Courses and Seminars of Interest to Graduate Students
offered by the
Mathematics Department
Spring, 2007

MA 558: Abstract Algebra II
Instructor: Prof. Ulrich, office: Math 618, phone: 49–41972, e-mail: `ulrich@math.purdue.edu`
Time: MWF 4:30
Prerequisite: Basic knowledge about commutative rings (such as the material of MA 557).
Description: The topics of the course will be introductory homological algebra and commutative algebra. We will study properties of commutative rings and their modules, with some emphasis on homological methods. The course should be particularly useful to students interested in commutative algebra, algebraic geometry, number theory or algebraic topology. Specific topics are: Derived functors, structure of injective modules, flatness, completion, dimension theory, regular sequences, Cohen-Macaulay modules.
Texts: No particular book is required, but typical texts are:
- J. Rotman, *An introduction to homological algebra*, Academic Press.
- H. Matsumura, *Commutative ring theory*, Cambridge.
- W. Bruns and J. Herzog, *Cohen-Macaulay rings*, Cambridge.
- D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, Springer.

MA 598A: Basic Algebra II
Instructor: Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail: `ram@math.purdue.edu`
Time: TTh 1:30-2:45
Description: Although this is called Part II, it will be independent of Part I, and will be run as a seminar where new graduate students can join in and find thesis topics. There are no prerequisites and all interested students are welcome. The first lecture will be on Tuedsay, January 16.
Text: Shreeram S. Abhyankar, *Lectures on Algebra I*, World Scientific.

MA 598C: Mexed and Least-Squares Finite Element Methods for Systems of Partial Differential Equations
Instructor: Prof. Cai, office: Math 810, phone: 49–41921, e-mail: `zcai@math.purdue.edu`
Time: TTh 1:30-2:45
Prerequisites: MA 598C or CS 615 or equivalent or consent of instructor.
Description: The original physical equations for mechanics of continua are systems of partial differential equations of first-order. There are many advantages to simulate these first-order systems directly. This can be done through either mixed or least-squares finite element methods. This course is an introduction to both techniques, with applications to Darcy's flow in porous media, elastic equations for solids, incompressible Newtonian fluid flow, and Maxwell's equations in electromagnetic. We shall focus on fundamental issues such as (mixed and least-squares) variational formulations and construction of finite element spaces of H^1, H(div), and H(curl). A review of fast iterative solvers such as multigrid and domain decomposition for algebraic systems resulting from discretization will also be presented.
A tentative list of contents:
1. Mathematical Models of Continuum Mechanics
2. Construction of Finite Element Spaces in H^1, H(div), or H(curl)
3. Mixed Variational Formulations
4. Least-Squares Variational Formulations
5. Finite Element Approximations
6. Iterative Solvers
7. FOSPACK – a computer package based on least-squares methods

References:
[1] F. Brezzi and M. Fortin, *Mixed and Hybrid Finite Element Methods*, Springer-Verlag, New York, 1991.
[2] V. Girault and P. Raviart, *Finite Element Methods for Navier-Stokes Equations: Theory and Algorithms*, Springer-Verlag, New York, 1986.
[3] P. Monk, *Finite Element Methods for Maxwell's Equations*, Oxford University Press, Oxford, 2003.
[4] W. Briggs, V. Henson, S. McCormick, *A Multigrid Tutorial*, Second Edition, SIAM, Philadelphia, 2000.
[5] research articles and my lecture notes.

MA 598D: Numerical Modeling and Inversion in Porous Media
Instructor: Prof. Santos, office: Math 808, phone: 41925, e-mail: `santos@math.purdue.edu`
Time: TTh 3:00-4:15
Prerequisites: MA 303, 304 or equivalent.
Description: Numerical simulation to model wave propagation and fluid flow in saturated porous media has applications in many branches of science and technology, such as seismic methods to analyze the concentration of gas hydrates in ocean bottom sediments, and nondestructive testing of materials using ultrasound techniques. The associated inverse problems can be solved combining the forward models with optimization algorithms in iterative fashion.

