Introduction to Partial Differential Equations

Instructor: Harold Donnelly
Course Number: MA52300
Credits: Three
Time: MWF 12:30 P.M.

Description

Integral curves and surfaces of vector fields, theory of quasi–linear and linear equations of first order, linear partial differential equations, characteristics, classification and canonical forms, equations of mathematical physics, Laplace's equation, wave equation, heat equation.

Text, E.C. Zachmanoglou and D.W. Thoe, Introduction to Partial Differential Equations, Dover, 1986.

Complex Analysis

Instructor: Louis de Branges
Course Number: MA53100
Credits: Three
MWF 9:30–10:20 A.M.

Description

This second course in complex analysis presumes a knowledge of MATH 53000 or the equivalent from the text of Lars Ahlfors. The aim of the course is the application of complex analysis to Fourier analysis. Fourier analysis is formulated in locally compact abelian groups by Lynn Loomis in ABSTRACT HARMONIC ANALYSIS. The present treatment applies to locally compact skew–fields as they appear in a proof of the Riemann hypothesis (preprint). This requires an introduction to algebraic number fields as in van der Waerden MODERN ALGEBRA. Skew–fields are algebras of quarternions with coordinates in a real algebraic number field. Quarternions with real numbers as coordinates create a pattern for generalization. Topics covered are the Laplace transformation and the Mellin transformation in relation to the Fourier transformation.

Ordinary Differential Equations and Dynamical Systems

Instructor: Aaron Yip
Course Number: MA54300
Credits: Three
Time: TTh 1:30–2:45 P.M.

Description

This is a beginning graduate level course on ordinary differential equations. It covers basic results for linear systems, local theory for nonlinear systems (existence and uniqueness, dependence on parameters, flows and linearization, stable manifold theorem) and their global theory (global existence, limit sets and periodic orbits, Poincare maps). Some further topics include bifurcations, averaging techniques and applications to mechanics and population dynamics.

Prerequisites: one undergraduate course in each of the following topics: linear algebra (for example, MA 265, 351), differential equation (for example, MA 266, 366), analysis (for example, MA 341, 440, 504), or instructor's consent.

Textbook: Perko: Differential Equations and Dynamical Systems (available online from Purdue)

Mathematical Continuum Physics

Instructor: John H. Cushman

Course Number: MA 598/EAPS 591

Credits: Three

Time: TTh 1:30 P.M.

Description

Lagrangian and Eulerian coordinate system representations are employed throughout all developments. We begin by constructing the fully non-linear strain tensor and analyze its component?s physical significance. This is followed by development of the integral, and subsequently local forms, of conservation of mass, balances of linear and angular momentum and conservation of energy. The 2nd ?law of thermodynamics is postulated for the entire body and employed to develop fully non-linear constitutive relations which are subsequently linearized near equilibrium for many classes of fluids and solids. Maxwell?s equations of electrodynamics are introduced, coupled with the conservation and balance laws and subjected to the 2nd ? law to obtain generalized field equations. Averaging principles are employed to obtain the conservation and balance laws for mixtures of species and phases of relevance to porous media. Applications are presented for swelling biopolymers (foods and cells), drug delivery substrates, geophysical media (soils, aquifers and petroleum reservoirs), electro-active polymers (soft robotics), and fuel cells (flow batteries). The common structure of all these examples is highlighted.

Profinite Groups and Group Cohomology

Instructor: Professor Ben McReynolds

Course Number: MA59800

Credits: Three

Time: T Th 10:30-11:45 AM

Description

This course will cover foundational material on the topics of profinite groups, profinite completions, and cohomology of groups. Some knowledge of groups and some experience with some cohomology/homology theory would be useful but not necessary. With the remaining lectures, we may cover Galois cohomology or cohomological dimension. I will let those that attend the class decide on some finishing topics.

Other: There will be no formal obligations required for the course. I will give some exercises but they will not be taken up.

