Courses and Seminars of Interest to Graduate Students offered by the Mathematics Department Spring, 2011

MA 54400: CRN:22177 REAL ANALYSIS AND MEASURE THEORY
Instructor: Prof. Danielli, office: Math 620, phone: 49–41920, e-mail: `danielli@math.purdue.edu`
Time: MWF 9:30
Prerequisite: MA 50400 or equivalent
Description: The purpose of this course is to develop the theory of Lebesgue measure and integrals in the setting of Euclidean spaces. Topics covered will include: functions of bounded variation and the Riemann-Stieltjes integral; outer measure and Lebesgue measure; Lebesgue measurable functions; the Lebesgue integral; Fubini's and Tonelli's theorems; Lebesgue's differentiation theorem; L^p spaces; approximation to the identity and maximal functions.
Text: R. L. Wheeden and A. Zygmund, *Measure and Integral*, Dekker

MA 54500: CRN:22178 FUNCTIONS OF SEVERAL VARIABLES AND RELATED TOPICS
Instructor: Prof. Garofalo, office: Math 616, phone: 49–41971, e-mail: `garofalo@math.purdue.edu`
Time: MWF 11:30
Description: Sobolev space are a fundamental tool in analysis and geometry, and they play a pervasive role in partial differential equations. In the first part of this course I will develop a complete theory for these spaces, including the geometrically relevant case $p = 1$, with De Giorgi's theory of functions of bounded variation and perimeters, and the case $p = \infty$, with the Rademacher-Stepanov characterization of Lipschitz continuity. In the second part of the course I will discuss the theory of convex functions, which culminates with the Busemann-Feller-Alexandrov theorem. If time allows I will present various fundamental applications of the theory developed. The only prerequisite for this course is knowledge of the Lebesgue integral.

MA 55800: CRN:22181 ABSTRACT ALGEBRA II
Instructor: Prof. Ulrich, office: Math 618, phone: 49–41972, e-mail: `ulrich@math.purdue.edu`
Time: TTh 3:00-4:15
Prerequisite: Basic knowledge about commutative rings (such as the material of MA 557).
Description: The topics of the course will be introductory homological algebra and commutative algebra. We will study properties of commutative rings and their modules, with some emphasis on homological methods. The course should be particularly useful to students interested in commutative algebra, algebraic geometry, number theory, or algebraic topology. Specific topics are: Derived functors, structure of injective modules, flatness, completion, dimension theory, regular sequences, Cohen-Macaulay rings and modules.
Texts: No particular book is required, but typical texts are:
- J. Rotman, *An introduction to homological algebra*, Academic Press.
- H. Matsumura, *Commutative ring theory*, Cambridge.
- W. Bruns and J. Herzog, *Cohen-Macaulay rings*, Cambridge.
- D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, Springer.

MA 57200: CRN:46826 INTRODUCTION TO ALGEBRAIC TOPOLOGY
Instructor: Prof. Ralph Kaufmann, e-mail: `rkaufman@math.purdue.edu`
Time: TTh 9:00-10:15
Description: The course is an introduction to algebraic topology. The focus will be on homology and cohomology theory. This subject is important to topology, but also to many other fields, such as differential, symplectic and algebraic geometry, number theory, mathematical physics, etc. We will treat the classical simplicial and singular homology and cohomology, but we also plan to cover CW complexes and differential forms.
Text: James R. Munkres *Elements of Algebraic Topology*

MA 59800: CRN:43028 INTRODUCTION TO MORSE THEORY
Instructor: Prof. Lee, office: Math 734, phone: 49–47919, e-mail: `yjlee@math.purdue.edu`
Time: TTh 3:00-4:15
Description: Morse theory underlies the spectacular developments in differential topology in mid 20th century by Bott, Milnor, Smale, et al., and continues to play a central role in current research on the topology of differential manifolds. This course is intended to be a first (graduate level) course in differential topology for students with basic knowledge of manifolds (at the level of MA 562), using Morse theory as a central theme. Depending on the level and interests of the students enrolled, this course could be made more advanced in which case we might go into more modern topics such as equivariant or Morse-Bott theory, Morse-Novikov theory, moment maps and symplectic reduction, Lefschetz fibration (of symplectic manifolds), Witten's interpretation, Floer theory etc. Please contact Prof. Lee to let her know your background and preferences if you are interested in taking this course. No textbook will be used but references will be given along the way.

