mathematics)
Syllabus: Fast numerical solutions of Black-Scholes and Dupire PDE's; implied volatility of European options via optimal control of PDE's; American options: optimal stopping, free boundary problems, symbolic and fast numerical solutions of obstacle problems; implied volatility for American options via optimal control of obstacle problems; optimal portfolio rules: stochastic control with and without constraints, Hamilton-Jacobi-Bellman PDE's, Monge-Ampere type PDE's, symbolic solutions.
Teaching Style: Sophisticated theories are presented in a practical style, which with the help of the programming capabilities of Mathematica, help the students develop good intuition about the real trading of stocks and options, as well as about the wide variety of mathematics involved.
Text: S. Stojanovic, *Computational Financial Mathematics using Mathematica*, Purdue University Preprint, August 2001; to be published by Birkhauser, Boston, 2002, ISBN 0-8176-4197-1

MA 598F: Mathematics of Finance
Time: TTh 10:30-11:45
Prerequisites: MA 519 (or equivalent) + MA 261 (or equivalent) + MA 440 (or equivalent); or consent of the instructor.
Description: We will provide an introduction to the mathematical tools and techniques of modern finance theory, in the context of Black-Scholes-style option pricing. The typical (pricing) question is: how much should you charge someone for allowing them the right to purchase a certain stock from you at a given price and given time in the future? The typical (Black-Scholes) assumption is that the differential of the (log of the) stock price is the sum of a constant term (r.dt, constant interest rate) and a random noise term (dW(t), a Brownian increment). Under this assumption, to answer the pricing question, the main mathematical tool is stochastic calculus and its connection to partial differential equations. These mathematics will be the object of a thorough introduction at an elementary level, without measure theory. This toolbag will enable us to derive the main pricing and hedging results in complete and incomplete markets, and to treat many examples of exotic and path-dependent options, as well as an introduction to stochastic optimal control and portfolio optimization.
Text: T. Bjork, *Arbitrage Pricing in Continuous Time*, Oxford, 1998.
Suggested additional reading: D. Lambertson and B. Lapeyre, *Stochastic Calculus applied to Finance*, Chapman and Hall/CRC, 1996, reprinted 2000 by CRC Press.; P. Wilmott, S. Howison, J. Dewynne, *The mathematics of financial derivatives. A student introduction.* Cambridge U.P. 1995. Chapters 11 to 16.

MA 598G: Advanced Probability and Options, with Numerical Methods
Instructor: Prof. F. Viens, office: Math 504, phone: 49-46035, e-mail: viens@stat.purdue.edu
Time: TTh 9:00-10:15
Prerequisites: Those who have not had MA 598 F as a prerequisite can still hope to enroll in the course by providing evidence of basic preparation in stochastic calculus, and by reading the material of the first 10 chapters of the textbook by Bjork, before class begins in January. Several students in the past have succesfully achieved this preparation.
Description: This is the second course in a two-course sequence on the mathematics of finance, and especially on option pricing. The material will be divided in two parts. First, we will cover theoretical issues regarding: (i) Interest rate term structure models; (ii) American options and stochastic optimal stopping; (iii) finite difference methods. Then we will examine in detail the numerical methods used to solve the partial differential equations and inequalities that determine the prices of options, including the Binomial, Monte-Carlo, and finite difference methods.

MA 611: Methods of Applied Mathematics I
Instructor: Prof. A. SaBarreto, office: Math 604, phone: 49-41965, e-mail: sabarre@math.purdue.edu
Time: MWF 11:30
Prerequisites: MA 511 or equivalent and MA 544.
Description: Banach and Hilbert spaces; linear operators; spectral theory of compact linear operators; applications to linear integral equations and to regular Sturm-Liouville problems for ordinary differential equations.

MA 631: Several Complex Variables
Instructor: Prof. L. Lempert, office: Math 734, phone: 49-41952, e-mail: lempert@math.purdue.edu
Time: TTh 1:30-2:45
Prerequisites: MA 530
Description: Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. Local theory of analytic sets (Weierstrass preparation theorem and consequnces). Functions and sets in the projective space P^n (theorems of Weierstrass and Chow and their extensions).