The course will present the differential and numerical models describing wave propagation and fluid flow in multiphase porous media and discuss the computer implementation of the algorithms. Applications of the numerical procedures to solve direct and inverse problems in porous media will also be presented.

MA 598S: Designing your own Course
Instructor: Dr. R. Saerens, office: Math 826, phone: 49–41906, e-mail: `saerens@math.purdue.edu`
Time: W 3:30 - 5:20 every other week, odd weeks only, 1 credit – pass/fail, Instructor Permission is Required.
Minimum Enrollment: 7 students. **Maximum Enrollment:** 12 students
Prerequisite: having taught a course in which you were responsible for writing some exams. If not meeting the prerequisite, send a brief e-mail to the instructor to make your case.
Note: Instructor permission needed. Priority will be given to Mathematics graduate students who plan to graduate by August 2008, meet the above prerequisite and put a (properly filled out) Form 23 in the instructor's mailbox before noon on October 20 - attach a note by when you plan to graduate if it is by August 2008. Your signed Form 23 will be returned to your mailbox by 5:00 pm on October 20. After that date, permission will be granted on a first-come first-serve basis for people meeting the prerequisite.
Description: The main objective of the course is to have the students (as a group) develop an Introduction to Real Analysis course for sophomores and juniors. The class will meet every other week for (at most) 100 minutes. Though various topics are scheduled for the different meetings, the content, etc. of the course will mostly be determined by the students. Homework will be assigned each class meeting and will be due the next week. Copies of the homework will be distributed and students are expected to have read and be ready to discuss the homeworks at the next class meeting.

Since this is a highly participatory course, grades will be based on attendance, preparation for and participation in the discussions, and the homework. Homework will not be graded for correctness but a certain amount of effort and thoughtfulness is expected.

MA 598T: Bridge to Research Seminar
Instructor: Prof. Milner, office: Math 628, phone: 49–41967, e-mail: `milner@math.purdue.edu`
Time: Monday 4:30
Description: The seminar has two main goals, both aimed at helping students early in their graduate career find their place in the department. The first is to help students discover what area of mathematics they might be interested in researching, as well as who they might like to work with. The second is to provide students with an opportunity to interact with faculty in a casual setting. This is achieved by having professors from the department give brief talks about their research area at a level that is accessible to those in their first and second year of graduate study.

MA 611: Methods of Applied Mathematics I
Instructor: Prof. Phillips, office: Math 706, phone: 49–41939, e-mail: `phillips@math.purdue.edu`
Time: TTh 10:30-11:45
Prerequisite: MA 511, 544
Description:
Banach spaces; linear operators; the Open Mapping Theorem and the Closed Graph Theorem; the Hahn-Banach Theorem; weak topologies; the Fredholm-Riesz-Schauder theory and elements of Spectral Theory for compact operators; Hilbert spaces; the Projection Theorem; the Riesz Theorem and the Lax-Milgram Lemma; self-adjoint operators. Applications to ordinary and partial differential equations.
Text: A. Friedman, *Foundations of Modern Analysis*, Dover
Additional recommended reference: Hl. Brezis *Analyse Functionnelle - Theorie et applications*, Masson.

MA 643: Methods of Partial Differential Equations II
Instructor: Prof. Bauman, office: Math 718, phone: 49–41945, e-mail: `bauman@math.purdue.edu`
Time: MWF 1:30
Prerequisite: MA 642
Description: Continuation of MA 642. Topics to be covered are L_p theory for solutions of elliptic equations, including Moser's estimates, Aleksandrov maximum principle, and the Calderon-Zygmund theory. Introduction to evolution problems for parabolic and hyperbolic equations, including Galerkin approximation and semigroup methods. Applications to nonlinear problems.
References: L.C. Evans *Partial Differential Equations*
Text: D. Gilbarg and N.S. Trudinger *Elliptic Partial Differential Equations of Second Order*