MA 59800, CRN: 52803, NUMERICAL SIMULATION OF WAVES IN POROUS MEDIA: THEORY AND APPLICATIONS, NOTE course cancelled Nov 17
Instructor: Prof. J. Santos, e-mail: `santos@math.purdue.edu`
Time: TTh 12:00-1:15
Description: The course will present the partial differential equations describing the propagation of seismic and electromagnetic waves in heteroge- neous fluid-saturated porous media. Finite element techniques to compute approximate solutions will be discussed and analyzed, including numeri- cal upscaling techniques. Computer implementation of the algorithms will be analyzed. The numerical procedures will be applied to detection and characterization of hydrocarbon reservoirs, seismic monitoring of CO_2 se- questration after injection and electroseismic and seismoelectric modeling among others.
Course Contents

1. Derivation of the constitutive relations and equations of motion for fluid-saturated porous media (Biot's media).
2. Determination of the elastic and viscodynamic coefficients in Biot's equations of motion in terms of the properties of the individual solid and fluid phases. Introduction of viscoelasticity employing the Corre- spondence Principle.
3. Analysis of the phase velocities and attenuation coefficients for the different types of body waves propagating in Biot's media.
4. Review of the Finite Element Method. Description of some finite el- ement spaces in 1D, 2D and 3D. Analysis of the interpolation error. Mixed finite element spaces methods for solving elliptic and Maxwell equations.
5. Numerical solution of Biot's equations of motion in bounded domains using the finite element method.
6. Derivation of numerical upscaling procedures in a highly heterogeneous Biot's medium to determine an equivalent viscoelastic medium at the macroscale. Application to wave propagation in patchy-saturated porous media.
7. Numerical solution of the coupled Biot's and Maxwell's equation in Biot's media using the finite element method. Seismo-electric and Elec- troseismic applications.

Description of Homework Assignements

1 Determination of the static and viscodynamic coefficients in Biot's equations of motion using fortran or similar computer language.
2 Calculation of the phase velocities and attenuation coefficients in fluid-saturated porous materials.
3 Numerical simulation of wave propagation in Biot's media. Appli- cation to the analysis of attenuation and dispersion effects in partially saturated porous media.
4 Computer implementation of numerical upscaling procedures in Biot's media. Application to seismic monitoring in CO2 sequestration sites.
5 Numerical modeling of coupled electromagnetic and seismic waves in fluid-saturated porous media.

MA 59800, CRN: 22196, BRIDGE TO RESEARCH SEMINAR
Instructor: Prof. S. Bell, office: Math 628, phone: 49–41967, e-mail: bell@math.purdue.edu
Time: M 4:30
Description: The seminar has two main goals, both aimed at helping students early in their graduate career find their place in the department. The first is to help students discover what area of mathematics they might be interested in researching, as well as who they might like to work with. The second is to provide students with an opportunity to interact with faculty in a casual setting. This is achieved by having professors from the department give brief talks about their research area at a level that is accessible to those in their first and second year of graduate study.

