Seminars and Advanced Graduate Courses offered by the Mathematics Department Spring, 2003

MA 545: Functions of Several Variables and Related Topics
Instructor: Prof. N. Garofalo, office: MATH 616, phone: 49-41971, e–mail: garofalo@math.purdue.edu
Time: TTh 10:30-11:45
Description: In this course we will start with some basic topics in harmonic analysis, such as e.g. the covering theorems of Vitali, Wiener, Besicovitch, and the Whitney decomposition. Several applications of these results will be presented, starting from the classical weak L^1 continuity of the Hardy-Littlewod maximal operator and the Sobolev and isoperimetric inequalities. We will analyze the powerful methods of real and complex interpolation and apply them to the study of some fundamental problems involving the Fourier transform. Particular attention will be devoted to the "restriction problem", which studies the possibility of restricting the Fourier transform of a function in a Lebesgue space L^p to a lower dimensional manifold. This is a problem which is at the forefront of present developments in the theory of partial differential equations. The role of curvature in this problem will be amply discussed.

This course will have a completely self-contained character, but the students will need to know the Lebesgue integral, and a few basic facts from calculus of several variables.

MA 572: Introduction to Algebraic Topology
Instructor: Prof. J. McClure, office: MATH 714, phone: 49-42719, e–mail: mcclure@math.purdue.edu
Time: MWF 1:30
Description: I will be teaching MA 572 (Introduction to Algebraic Topology) in a way which is designed to be useful for students in other fields, especially algebraic geometry, commutative algebra, and several complex variables. The first part of the course will be about line integrals in open subsets of the plane and their paths of integration.
Prerequisites: Basic point-set topology, up to compactness and connectedness. Some knowledge of the fundamental group is also desirable.
Texts: 1. Fulton, *Algebraic Toplogy*
2. Greenberg and Harper, *Algebraic Topology: A First Course*

MA 598A: Using Algebraic Geometry
Instructor: Prof. A. Gabrielov, office: MATH 648, phone: 49-47911, e–mail: agabriel@math.purdue.edu
Time: MWF 8:30
Prerequisite: Linear Algebra; no previous knowledge of algebraic geometry is expected.
Description: This graduate course, suitable also for advanced undergraduates, will introduce some basic ideas from Algebraic Geometry, emphasizing computational aspects and interactions with linear algebra and combinatorics.

We will cover roughly chapters 1-3 and 7 of the text: *Using Algebraic Geometry* by Cox, Little and O'Shea, with some preliminaries from *Ideals, Varieties, and Algorithms* by Cox, Little and O'Shea, and *Introduction to Groebner Bases* by Adams and Loustaunau.

Our principal subject will be solving systems of polynomial equations, both algebraically and geometrically. Two principal computational approaches are based on Groebner bases and resultants. For sparse systems solving, connection with polytopes and toric varieties will be discussed.

Some homework will use Maple, but no previous experience with Maple is required.

MA 598D: Numerical Partial Differential Equations
Instructor: Prof. Santos, office: Math 808, phone: 49-41925, e-mail: santos@math.purdue.edu
Time: TTh 3:00-4:15
Prerequisite: MA 523 or consent of instructor
Description: The objective of the course is to teach the methodology for developing efficient and accurate algorithms for the numerical solution of partial differential equations in problems arising in applied mathematics, physics, geophysics and engineering.

The course will begin with polynomial approximation theory in Sobolev spaces. Then the theory and application of mixed finite element method for the solution of second order elliptic and parabolic problems will be covered, including the computer implementation of the algorithms to simulate flow in porous media.

Iterative procedures for the numerical simulation of waves in dispersive media and the approximate solution of Maxwell's equations as well as the associated inverse problems will be covered at the discretion of the instrutcor and depending on time limitations.
Reference Textbooks: 1. S. C. Brenner and L. R. Scott, *The Mathematical Theory of Finite Element Methods*, Springer, 1994.
2. P. G. Ciarlet, *The Finite Element Method for Elliptic Problems*, North-Holland, 1980.