MA 639: Stochastic Processes II
Instructor: Prof. J. Ma, office: Math 620, phone: 49-41973, e-mail: majin@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisites:
Description: This is the second part of the two-semester course (the first part was MA638 offered in Fall 2001). In this part I will mainly focus on the theory of stochastic differential equations and their applications in stochastic control and mathematical finance.

Detailed topics include:

A. Martingale theory and stochastic integrations: Brownian motion and Brownian filtrations, stochastic integration (for general martingales), Ito's formula(s), Martingale representation theorem, Girsanov transformations,...
B. Stochastic differential equations: general theory on stochastic differential equations, strong solutions, weak solutions, backward stochastic differential equations,...
C. (optional) Applications of stochastic differential equations: basics in stochastic control, linear and nonlinear filtering, finance,...

This course will provide a solid foundation and many necessary tools for further study in various directions of stochastic analysis.

MA 643: Methods of Linear and Nonlinear Partial Differential Equations II
Instructor: Prof. N. Garofalo, office: Math 616, phone: 49-41971, e-mail: garofalo@math.purdue.edu
Time: TTh 10:30-11:45
Description: This course will be a continuation of MA 642, only in the sense that the material covered in the Fall semester constitutes a useful exposure to some of the tools and ideas used in pde's. In reality, the course will have a self-contained character, and can be profitably attended also by students who have not taken MA 642 in the Fall, provided that they are willing to make a serious effort. We will continue the study of some of the main trends in pde's of the second order, with special emphasis on topics which lie at the interface of classical analysis and geometry. Topics to be covered are L^p theory for solutions of elliptic equations, including Moser's estimates, Alexandrov maximum principle, and the Calderon-Zygmund theory. We will discuss the Yamabe problem, its solution, and its ramifications. We will also give an introduction to the theory of minimal surfaces and generalized perimeters, and present some of the basic results in this subject.

MA 661: Modern Differential Geometry
Instructor: Prof. L. Tong, office: Math 748, phone: 49-43173, e-mail: tong@math.purdue.edu
Time: MWF 10:30
Prerequisites: Some knowledge of manifolds and topology such as those in MA 562 and 572.
Description: The following topics will be discussed in this course: the theory of characteristic classes and equivariant cohomology based on differential forms, various types of index theorems and localization techniques. The subject matters are of interest to students in algebraic geometry as well as in differential geometry.

MA 690A: Topics in Algebraic Geometry
Instructor: Prof. S. Abhyankar, office: Math 600, phone: 49-41933, e-mail: ram@math.purdue.edu
Time: TTh 3:00-4:15
Description: Various topics of current interest will be discussed. First meeting will be on January 29.

MA 690B: Topics in Commutative Algebra
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49-41980, e-mail: heinzer@math.purdue.edu
Time: MWF 12:30
Description: A continuation of MA 690B from Fall, 2001 semester.

MA 690C: Graded Free Resolutions
Instructor: Prof. I. Peeva, office: Math 848, phone: 49-41923, e-mail: ipeeva@math.purdue.edu
Time: MWF 2:30
Prerequisites: A course in Commutative Algebra
Description: The structure of a finitely generated module T over a commutative noetherian ring R can be described by a free resolution. The idea to associate a free resolution to T was introduced in a paper of Hilbert; he proved that if R is a polynomial ring, then any finitely generated R-module has a finite free resolution. In essence constructing a free resolution consists of solving systems of R-linear equations.

The course will cover material concerning the following topics:
1) an introduction to the basic properties of graded free resolutions
2) resolutions of semigroup (toric) ideals
3) resolutions of monomial ideals
4) resolutions over complete intersections.

MA 690E: Sheaf Theoretical Methods in Algebraic Geometry
Instructor: Prof. D. Arapura, office: Math 642, phone: 49-41983, e-mail: dvb@math.purdue.edu
Time: TTh 12:00-1:15
Description: In this course I plan to introduce sheaf theoretical methods including sheaf cohomology. My emphasis will be on the application of these ideas to algebraic geometry, however I will try to do things such as the de Rham theorem, which would be of interest to others. So if anyone wants to sit in for the first few weeks, that's fine with me. Eventually knowledge of the basic theory of algebraic varieties will be necessary. I'm not going to assume any prior knowledge of homological algebra or algebraic topology (although it certainly wouldn't hurt).