MA 59800: CRN:43026 PSEUDODIFFERENTIAL OPERATORS
Instructor: Prof. Sa Barreto, office: Math 410, phone: 49–41965, e-mail: `sabarre@math.purdue.edu`
Time: TTh 1:30-2:45
Prerequisite: Knowledge of the calculus of pseudodifferential operators with $S^m_{1,0}$ symbols.
Description: This is a continuation of the course of the same title that was taught during the fall 2010. We will cover the Weyl calculus and semiclassical pseudodifferential operators. As an application we will discuss Carleman esimates and unique continuation for elliptic and parabolic operators. We will also discuss the FBI transform. If we have time we will study Hormander's refinement of Tataru's theorem for uniqueness for differential operators with partially analytic coefficients.
References: 1) *Introduction to Carleman estimates for elliptic and parabolic operators. Applications to unique continuation and control of parabolic equations.* Jerome Le Rousseau and Gilles Lebeau. http://hal.archives-ouvertes.fr/hal-00351736/fr/
2) *On the uniqueness of the Cauchy problem under partial analyticity assumptions.* Lars Hormander. Geometric optics and related topics, Birkhauser, 1997.
3) *An introduction to Semiclassical and Microlocal Analysis.* Andre Martinez, Springer, 2002.

MA 59800: CRN:45729 STOCHSTIC ODE'S AND PDE'S IN PHYSICAL SCIENCES
Instructor: Prof. Cushman, office: Math 416, phone: 49–48040, e-mail: `jcushman@math.purdue.edu`
Time: TTh 1:30-2:45
Description: The course is designed to introduce the student to the engineering/physical aspects of stochastic ode's (both Markovian and non-Markovian) and pde's (with random spatially distributed material properties). Classical results on Markovian sode's such as Ito calculus, and Langevin and Fokker-Planck equations will be discussed as well as non-Markovian processes such as fractional Brownian motion, continuous time random walks and related subordinated processes and non-linear clocks. Time permitting, parabolic and elliptic problems with random coefficients will be discussed in the spirit of moment expansions. Applications in the physical and engineering sciences will be focused on throughout the course.
References: 1. C. W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry and Natural Sciences.* 2nd edition
2. J. H. Cushman, *The Physics of Fluids in Hierarchical Porous Media: Angstroms to Miles.* 1997

MA 59800: CRN:54988 MODULAR FORMS
Instructor: Prof. Shahidi, office: Math 650, phone: 49–41917, e-mail: `shahidi@math.purdue.edu`
Time: MWF 9:30
Description: The course will be a study of classical modular forms on upper half plane using Shimura or some other recent books on the subject. I will try to include also Maass forms which are no longer holomorphic and their spectral theory using Iwaniec, as well as connections with p-adic L-functions. These are basic to any number theorist who wants to work in automorphic forms. I will try to keep prerequisite to a minimum and basically no more than MA553 and MA554.

MA 61100: CRN:34046 METHODS OF APPLIED MATH I
Instructor: Prof. Phillips, office: Math 706, phone: 49–41939, e-mail: `phillips@math.purdue.edu`
Time: TTh 9:00-10:15
Prerequisite: MA 51100, 54400
Description: This course develops the functional analysis needed to study differential equations and applied math. Examples and applications will be given using Sobolev spaces and Sturm-Liouville theory. Topics include Banach and Hilbert spaces; convex analysis; weak topologies; linear operators; Lax-Milgram theorem; compact operators; Riesz-Fredholm theory; spectral theory of compact operators.
References: Prof. Sylva Serfaty *Functinal Analysis Notes Fall 2004* math.nyu.edu/ vilensky/Functional_Analysis.pdf
Text: Avner Friedmen *Foundations of Modern Analysis*

MA 61500: CRN:43130 NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS I
Instructor: Prof. Cai, office: Math 412, phone: 49–41921, e-mail: `zcai@math.purdue.edu`
Time: TTh 12:00-1:15
Prerequisite: MA 51400, 52300
Description: Finite element method for elliptic partial differential equations; weak formulation; finite-dimensional approximations; error bounds; algorithmic issues; solving sparse linear systems; finite element method for parabolic partial differential equations; backward difference and Crank-Nicholson time-stepping; introduction to finite difference methods for elliptic, parabolic, and hyperbolic equations; stability, consistency, and convergence; discrete maximum principles.

MA 63100: CRN:52786 SEVERAL COMPLEX VARIABLES. NOTE: course cancelled Nov 17
Instructor: Prof. Catlin, office: Math 744, phone: 49–41958, e-mail: `catlin@math.purdue.edu`
Time: MWF 11:30
Prerequisite: MA 53000
Description: Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. Local theory of analytic sets (Weierstrass preparation theorem and consequences). Functions and sets in the projective space Pn (theorems of Weierstrass and Chow and their extensions).

