Real Analysis and Measure Theory
Instructor: Patricia Bauman (baumanp@purdue.edu, 4-1945)
Course Number: MA 54400
Credits: 3
Time: MWF 10:30am-11:20am

Description

Metric space topology; continuity, convergence; equicontinuity, compactness; bounded variation; Helly selection theorem; Riemann-Stieltjes integral; Lebesgue measure; abstract measure spaces; LP spaces, Holder and Minkowski inequalities; Riesz-Fischer theorem.

Introduction to Functional Analysis
Instructor: Prof. Rodrigo Bañuelos (banuelos@purdue.edu, 4-1977)
Course Number: MA 54500
Credits: 3
Time: MWF, 9:30am-10:20am

Description

This course will cover some of the basic tools of analysis that are extremely useful in many areas of mathematics, including PDE's, stochastic analysis, harmonic analysis and complex analysis. Specific topics covered in the course include: "Geometric lemmas" (Vitali, Wiener, etc.) and "geometric decomposition theorems" (Whitney, etc.) and their applications to differentiation theory and to the Hardy–Littlewood maximal function; convolutions; approximations to the identity and their applications to boundary value problems in R^d with L^p-data; the Fourier transform and its basic properties on L^1 and L^2 (including Plancherel's theorem); interpolation theorems for linear operators (Marcinkiewicz, Riesz–Thorin); the basic (extremely elegant and useful) Calderón-Zygmund singular integral theory and some of its applications; the Hardy-Littlewood-Sobolev inequalities for fractional integration and powers of the Laplacian and other elliptic operators; the inequalities of Nash and Sobolev viewed from the point of the heat semigroup in R^d.

TEXT BOOKS: No text book is required. The course follows my lecture notes *"Lectures in Analysis".* Recommended are: (1) E. M. Stein, *"Singular Integrals and Differentiability Properties of Functions"*, (2), L. Grafakos *"Modern Fourier Analysis"*, (3) E. H. Lieb and M. Loss, *"Analysis"*.

Introduction to Functional Analysis
Instructor: Prof. Marius Dadarlat (mdd@purdue.edu, 4-1940)
Course Number: MA 54600
Credits: 3
Time: MWF, 1:30pm-2:20pm, UNIV 103

Description

1. Banach spaces

2. Hilbert spaces
3. Linear Operators and functionals
4. The Hahn-Banach Theorem
5. Duality
6. The Open Mapping Theorem
7. The Uniform Boundedness Principle
8. Weak Topologies
9. Spectra of operators
10. Compact operators
11. Banach algebras and C*-algebras
12. Riesz calculus
13. Fredholm index
14. Gelfand transform
15. Spectral theorem for normal operators
16. Applications: differential operators, Peter-Weyl theorem

No specific textbook is needed. These topics are covered by most books on functional analysis. Good references are: Gert Pedersen, *Analysis Now*, (Graduate Texts in Mathematics 118) and John B. Conway, *A course in Functional Analysis*, 2nd edition, Springer-Verlag 1990.

P-ADIC L-FUNCTIONS

Instructor: Prof. Freydoon Shahidi (fshahidi@purdue.edu, 4-1917)
Course Number: MA 59800:
Credits: 3
Time: TTh 10:30am-11:45am

Description

I hope to cover the theory of p-adic L-functions attached to Dirichlet characters and introduce the Iwasawa algebra. The subjects to be covered are: Generalities about p-adic numbers, p-adic distributions and measures, Bernoulli numbers and distributions, Kummer and von Staudt congruences, p-adic L-functions attached to Dirichlet characters, Iwasawa algebra, proof of Iwasawa-Serre's Theorem and connections with the main conjecture.

While p-adic interpolation is available only for a few automorphic L-functions, there is a vast body of litrature on arbitrary(complex) automorphic L-functions and time permitting I hope to discuss some of it later on. They are both very central in the developement of modern number theory.

Prerequisite: MA584(Algebraic Number Theory) and some familiarity with class field theory-MA684 is more than enough.