MA 693B: Scattering Theory
Instructor: Prof. L. de Branges, office: Math 800, phone: 49-46057, e-mail: branges@math.purdue.edu
Time: MWF 10:30
Description: This course in linear and complex analysis is concerned with the invariant subspaces of continuous and contractive transformations in Krein spaces which have contractive adjoints. A canonical model of a transformation is constructed in a Krein space whose elements are power series with coefficients in a Hilbert space. The space is characterized as the state space of a canonical conjugate isometric linear system. A construction is made of canonical conjugate isometric linear systems with given transfer function. The transfer function is assumed to be a power series with operator coefficients which defines a Toeplitz multiplication in the space of square summable power series with coefficients in a Hilbert space. The multiplication is not assumed to be everywhere defined and bounded. The main transformation admits an invariant subspace which is a Hilbert space and whose orthogonal complement is the anti-space of a Hilbert space. Factorizations of transfer functions result from invariant subspaces of the main transformation. The converse construction of invariant subspaces from factorizations is of interest when the state

space is a Hilbert space. The existence of a nontrivial proper closed invariant subspace is obtained for a contractive transformation of a Hlbert space into itself which is not a scalar multiple of the identity transformation. A close relationship to the Nevanlinna factorization theory for functions which are meromorphic and of bounded type in the unit disk is found when the coefficient space is one-dimensional. Applications are given to the estimation theroy of functions which are analytic and injective in the unit disk and which have a fixed point at the origin. Students are expected to have the mathematical maturity which ususally results from a course in functional analysis (MA 546).

MA 693D: Infinite Discrete Groups
Instructor: Prof. M. Dadarlat, office: Math 708, phone: 49-41940, e-mail: mdd@math.purdue.edu
Time: MWF 1:30
Prerequisites: Some prior exposure to real analysis and/or functional analysis would be certainly helpful but not absolutely necessary.
Description: This will be an introduction to analysis on groups and geometric group theory. For the sake of simplicity, we will focus mainly on discrete groups. Topics include: elementary group representation theory, amenability and other growth conditions, groups with property T of Kazhdan and their applications. If time permits we may discuss certain invariants such as the ℓ^2 Betti numbers and the K-theory of group algebras.

MA 693E: Potential Theory
Instructor: Prof. A. Eremenko, office: Math 612, phone: 49-41975, e-mail: eremenko@math.purdue.edu
Time: TTh 9:00-10:15
Prerequisites: Complex Analysis (MA 530) and Measure Theory (MA 544).
Description: Potential theory is one of the major areas of analysis. Classical Potential Theory (= Laplace and Poisson equations, Newtonian and Logarithmic potentials) originates in Mathematical physics, namely in problems of gravity and electrostatics. As it frequently happens with theories coming from physics, potential theory and its generalizations became indispensible powerful tools in almost all parts of Analysis, including the theory of analytic functions (of one and several complex variables), PDE, Probability, Approximation theory and Holomorphic Dynamics.

The purpose of the course is to introduce the basic notions and facts about subharmonic functions, Newtonian and Logarithmic potentials, to lay a necessary basis for further study of advanced modern generalizations such as nonlinear potential theory and pluripotential theory. Some applications of this classical theory will be also discussed including the recent ones in holomorphic dynamics.

Contents:

1. Review of harmonic functions
2. Electrostatics
3. Energy
4. Equilibrium distributions
5. Dirichlet's Problem and Green's Function. Capacity
6. Subharmonic functions
7. Riesz Representation Theorem
8. Symmetrization and extremal problems
9. Uniform Approximation
10. Applications to Banch Algebras
11. Applications to Holomorphic dynamics.

Recommended literature: 1. Th. Ransford, *Potential theory in the complex plane* (main text).
2. J. Wermer, *Potential Theory.*
3. L. Hormander, *Notions of Convexity.*
4. M. Brelot, *Elements de la theorie du potentiel.*