MA 64300: CRN:52797 METHODS OF PARTIAL DIFFERENTIAL EQUATIONS II
Instructor: Prof. Phillips, office: Math 706, phone: 49–41939, e-mail: `phillips@math.purdue.edu`
Time: TTh 12:00-1:15
Prerequisite: MA 64200
Description: Continuation of MA 64200. Topics to be covered are L_p theory for solutions of elliptic equations, including Moser's estimates, Aleksandrov maximum principle, and the Calderon-Zygmund theory. Introduction to evolution problems for parabolic and hyperbolic equations, including Galerkin approximation and semigroup methods. Applications to nonlinear problems.
References: L.C. Evans *Partial Differential Equations*
Text: Gilbarg and Trudinger *Elliptic Partial Differential Equations of Second Order*

MA 66100: CRN:43124 MODERN DIFFERENTIAL GEOMETRY
Instructor: Prof. Donnelly, office: Math 716, phone: 49–41944, e-mail: `hgd@math.purdue.edu`
Time: MWF 12:30
Prerequisite: MA 56200.
Description: A foundational course in Riemannian geometry. Topics include the Levi–Civita connection, geodesics, normal coordinates, Jacobi fields. Emphasis on curvature and its relation with topology. Familiarity with differentialble manifolds, tensor fields, and differential forms is assumed.
Text: John M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, Springer–Verlag, 1997

MA 66400: CRN:52798 ALGEBRAIC CURVES AND FUNCTIONS II
Instructor: Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail: `ram@math.purdue.edu`
Time: TTh 3:00-4:15
Description: Algebraic geometry, concerned with solutions of systems of polynomial equations, and their graphical representations, has for long been regarded as a very abstract area of mathematics. However, recently, with the advent of high-speed computers, applications have come about in such diverse areas of science and engineering such as theoretical physics, chemical, electrical, industrial and mechanical engineering, computer aided design (CAD), computer aided manufacturing (CAM), optimization, and robotics. These application areas are also increasingly posing fundamental open mathematical questions. This course is intended as an introduction to various relevant topics in algebraic geometry such as:

- Analysis and resolution of singularities
- Rational and polynomial parametrization
- Intersections of curves and surfaces
- Polynomial maps
- Fundamental Groups and Galois groups
- Expansions of polynomials of any degree in terms of sequences of other polynomials.
- Resultants and solutions of higher degree polynomial equations in several variables.
- Divisors, Differentials, and Genus Formulas.

The lectures will be expository in nature and so will be accessible to everyone. Thus there are no formal prerequisites and all interested students are welcome. Although this will be a continuation of Part I (MA 66300) which I am teaching this Fall, new interested students can follow it with a little extra work.
Text: 1. Shreeram S. Abhyankar *Algebraic Geometry for Scientists and Engineers*, Amer Math Soc
2. Shreeram S. Abhyankar *Resolution of Singularities and Embedded Algebraic Surfaces*, Springer Verlag

MA 68400: CRN:52799 CLASS FIELD THEORY (NOTE: Course Cancelled Nov 2)
Instructor: Prof. Shahidi, office: Math 650, phone: 49–41917, e-mail: `shahidi@math.purdue.edu`
Time: MWF 9:30

MA 69000: CRN:22204 FOURIER ANALYSIS ON NUMBER FIELDS AND TATE'S THESIS
Instructor: Prof. Takeda, office: Math 736, phone: 49–41954, e-mail: `stakeda@math.purdue.edu`
Time: MWF 2:30
Description: The 2010 Abel Prize has been awarded to the prominent number theorist and arithmetic geometer, John Tate. Apparently, his mathematical contributions are immense. Amony many other things, his Ph.D thesis, supervised by Emil Artin, is still a masterpiece in modern number theory, though it was written over a half century ago. In this Ph.D thesis, commonly known as "Tate's thesis", Tate developed a new method of proving the meromorphic continuation and functional equation of the Hecke L-function by using Fourier analysis on locally compact topological spaces. From today's point of view, his method can be considered as a theory of automorphic L-functions for $GL(1)$, and hence a must learn subject for anyone interested in number theory and/or automorphic forms and representations.