MA 644: Calculus of Variation
Instructor: Prof. Danielli, office: Math 802, phone: 49–41920, e-mail: `danielli@math.purdue.edu`
Time: TTh 9:00-10:15
Prerequisite: The course will be essentially self-contained. The only prerequisites are familiarity with the Lebesgue integral and the first properties of L^p spaces, as well as a basic knowledge of Sobolev spaces.
Description: The fundamental problem of the calculus of variations is the minimization of the integral functional

$$\mathcal{F}(u,\Omega) = \int_\Omega F(x,u,Du(x))\,dx$$

among all the functions u taking prescribed boundary values on the boundary Ω. In this course we will focus on its solution via the so-called *direct methods*, which consist in proving the existence of the minimum of the integral functional $\mathcal{F}$ directly, rather than resorting to its Euler equation. The central idea is to consider $\mathcal{F}$ as a real-valued mapping on the manifold of functions taking on $\partial\Omega$ the given boundary values and applying to it a generalization of Weierstrass' theorem on the existence of the minimum of a continuous function. There are two main issues in this approach. The first one is that semicontinuity, rather than continuity, is the key assumption to apply Weierstrass theorem to the functional $\mathcal{F}$, whereas the second one is to identify the Sobolev spaces as the proper function spaces for compactness results to hold. On the other hand, the solution of the existence problem in the Sobolev class opens up a series of questions about the regularity of the minimizers. This was a long-standing open problem, until the way to its solution (in a non-direct fashion, since it involves the Euler equation) was opened by the celebrated De Giorgi-Nash-Moser result concerning the Hölder continuity of solutions to uniformly elliptic PDEs in divergence form with bounded and measurable coefficients. A first step towards the use of direct methods in the regularity issue came from a 1982 paper by Giaquinta and Giusti, who proved the Hölder continuity of *quasi-minima*, that is functions u for which

$$\mathcal{F}(u,K) \le Q\mathcal{F}(u+\varphi,K)$$

for every φ with compact support $K \subset \Omega$. The notion of quasi-minima reduces of course to the one of minimum when $Q = 1$, but it is substantially more general, since it includes solutions of linear and nonlinear elliptic equations and systems.

The course will follow the path outlined above, and it will combine the classical approach to the subject with its latest developments. In particular, emphasis will be placed on presenting an unified treatment of the regularity of the minima of functionals in the calculus of variations, and of the solutions to elliptic equations and systems in divergence form. We will also explore the connections with the study of minimal surfaces and applications to optimal control theory.
Text: E. Giusti, *Direct methods in the calculus of variations*, World Scientific, 2002.

MA 684: Class Field Theory
Instructor: Prof. Shahidi, office: Math 650, phone: 49–41917, e-mail: `shahidi@math.purdue.edu`
Time: MWF 9:30
Prerequisite: MA 584 (algebraic number theory).
Description: This is the second semester of our two semester courses in number theory. Class field theory is considered the crowning achievement of the number theory in the 20–th century and the first and classical example of general reciprocity laws which are now generally called the Langlands Program which aims for its non–abelian generalizations. I will use zeta functions to prove the first inequality and this may be a good chance to learn about Dirichlet and more generally L–functions: Ideles, adeles, L–functions, Artin symbol, reciprocity, local and global class field theory, Kronecke-Weber theorem.
Text: S. Lang, *Algebraic Number Theory*, Addison Wesley, 1970.

MA 690A: Algebraic Geometry II
Instructor: Prof. Arapura, office: Math 642, phone: 49–41983, e-mail: `dvb@math.purdue.edu`
Time: TTh 3:00-4:15
Description: This is a second course in algebraic geometry. So I will that assume everyone knows the definitions of quasi-projective varieties, regular maps, and perhaps sheaves and schemes, and commutative algebra at the Atiyah-MacDonald level. It would also be nice if people have seen (co)homology before in some form (singular, de Rham or sheaf), but if have to do it I can go through the basic homological ideas very quickly. However, my main goal is to concentrate on the more geometric side of the subject: curves, surfaces and special varieties in higher dimensions. I will alternate between purely algebraic methods valid over general fields, and complex analytic or topological methods over $\mathbb{C}$. Often things which seem complicated in one setting are simpler in the other. For example, the genus of a curve over $\mathbb{C}$ is just the number of holes; in general, it's the dimension of a certain sheaf cohomology group. The topics that I'm going to cover would include Riemann-Roch in low dimensions, the first Chern class (over $\mathbb{C}$), the adjunction and Riemann-Hurwitz formulas, and intersection theory for curves on a surface. I haven't decided what we'll do after that (if there is an "after"), perhaps Jacobian varieties. We'll see.