MA 598E: Elliptic Curves
Instructor: Prof. J. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 553
Description: The course will survey some elementary algebraic, geometric, analytic and arithmetic features of elliptic curves (nonsingular projective plane cubics, or abstract curves of genus 1). Topics: uniformization; addition theorem for elliptic integrals, and its avatar as the group law on an elliptic curve; finite generation of the group of rational points (Mordell); integer points; elliptic curves over finite fields; complex multiplication; two-dimensional Galois representation of torsion subgroups.

Elliptic curves play a large role in the study of Diophantine equations. The course takes the first few steps of a thousand-mile journey to Wiles's proof of Fermat's Last Theorem. Some examples of more ancient vintage (first three discovered by Fermat):

- The only rational (x, y) satisfying $x^3 + y^3 = 1$ are $(1, 0)$ and $(0, 1)$.
- There do not exist four integer squares in arithmetic progression [$\leftrightarrow$ the only rational x, y satisfying $y^2 = x(x+1)(x+4)$ are $(0,0)$, $(-1,0)$, $(-4,0)$, $(-2,\pm 2)$, $(2,\pm 6)$].
- There do not exist four integer squares in arithmetic progression [$\leftrightarrow$ the only rational x, y satisfying $y^2 = x(x+1)(x+4)$ are $(0,0)$, $(-1,0)$, $(-4,0)$, $(-2,\pm 2)$, $(2,\pm 6)$].
- Find all right triangles whose sides have integer lengths $a < b < c$ such that both c and $a + b$ are squares [$\leftrightarrow$ find all rational (x, y) satisfying $y^2 = x^3 - 2x$]. The smallest such a, b, c have thirteen digits each.
- For a given squarefree integer n, is there a right triangle with sides of rational length, whose area is n [$\leftrightarrow$ does $y^2 = x^3 - n^2x$ have nontrivial rational solutions]? This question, dating back thousands of years, was answered (almost) only in the 1980s. (No for $n = 1, 2, 3, 4$, yes for $n = 5, 6, 7$, and 358 other $n < 1000$, including 157, where the simplest hypotenuse has 48-digit numerator and 46-digit denominator.)

Text: J. Silverman and J. Tate, *Rational points on elliptic curves.*

MA 598F: Mathematics of Finance
Instructor: Prof. J. Ma, office: Math 620, phone: 49–41973, e-mail: majin@math.purdue.edu
Time: TTh 1:30-2:45
Prerequisites: MA 519 (or equivalent) + MA 261 (or equivalent) + MA 440 (or equivalent); or consent of the instructor.
Description: We will provide an introduction to the mathematical tools and techniques of modern finance theory, in the context of Black-Scholes-style option pricing. The typical (pricing) question is: how much should you charge someone for allowing them the right to purchase a certain stock from you at a given price and given time in the future? The typical (Black-Scholes) assumption is that the differential of the (log of the) stock price is the sum of a constant term (r.dt, constant interest rate) and a random noise term (dW(t), a Brownian increment). Under this assumption, to answer the pricing question, the main mathematical tool is stochastic calculus and its connection to partial differential equations. These mathematics will be the object of a thorough introduction at an elementary level, without measure theory. This toolbag will enable us to derive the main pricing and hedging results in complete and incomplete markets, and to treat many examples of exotic and path-dependent options, as well as an introduction to stochastic optimal control and portfolio optimization.
Text: T. Bjork, *Arbitrage Pricing in Continuous Time*, Oxford, 1998.
Suggested additional reading: D. Lambertson and B. Lapeyre, *Stochastic Calculus applied to Finance*, Chapman and Hall/CRC, 1996, reprinted 2000 by CRC Press.; P. Wilmott, S. Howison, J. Dewynne, *The mathematics of financial derivatives. A student introduction.* Cambridge U.P. 1995. Chapters 11 to 16.