The aim of this course is to understand Tate's thesis from scratch. Accordingly, all the necessary tools and concepts such as local fields, adeles, Haar measure, etc, will be explained through the course. Only modest backgrounds in algebra, analysis and topology usually covered in first year graduate courses are needed. Although it would be ideal if the students have taken MA 58400, it is not a prerequisite for this course. This course is Intended for anyone interested in number theory and/or automorphic representation theory.
Text: Dinakar Ramakrishana and Robert J. Valenza, *Fourier Analysis on Number Fields*, (GTM 186), Springer

MA 69000: CRN:22210 TOPICS IN LINEAR ALGEBRAIC GROUPS
Instructor: Prof. Jiu-Kang Yu, office: Math 604, phone: 49–67414, e-mail: `jyu@math.purdue.edu`
Time: MWF 8:30
Prerequisite: A first course in linear algebraic group
Description: This course is intended for advanced students working on number theory or geometry who use substantial Lie theory. We will cover various topics about conjugacy classes, invariant theory, and representation theory, such as Kac coordinates, the Dynkin-Kostant classification, Bala-Carter theory, Vinberg theory, and Springer correspondence.

MA 69200: CRN:52831 APPLIED INVERSE PROBLEMS
Instructor: Prof. de Hoop, office: Math 422, phone: 49–66439, e-mail: `mdehoop@math.purdue.edu`
Time: TTh 10:30-11:45

MA 69200: CRN:22211 SPARSE AND STRUCTURED MATRIX COMPUTATIONS
Instructor: Prof. Xia, office: Math 442, phone: 49–41922, e-mail: `xiaj@math.purdue.edu`
Time: TTh 1:30-2:45
Prerequisite: numerical linear algebra or similar, or CS 515, or consent of instructor
Description: This course is intended for graduate students with basic knowledge in matrix computations. This course discusses advanced techniques for sparse and structured matrix computations, including both sparse matrix theory and numerical algorithms that are widely used in large scale matrix computations and high performance scientific computing. Some state-of-the-art sparse matrix methods will also be covered. The main topics include graph theory for sparse matrices, multifrontal type sparse linear system solution, multigrid, hybrid methods, structured/semiseparable matrices, etc.
References: 1. Golub and Van Loan, *Matrix Computations*, Johns Hopkins
2. Demmel, *Applied Numerical Linear Algebra*, SIAM
3. Saad, *Iterative methods for sparse linear systems*, SIAM
4. Vandebril, Van Barel, and Mastronardi, *Matrix Computations and Semiseparable Matrices*, volume 1-2, Johns Hopkins

MA 69300: CRN:52833 RIEMANN MAPPING. NOTE: course cancelled December 10
Instructor: Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: `branges@math.purdue.edu`
Time: MWF 9:30
Description: Riemann mapping is a persitent source of research in complex analysis which is now generalized to functions of a quaternion variable. The quaternion skew-plane is a skew-field rich in subfields isomorphic to the complex plane. An analytic function of a variable in the skew-plane is treated by analytic functions in subplanes. Classical theorems of complex analysis are generalized to a new context in which the classical proofs apply. The Riemann mapping theorem applies in the skew-plane as conjectured by Poincare. The structure of mapping functions and the estimates of coefficients apply the 1984 proof of the Bieberbach conjecture. Riemann mapping for a skew-plane is preparation for Fourier analysis appearing in the proof of the Riemann hypothesis. No previous experience of complex analysis is required, but doctoral students are advised to complete qualifying examinations before taking the course.