There won't be an official textbook for the course, but most of material will be contained in the union of [GH] and [H]. In case you don't already have these books, I will try to put them on reserve.
References: [GH] P. Griffiths, J. Harris, *Principles of Algebraic Geometry*
[H] R. Hartshorne, *Algebraic Geometry*

MA 690B: Topics in Commutative Algebra
Instructor: Prof. Heinzer, office: Math 636, phone: 49–41980, e-mail: `heinzer@math.purdue.edu`
Time: MWF 3:30
Description: The course is planned to be a continuation of MA 690B of this fall and will cover additional material from the text by W. Bruns and J. Herzog titled *Cohen-Macaulay Rings*, revised edition. Students enrolled in the course will be encouraged to actively participate by presenting material and exercises from the text.

MA 690C: Integral closures of rings, ideals and modules
Instructor: Prof. Ulrich, office: Math 618, phone: 49–41972, e-mail: `ulrich@math.purdue.edu`
Time: MWF 2:30
Prerequisite: Basic knowledge about commutative rings (such as the material of MA 557)
Description: The concepts of integral dependence and integral closure originated in number theory, but are of central importance in commutative algebra and algebraic geometry as well. We will study the topic from a commutative algebra point of view, mainly using material from a recent book by Huneke and Swanson. The course is in some sense a continuationof MA 650, but will be accessible to anybody with basic knowledge in commutative algebra.
Recommended texts: 1. C. Huneke and I. Swanson, *Integral closure of ideals, rings, and modules*, Cambridge Univ. Press, to appear.
2. W. Vasconcelos, *Integral closure*, Springer.

MA 690D: Linear Algebraic Groups
Instructor: Prof. Yu, office: Math 738, phone: 49–41946, e-mail: `jyu@math.purdue.edu`
Time: TTh 10:30-11:45
Prerequisite: MA 553 and 554.
Description: This course covers the structure theory of linear algebraic groups: Tits systems, Bruhat decompositions, nilpotent groups, tori, radicals, semisimple and reductive groups, classification of reductive groups by root data. Optional topics may include classification over a general perfect ground field, and classification of irreducible rational representations.
References: T. A. Springer, *Linear Algebraic Groups*, Birkhauser, 2nd edition.

MA 690G: Modular Forms II
Instructor: Prof. Lipman, office: Math 750, phone: 49–41994, e-mail: `lipman@math.purdue.edu`
Time: MWF 10:30
Description: This is a continuation of the present course on Modular Forms (MA690F).

MA 692A: Special Topics in Mathematical Biology
Instructor: Prof. Z. Feng, office: Math 814, phone: 49–41915, e-mail: zfeng@math.purdue.edu
Time: MWF 2:30
Description: This course covers various topics in mathematical biology. The focus will be on the application of mathematical methods and concepts to the description and analysis of biological processes. The mathematical contents consist of difference and differential equations and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, epidemiology, coevolutionary systems, structured population models, nonlinear dynamics, and stochastic simulations.

MA 692B: Analysis of Numerical Methods I: Geometric Numerical Integration
Instructor: Prof. Leok, office: Math 430, phone: 49–63578, e-mail: mleok@math.purdue.edu
Time: TTh 12:00-1:15
Description: Many differential equations of interest in the physical sciences and engineering exhibit geometric properties that are preserved by the dynamics. Recently, there has been a trend towards the construction of numerical schemes that preserve as many of these geometric invariants as possible.

Such methods are of particular interest when simulating mechanical systems that arise from Lagrangian or Hamiltonian mechanics, wherein the preservation of physical invariants such as the energy, momentum and symplectic form can be important when simulating long–time dynamics of such systems.