No books are required, but here are some recommended ones:

N. Koblitz: P-adic Numbers, P-adic Analysis and zeta functions, GTM 58 L. Washington, Intro. to Cyclotomic Fields, GTM 83 S. Lang, Cyclotomic Fields I & II, GTM 121 D. Bump, Automorphic Forms, Cambridge U. Press F. Shahidi: Eisenstein Series and Automorphic L-functions, AMS Colloquium 58

STOCHASTIC CALCULUS

Instructor: Prof. Fabrice Baudoin (fbaudoin@purdue.edu)
Course Number: MA 59800
Credits: 3
Time: MWF 11:30am-12:30pm

Description

This course will cover the theory of stochastic integration and its applications . We will focus on the following topics:

1) Martingales in continuous time;
2) Brownian motion;
3) integration;
4) Stochastic differential equations;
5) Malliavin calculus.
6) Introduction to financial mathematics

Lecture notes are posted on my blog http://fabricebaudoin.wordpress.com/

Prerequisites: *Basic probability theory: Random variables, Central limit theorem, Law of large numbers, Conditional expectations*

Mathematical Continuum Physics
Instructor: John H. Cushman (jcushman@purdue.edu, 4-48040)
Course Number: EAPS 591/Math59800
Credits: 3
Time: TTh 1:30pm-2:45pm

Description

Prerequisite: Consent of Instructor. The course covers, infinitesimals and tensor notation, deformations and strain, infinitesimal strain, rates of strain, thermodynamics, balance and conservation laws, frame indifference and constitutive axioms and the role of the 2nd law, linearization about equilibrium, local and non-local theories for viscous fluids, elastic bodies, viscoelastic bodies, plasticity, heat conducting bodies, quasi-static electrodynamics with application to carbon nanotube electrode brushes and fuel cells. Other applications to be discussed include swelling porous bodies, shale, blood rheology and living cells and membranes.

Introduction to Geometric Measure Theory
Instructor: Monica Torres (torres@math.purdue.edu, 4-1969)
Course Number: MA 59800
Credits:
Time:

Description

Geometric Measure Theory is widely applied to many areas of Analysis and Partial Differential Equations. This class will be an introduction to Geometric Measure Theory and the topics that will be covered include:

Radon measures

Hausforff measures

Riesz's theorem and vector-valued Radon measures

Sets of finite perimeter and the existence of minimizers in geometric variational problems (i.e. minimal surface).

Approximation of sets of finite perimeter

Coarea formula

Isoperimetric inequality

Reduced boundary and De Giorgi's structure theorem

Rectifiable sets and blow-ups of Radon measures

Fine properties of functions of bounded variation (BV)

Traces of BV functions and the Gauss-Green formula

Traces and Gauss-Green formulas for divergence-measure fields

We will use the following books:

Sets of finite perimeter and geometric variational problems: An introduction to geometric measure theory, by Francesco Maggi, Cambridge studies in advanced mathematics (135), 2012.

Minimal surfaces and functions of bounded variation, by Enrico Giusti, Monographs in Mathematics (80), Boston, 1984.

Measure theory and fine properties of functions, by L.C. Evans and R. Gariepy, Studies in Advanced Mathematics, CRC Press, 1992.

Prerequisites:

Readers require only basic measure theory (as in MA544 or equivalent).

HOPF ALGEBRAS IN TOPOLOGY, GEOMETRY AND PHYSICS

Instructor: Prof. Ralph Kaufmann (rkaufman@math.purdue.edu, 4-1205)
Course Number: MA59800
Credits: 3
Time: 9:00am-10:45am

Description

algebras are a classical object, which underlie many constructions in mathematics. By definition they are algebraic objects, but they appear naturally as functions on a group. More recently they have started to play an increasing role in physics and number theory. We will start with the basic algebraic definitions, examples and main theorems. After this we will move to the topological/geometric side and consider how these structures arise in these contexts. The next topic of study will be their occurrence in analysis and mathematical physics through the work of Connes and Kreimer. Finally we aim to consider their role in number theory, through the work of Connes-Moscovici and their role in motivic theory as explained by Goncharov, Deligne and more recently Francis Brown.

METHODS OF PARTIAL DIFFERENTIAL EQUATIONS II

Instructor: Prof. Arshak Petrosyan (arshak@purdue.edu, 4-1932)
Course Number: MA 64300 CRN:
Credits: 3
Time: 1:30pm-2:45pm

Description

Continuation of MA 64200. Topics to be covered are L^p theory for solutions of elliptic equations, including Moser?s estimates, Aleksandrov maximum principle, and the Calderon-Zygmund theory. Introduction to evolution problems for parabolic and hyperbolic equations, including Galerkin approximation and semigroup methods. Applications to nonlinear problems.

References:

1) D. Gilbarg and N. S. Trudinger, Elliptic Partial Differential Equations of Second Order, 2nd edition

2) G. M. Lieberman, Second Order Parabolic Differential Equations.

3) L. C. Evans, Partial Differential Equations, 2nd edition

CALCULUS OF VARIATIONS

Instructor: Prof. Donatella Danielli (dgarofal@purdue.edu, 4-1920)

Course Number: MA 64400

Credits: 3

Time: MWF 9:30am-10:30am

Prerequisite: The course will be essentially self-contained. The only prerequisites are familiarity with the Lebesgue integral and the first properties of L^p spaces, as well as a basic knowledge of Sobolev spaces.