MA 598G: Advanced Probability and Options, with Numerical Methods
Instructor: Prof. F. Viens, office: Math 504, phone: 49-46035, e-mail: viens@stat.purdue.edu
Time: TTh 9:00-10:15
Prerequisites: Those who have not had MA 598 F as a prerequisite can still hope to enroll in the course by providing evidence of basic preparation in stochastic calculus, and by reading the material of the first 10 chapters of the textbook by Bjork, before class begins in January. Several students in the past have succesfully achieved this preparation.
Description: This is the second course in a two-course sequence on the mathematics of finance, and especially on option pricing. The material will be divided in two parts. First, we will cover theoretical issues regarding: (i) Interest rate term structure models; (ii) American options and stochastic optimal stopping; (iii) finite difference methods. Then we will examine in detail the numerical methods used to solve the partial differential equations and inequalities that determine the prices of options, including the Binomial, Monte-Carlo, and finite difference methods.

MA 598N: Mathematical Modeling of Nonlinear Waves
Instructor: Prof. M. Chen, office: Math 818, phone: 49–41964, e-mail: chen@math.purdue.edu
Time: MWF 1:30
Description: This is an introductory course in the modern theory of nonlinear wave propagation. The course assumes some knowledge of Sobolev spaces and the Fourier transform. Most topics will be developed from scratch, however. It is suitable for graduate students or well–prepared undergraduate students. The course will cover material from the topics listed below.
TOPICS.

1. Derivation of model equations for long waves. One–way models. Two–way models. Weakly three–dimensional models.
2. Initial–value problems.
3. Boundary–value problems.
4. Solitary waves and other travelling–wave phenomena.
5. Numerical simulations. Algorithnms. Analysis.
6. Comparison between various models.
7. Stability and instability – singularity formation.
8. Dissipative effects. Long–time asymptotics of solutions. Comparison with laboratory data.
9. Applications in coastal engineering.

MA 598S: Introduction to Spectral Methods for Scientific Computing
Instructor: Prof. J. Shen, office: Math 848, phone: 49–41923, e-mail: shen@math.purdue.edu
Time: Note new time TTh 10:30-11:45
Prerequisite: A good knowledge of calculus, linear algebra, numerical analysis and some basic programming skills are essential. Some knowledge of real analysis and functional analysis will be helpful but not necessary.
Description: This is an introduction course on spectral methods for solving partial differential equations (PDEs). We shall present some basic theoretical results on spectral approximations as well as practical algorithms for implementing spectral methods. We shall specially emphasize on how to design efficient and accurate algorithms for solving PDEs of current interest.

The course is suitable for advanced undergraduate students in mathematics and graduate students in mathematics and engineering.
Some of the topics are:

- Basic results for polynomial approximations.
- Galerkin method using Legendre and Chebyshev polynomials.
- Collocation method using Legendre and Chebyshev polynomials.
- Fast elliptic solvers using the spectral methods.
- Applications to various PDEs of current interest.

Requirement: There will be no exam. Grades will be based on homework and programming projects.
No text book is required. Typed lecture notes for most topics will be distributed.
Reference Books:

1. C. Bernardi and Y. Maday, *Spectral Method*, in "Handbook of Numerical Analysis, V. 5 (Part 2)" eds. P. G. Ciarlet and L. L. Lions, North–Holland, 1997.
2. I. Szegö, *Orthogonal Polynomials*, AMS, 1975.
3. D. Gottleib, S. A. Orszag, *Numerical Analysis of Spectral Methods: Theory and Applications*, SIAM–CBMS (1977).

MA 598U: Stochastic PDEs and fractional Brownian Motion
Instructor: Prof. F. Viens, office: Math 504, phone: 49-46035, e-mail: viens@stat.purdue.edu
Time: TTh 10:30-11:45
Audience: PhD and Advanced MS students in Mathematics and in Statistics, or anyone with a strong interest in Stochastic Analysis.
Description: This course is a continuation of the course MA/STAT598T that is currently being taught by Viens. However, students who are not attending MA/STAT598T will be able to jump into the proposed course by taking a short self-taught crash course on the basics of Stochastic PDEs, equivalent to the first two weeks of MA/STAT598T.