This course will begin with an overview of classical numerical integration schemes, and their analysis, followed by a more in depth discussion of the various geometric properties that are of importance in many practical applications, followed by a survey of the various geometric integration schemes that have been developed in recent years. Issues pertaining to the analysis and implementation of such schemes will also be addressed.

Course Topics:
1. Examples and Motivating Numerical Experiments.
2. Overview of Numerical Integration Schemes.
3. Conservation of First Integrals and Methods on Manifolds
4. Symmetric and Reversible Methods.
5. Symplectic Integration of Hamiltonian Systems.
6. Additional Topics in Structure Preservation.
7. Numerical Implementation of Structure Preservation Algorithms.
8. Backward Error Analysis.

Text: Hairer, Lubich and Wanner, *Geometric Numerical Integration*, 2nd Edition, Springer-Verlag, 2006.
References: Leimkuhler and Reich, *Simulating Hamiltonian Dynamics*, Cambridge University Press, 2005.

MA 692C: Classical Mechanics
Instructor: Prof. Lempert, office: Math 728, phone: 49–41952, e-mail: lempert@math.purdue.edu
Time: MWF 1:30
Prerequisite: Some exposure to analysis in $\mathbb{R}^n$ and to ordinary differential equations (MA 304 and MA 543); in the latter half of the course some differential geometry such as manifolds and tangent bundles.
Description:

All through its history mathematics has been greatly influenced by problems in mechanics, celestial or otherwise. Mechanics has been instrumental in the development of Riemannian and symplectic geometry, ordinary and partial differential equations, dynamical systems, etc. While it is now accepted that the physical world is described by quantum (and more advanced) physics rather than classical mechanics, modern physical theories are still built on mechanics.

The course will introduce the audience to fundamental notions of the subject and to some highlights, including rather recent ones.

Contents: Newtonian description of mechanics, conservative systems, motion in central fields, Kepler's problem. Lagrangian mechanics, principle of least action, Legendre transformation, Hamiltonian, Noether's theorem, preservation of phase volume, Poincare's recurrence theorem. Constraints, mechanics on manifolds, geodesics on surfaces of revolution. Small oscillations. Hamiltonian mechanics, symplectic manifolds, Gromov's uncertainty principle. Hamilton–Jacobi theory, generating functions, complete integrability, Kolmogorov–Arnold–Moser theory.
Reference: Arnold, *Foundation of Classical Mechanics*, Springer.

MA 693B: Radon = Riemann
Instructor: Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: `branges@math.purdue.edu`
Time: MWF 9:30
Description: The Radon transformation relates Fourier analysis on a plane to Fourier analysis on a line. The maximal dissipative property of the transformation is an underlying concept in the proof of the Riemann hypothesis. The transformation is now used to generate the theta functions and zeta functions which are used to formulate the Riemann hypothesis. Motivation for the proof of the Riemann hypothesis results from a consistent application of the principles applied in the proof. Open to all serious student so of analysis.

MA 694A: Free Boundary Problems of Obstacle Type
Instructor: Prof. Petrosyan, office: Math 610, phone: 49–41932, e-mail: `arshak@math.purdue.edu`
Time: TTh 12:00-1:15
Prerequisite: Minimal MA 523. MA 642 or the consent of the instructor.
Description: Free boundaries are apriori unknown sets, coming up in solutions of partial differential equations and variational problems. Typical examples are the interfaces and moving boundaries in problems on phase transitions and fluid mechanics. Main questions of interest are the regularity (smoothness) of free boundaries and their structure.

A well-known (and well-studied) example is the obstacle problem of minimizing the energy of the membrane subject to remaining above a given obstacle: the free boundary is the boundary of the contact set. The objective in this course is to give an introduction to the theory of the regularity of the free boundaries in problems of the obstacle type, pioneered in the works of Luis Caffarelli, et al.

We are going to discuss classical and more recent methods in such problems, including the optimal regularity of solutions, monotonicity formulas, classification of global solutions, geometric and energy criterions for the regularity of the free boundary, singular points.
Text: A. Petrosyan, H. Shahgholian, N. Ural'tseva, *Regularity of free boundaries in obstacle-type problems*, lecture notes, unpublished.