MA 69300: CRN:52834 PERSPECTIVES IN ANALYSIS AND GEOMETRY
Instructor: Prof. Garofalo, office: Math 616, phone: 49–41971, e-mail: `garofalo@math.purdue.edu`
Time: MWF 10:30
Description: This course is intended as a bridge between Analysis and Geometry. It aims at presenting, in a self-contained fashion, various tools and ideas which, besides being integral part of these fields themselves, also constitute an important common ground of modern research. Although the main purpose of the course is introducing the student to various classical problems and to their, perhaps not always classical, solution, I also intend to present several generalizations which are the subject of active research in these fields.

MA 69400: CRN:34103 HEAT KERNEL ASYMPTOTICS AND LOCAL INDEX THEORY
Instructor: Prof. Baudoin, office: Math 438, phone: 49–41406, e-mail: `fbaudoin@math.purdue.edu`
Time: MWF 8:30
Prerequisite: Basics on Brownian motion and differential geometry
Description: The purpose of this is to provide to the student introduction to the theory of heat kernels on bundles with a view toward local index theory.

In the first Part, we remind some basic facts about stochastic differential equations and introduce the language of vector fields. In the second Part, we study stochastic Taylor expansions by means of the so-called formal Chen series. In the third Part, we focus on the applications of stochastic Taylor expansions to the study of the asymptotics in small times of heat kernels on vector bundles. Finally, in the fourth Part, we make the connection between heat kernel analysis and index theory. We will in particular prove the Chern-Gauss-Bonnet theorem. If time allows it, we will also present the heat proof of the local index theorem for the Dirac operator on the spin bundle.

Lecture notes will be posted on my webpage.

MA 69600, CRN: 34038, TOPICS IN COMPLEX GEOMETRY
Instructor: Prof. Yeung, office: Math 712, phone: 49–41942, e-mail: `yeung@math.purdue.edu`
Time: MWF 10:30
Prerequisite: MA 56200, 53000
Description: The purpose of the course is to discuss some topics of classical and current interests in Kaehler geometry, bring in some analytic techniques and study their applications in complex manifolds and algebraic geometry. The followings are some tentative topics to be discussed. We will try to begin from an elementary level.

1. Geometric results related to hypergeometric differential equations.
2. Variation of Hodge structures.
3. Introduction to moduli spaces.
4. Introduction to complex hyperbolicity and value distribution theory.
5. Some aspects of arithmetics and geometry.

References: 1. Kodaira, K., Morrow, J., *Complex manifolds*
2. Griffiths, P., Harris, J., *Principle of algebraic geometry*
3. Mok, N., *Metric rigidity theorems on locally hermitian symmetric spaces.*

Seminars

Algebra and Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Applied Math Lunch Seminar, Prof. Buzzard
Time: Friday 11:30

Automorphic Forms and Representation Theory Seminar, Prof. Goldberg
Time: Thursdays, 1:30

Bridge to Research Seminar, Prof. Bell
Time: Mondays 4:30

Commutative Algebra Seminar, Profs. Heinzer and Ulrich
Time: Wednesdays 4:30

Computational and Applied Math Seminar, Prof. Shen
Time: Fridays 3:30

Computational Finance Seminar, Profs. Figueroa-Lopez and Viens
Time: Mondays 3:30

Function Theory Seminar, Prof. Eremenko
Time: Wedneday 1:30 pm.

Functional Analysis Seminar, Prof. de Branges
Time: Thursday 9:30

Geometric Analysis Seminar, Prof. Yeung
Time: Monday 3:30

Number Theory Seminar, Prof. Goins
Time: Thursday, 3:30

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30

PDE Seminar, Prof. Bauman
Time: Thursdays, 3:30

Probability Seminar
Time: Tuesday 10:30

Spectral and Scattering Theory Seminar, Prof. Sá Barreto
Time: Thursday 4:30

Symplectic Geometry Seminar, Profs. Albers and Lee
Time: Thursday 5:00

Topology Seminar, Profs. Kaufmann and McClure
Time: Thursday 3:30

Working Algebraic Geometry Seminar, Profs. Arapura and Matsuki
Time: Wednesday 3:30-5:00