The course will follow the same format at MA/STAT598T. It will begin with some lectures by the instructor on basic topics on fractional Brownian motion and more basics of Stochastic PDE. These lectures will be useful to understand most of the current literature whose reading and in-class discussion will constitute more than half of the course. Because the papers on fractional Brownian motion that will be read are at the very forefront of stochastic analysis, it is anticipated that in-class discussions will be quite substantial. Consequently, the instructor reserves the right to not require that students make in-class presentations on their readings, unlike in MA/STAT598T.

The topics to be covered will include: stochastic integration with respect to fractional Brownian motion, by pathwise and by stochastic methods; infinite-dimensional fractional Brownian motion, and relation to stochastic PDEs; deep results on Gaussian regularity theory, with application to stochastic PDEs; non-Gaussian regularity theory and probability on Banach spaces; discussion of open questions on the relation between regularity theory and fractional Brownian motion; discussion of open questions on fractional Brownian motion and Lyapunov exponents.

MA 598W: Linear Algebraic Groups
Instructor: Prof. C. Wilkerson, office: Math 450, phone: 49–41955, e-mail: wilker@math.purdue.edu
Time: MWF 10:30
Description: Linear algebraic groups are the algebraic analogue of Lie groups. They occur in nature as the automorphisms of algebraic structures and as such are often importent in building moduli spaces for the classification of these structures. The course will contain a short introduction to some of the algebraic geometry needed, so the prequisites are not large – probably Math 557 would be enough, or come talk to the instructor.
Text: T. Springer, *Algebraic Groups*, 2nd Edition.

MA 611: Methods of Applied Mathematics I
Instructor: Prof. D. Danielli, office: Math 802, phone: 49–41920, e-mail: danielli@math.purdue.edu
Time: TTh 12:00-1:15
Prerequisite: MA 511 or equivalent and MA 544
Description: This course will cover elements of functional analysis in Banach and Hilbert spaces, including: linear operators; weak topology; Fredholm-Riesz-Schauder theory and spectral theory for compact linear operators. Applications to linear integral equations and partial differential equations will be emphasized.
Text: A. Freidman, *Foundations of Modern Analysis*, Dover, 1982.
Recommended additional reading: H. Brezis, *Analyse Fonctionnelle*, Masson, 1983.

MA 631: Several Complex Variables
Instructor: Prof. D. Catlin, office: Math 744, phone: 49–41958, e-mail: catlin@math.purdue.edu
Time: Note new time MWF 1:30
Prerequisite: MA 530
Description: Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. Local theory of analytic sets (Weierstrass preparation theorem and consequences). Functions and sets in the projective space P^n (theorems of Weierstrass and Chow and their extensions).
Text: Hörmander, *Introduction to Complex Analysis of Several Variables*, North Holland, 1990.

MA 643: Methods of Linear and Nonlinear Partial Differential Equations II
Instructor: Prof. P. Bauman, office: Math 718, phone: 49–41945, e-mail: bauman@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 642
Description: This is a continuation of Math 642. Topics include Moser's estimates for second-order pde's in divergence form, Schauder L^p estimates for equations in nondivergence form, including Calderon-Zygmond theory, and applications to nonlinear elliptic problems. Additional topics include the Alexandrov maximum principle, fully nonlinear equations, time-dependent parabolic and hyperbolic boundary-value problems, and applications to nonlinear evolution problems.
Texts: 1. Gilbarg and Trudinger, *Elliptic Partial Differential Equations of Second Order*, 1998 Edition.
2. Lawrence C. Evans, *Partial Differential Equations* 1993.