Description: The calculus of variations is one of the classical subjects in mathematics. Besides its links with other branches of mathematics (such as geometry, optimal control theory, and differential equations), it is widely used in physics, engineering, economics, and biology. The essence of the calculus of variations is to identify necessary and sufficient conditions that guarantee the existence of minimizers for integral functionals of the type

$$\mathcal{F}(u;\Omega) = \int_\Omega F(x,u,Du)\ dx.$$

In this course we will focus on the so-called direct methods, which consist in proving the existence of the minimum of $\mathcal{F}$ directly, rather than resorting to its Euler equation. The central idea is to consider $\mathcal{F}$ as a real-valued mapping on the manifold of functions taking on $\partial\Omega$ given boundary values, and applying to it a generalization of Weierstrass? theorem on the existence of the minimum of a continuous function. One of the main issues in this approach is to identify the Sobolev spaces as the proper function spaces for compactness results to hold. In turn, the solution of the existence problem in the Sobolev class opens up a series of questions about the regularity of the minimizers. This was a long-standing open problem, until the way to its solution (in a non-direct fashion, since it involves the Euler equation) was opened by the celebrated De Giorgi-Nash-Moser result concerning the Hölder continuity of solutions to uniformly elliptic PDEs in divergence form with bounded and measurable coefficients. A first step towards the use of direct methods in the regularity issue came from a 1982 paper by Giaquinta and Giusti, who proved the Hölder continuity of quasi-minima, that is functions u for which

$$\mathcal{F}(u;K) \leq Q\mathcal{F}(u+\phi;K)$$

for every ϕ with compact support $K \subset \Omega$. The notion of quasi-minima reduces of course to the one of minimum when $Q = 1$, but it is substantially more general, since it includes solutions of linear and nonlinear elliptic equations and systems. The course will follow the path outlined above, and it will combine the classical approach to the subject with its latest developments. In particular, we will analyze some beautiful examples, such as the obstacle problem, which lead to the theory of free boundary problems.

MODERN DIFFERENTIAL GEOMETRY

Instructor: Prof. Harold Donnelly (hgd@purdue.edu, 4-1944)
Course Number: MA 66100
Credits: 3
Time: TTh 3:00-4:15 pm

Prerequisite: MA 56200.

Description: A foundational course in Riemannian geometry. Topics include the Levi?Civita connection, geodesics, normal coordinates, Jacobi fields. Emphasis on curvature and its relation with topology. Familiarity with differentialble manifolds, tensor fields, and differential forms is assumed. Text: John M. Lee, Riemannian Manifolds: An Introduction to Curvature, Springer Verlag, 1997

TOPICS IN COMPLEX GEOMETRY

Instructor: Prof. Sai Kee Yeung (yeung@math.purdue.edu, 4-1942)
Course Number: MA 69600
Credits: 3
Time: MWF 12:30pm-1:30pm

Prerequisite: 562, 525

Description: Here are the tentative topics to be discussed.

1. Introduction to Kaehler geometry.
2. Introduction to symmetric spaces.
3. Introduction to heat equations, harmonic maps and rigidity.
4. Introduction to K"ahler-Ricci flow.
5. Introduction to Riemann Zeta functions.

Reference: I would provide reference as the class proceeds.

INTEGRAL CLOSURES AND MULTIPLICITIES

Instructor: Prof. Bernd Ulrich (ulrich@math.purdue.edu, 4-1972)
Course Number: MA 69000
Credits: 3
Time: TTh 3:00-4:15 pm

Prerequisite: MA 56200.

Description: The course deals with integral closures of ideals and modules. Integral closures play an important role, for instance, in the study of Hilbert functions, in intersection theory, and in singularity theory. Of particular importance are numerical criteria for integral dependence, in terms of suitably defined generalized multiplicities. Such criteria provide conditions for families of analytic sets to be Whitney equisingular. A crucial tool for studying integral closures and multiplicities is the Rees algebra, an algebraic construction that also arises in the process of resolving singularities.

The course serves in some sense as a continuation of MA 65000. However, it should be accessible to anybody with basic knowledge in commutative algebra.

Basic knowledge about commutative rings (such as the material of MA 557).

Suggested texts for some of the material:

- C. Huneke and I. Swanson, Integral closure of ideals, rings, and modules, Cambridge Univ. Press

- W. Vasconcelos, Integral closure, Springer

- W. Vasconcelos, Arithmetic of blowup algebras, Cambridge Univ. Press.