MA 696A: K*–Actions Toric Varieties and Birational Geometry
Instructor: Prof. Wlodarczyk, office: Math 602, phone: 49–62835, e-mail: `wlodar@math.purdue.edu`
Time: TTh 12:00-1:15
Prerequisite: Basic knowledge about algebraic geometry (like R.Hartshorne *Algebraic Geometry* Chapter I or similar).
Description: The purpose of this course is to give a survey on various techniques used in birational geometry and its interactions with invariant theory and toric geometry. We will introduce C^*-actions , good and geometric quotients,. Bialynicki-Birula decomposition, elements of geometric invariant theory, birational cobordisms (techniques inspired by topological cobordisms), and elements of Mori theory. In the course we introduce and briefly discuss the theory of toric varieties as the illustration of the above mentioned techniques with particular emphasis on Mori theory , Morelli cobordisms and C^* - actions and the theory of valuations. One of the main goals will be the proof of the Weak Factorization Theorem which states that any birational map between smooth projective varieties is a composition of blow-ups and blow-downs along smooth centers. The focus of this course is to give an intuition about the interplay of different areas of algebraic geometry.
Texts: 1.) J.Wlodarczyk, *Algebraic Morse Theory and Factorization of Birational Maps.*
2.) J.Wisniewski, *Toric Mori Theory and Fano Manifolds.*
References: 1.) Bialynicki-Birula, *Some theorems on group actions*
2.) Tadao Oda, *Convex bodies and Toric Varieties*
3.) Kenji Matsuki, *Introduction to Mori Theory*
4.) Igor Dolgachev, *Lectures on Invariant Theory*

MA 696B: Topics in Complex Geometry
Instructor: Prof. Yeung, office: Math 712, phone: 49–41942, e-mail: `yeung@math.purdue.edu`
Time: MWF 10:30
Prerequisite: MA 530, MA 562. Some basic understandings in algebraic geometry and several complex variables will be helpful.
Description: In this course, some techniques and results in complex analysis and differential geometry will be studied. Tentatively, aspects of the following topics will be covered.

1. Ricci flow and Kähler-Ricci flow in geometry.
2. Hichin-Kobayashi correspondances (stability and special metrics on bundles).
3. Classical results related to Zeta functions.

Probably only parts of the materials would be covered, depending on the progress of the course.

Seminars

Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Applied Inverse Problems Seminar, Prof. de Hoop
Time: Tuesdays and Thursdays 10:30

Automorphic Forms and Representation Theory Seminar, Prof. Yu
Time: Thursdays, 1:30

Bridge to Research Seminar, Prof. Milner
Time: Mondays 4:30

Commutative Algebra Seminar, Prof. Ulrich
Time: Wednesdays 4:30-5:20

Computational and Applied Math Seminar, Prof. Shen
Time: Fridays 3:30

Computational Finance Seminar, Prof. Ma
Time: Fridays 2:30

Function Theory Seminar, Prof. Eremenko
Time: flexible.

Geometric Analysis Seminar, Prof. Lempert
Time: Monday 4:30

Foundations of Analysis Seminar, Prof. de Branges
Time: Thursday 9:30

Joint Geometry/Topology Seminar, Prof. Lee
Time: To Be Announced

Mathematical Biology, Prof. Feng
Time: Fridays, 2:30

Number Theory, Prof. Goins
Time: Thursday, 3:30

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30

PDE Seminar, Prof. Phillips
Time: Thursdays, 3:30

Probability Seminar, Prof. Viens
Time: Mondays 3:30

Spectral and Scattering Theory Seminar, Prof. Sá Barreto
Time: Thursdays 4:30

Tea Time Seminar on Applied Analysis, Prof. Petrosyan
Time: Tuesdays, 3:00-4:15

Topology Seminar, Prof. McClure
Time: Tuesday 3:30

Working Algebraic Geometry Seminar, Profs. Arapura and Matsuki
Time: Wednesday 3:30-5:00