MA 646: Banach Algebras and C*–algebras
Instructor: Prof. M. Dadarlat, office: Math 708, phone: 49–41940, e-mail: mdd@math.purdue.edu
Time: TTH 1:30-2:45
Prerequisite: MA546 or the agreement of the instructor.
Description: The course will give an introduction based on examples to the theories of C*-algebras and von Neumann algebras viewed as noncommutative counterparts of topology and of measure theory, respectively. These algebras are basic tools in the noncommutative geometry of Alain Connes.

While in an introductory course we will have time to explore only a limited number of topics, operator algebras is a well developed theory which has had profound applications to various areas, including knot theory and low-dimensional topology, index theory on manifolds, representation theory and the classification of dynamical systems.

The grade will be based on attendance and a (small) number of homework assignments.
Text: Ken Davidson, *C*-algebras by Example*

MA 661: Modern Differential Geometry
Instructor: Prof. L. Tong, office: Math 748, phone: 49–43173, e-mail: tong@math.purdue.edu
Time: MWF 11:30
Prerequisite: MA 544 and 554
Description: This course will be focused on complex manifolds. The core topics will include : Dolbeault complex, vector bundles, sheaves and cohomology, harmonic forms and Hodge theory, Kahler manifolds. Depending on availability of time, more specialized topics such as deformation theory or embedding theorems may be explored.

MA 690B: Topics in Commutative Algebra
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49-41980, e-mail: heinzer@math.purdue.edu
Time: MWF 3:30
Description: This is a continution of MA 690B in the Fall 2002 semester.

MA 692F: Special Topics in Mathematical Biology
Instructor: Prof. Z. Feng, office: Math 814, phone: 49-41915, e-mail: zfeng@math.purdue.edu
Time: MWF 1:30
Prerequisite: Calculus and an introductory course in linear algebra.
Description: This course is an introduction to the application of mathematical methods and concepts to the description and analysis of biological processes. The mathematical contents consist of difference and differential equations and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, epidemiology, coevolutionary systems, structured population models, nonlinear dynamics and chaos, stochastic processes, and introduction to Mathematica and MATLAB (computer packages). Bio-mathematical research projects (in small group) may be carried out.
Text and class materials:

1. Kot *Elements of Mathematical Ecology*, Cambridge University Press. Cambridge, U.K. 2001.
2. Class notes and Handouts and Articles on reserve.
3. Brauer and Castillo-Chavez, *Mathematical Moldes in Population Biology and Epidemiology* (optional).

MA 693B: Complex Analysis
Instructor: Prof. L. de Branges, office: Math 800, phone: 49-46057, e-mail: branges@math.purdue.edu
Time: MWF 10:30
Description: The relationship between analytic functions and their power series representation in the unit disk is an underlying theme of complex analysis which is treated by James Rovnyak and the present instructor in a text on *Square Summable Power Series*, Holt, 1966. Hilbert space concepts are introduced which have since proved their value in the proof of the Bieberbach Conjecture and in the invariant subspace theory of contractive transformations in the canonical model due to Arne Beurling. The Nevanlinna factorization theory of meromorphic functions which are of bounded type in the unit disk are reached in a Krein space generalization of the canonical model. A level of mathematical maturity sufficient for qualifying examinations in analysis is presumed.

Seminars

Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–5:45

Working Algebraic Geometry Seminar, Prof. Arapura
Time: Wednesday 3:30

PDE Seminar, Prof. Bauman
Time: Tuesday 9:30

Liftoff Seminar, Prof. Bell
Time: Wednesday 4:30

A Seminar in Analysis, Prof. de Branges
Time: Thursday 10:30

Geometric Analysis Seminar, Prof. Buzzard
Monday 3:30

Function Theory Seminar, Prof. Eremenko
Time: Tuesday 3:30

Automorphic Forms Seminar, Prof. Goldberg
Time: Thursday 1:30

Computational and Applied Mathematics Seminar, Prof. Shen
Time: To be determined

Commutative Algebra Seminar, Prof. Heinzer
Time: Monday and Wednesday 4:30

Computational Finance Seminar, Prof. Viens
Time: Friday 2:30