Stochastic Calculus
Instructor: Professor Samy Tindel
Course Number: MA59800
Credits: Three
Time: T Th 4:30-5:45 PM
Email: stindel@purdue.edu
Webpage: https://www.math.purdue.edu/∼stindel

Description

The central object of this course is Brownian motion. This stochastic process (denoted by $W = \{W_t;\, t \geq 0\}$ in the sequel) is used in numerous concrete situations, ranging from engineering to finance or biology. It is also of crucial interest in probability theory, owing to the fact that this process is Gaussian, martingale and Markov at the same time. This very rich structure converts Brownian motion into a fascinating object, but it should also be pointed out that the paths of W are irregular (in particular nowhere differentiable), as depicted in the figure below. Our first aim will thus be a good description of Brownian motion. We will then construct a differential calculus with respect to W. Specifically, our two main tasks will be:

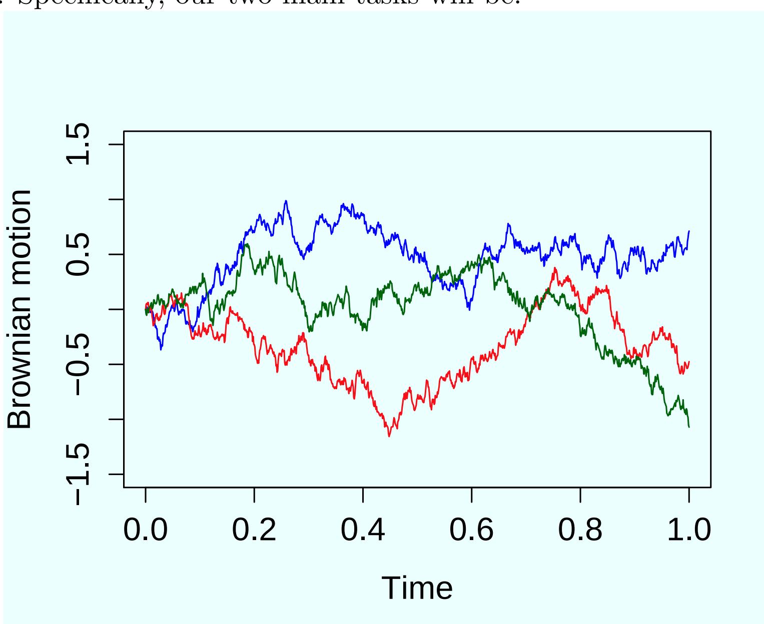

Figure 1: Three independent Brownian paths

(i) Give a formula for the differential $df(W_t)$ when f is smooth enough. This fundamental relation is called Itô's formula.

(ii) Solve differential equations driven by W, of the form $dX_t = b(X_t)\, dt + \sigma(X_t)\, dW_t$, and give elementary properties for their solutions.

(iii) If time constraints allow it, give an introduction to the analysis on Wiener's space.

Outline of the course

1. Notions of probability theory.
2. Brownian motion.
3. Itô's formula.
4. Stochastic differential equations.

Bibliography:

1. R. Durrett: Stochastic calculus. A practical introduction. Probability and Stochastics Series. CRC Press, 1996.
2. I. Karatzas, S. Shreve: Brownian motion and stochastic calculus. Second edition. Graduate Texts in Mathematics, 113. Springer-Verlag, 1991.
3. D. Revuz, M. Yor: Continuous martingales and Brownian motion. Third edition. Springer-Verlag, 1999.

Meth Applied Math I

Instructor: Donatella Danielli
Course Number: MA 61100
Credits: Three
Time: MWF 11:30–12:20 P.M.

Description

The purpose of this course is to present the most fundamental theorems of operator theory and functional analysis, keeping applications in mind. Topics covered will include metric spaces; Banach spaces; analytic and geometric formulations of the Hahn-Banach theorem; the principle of uniform boundedness, the closed graph theorem and the open-mapping theorem; weak topologies; reflexive and separable spaces; Hilbert spaces; compact operators and the Fredholm-Riesz-Schauder theory; spectral theory for self-adjoint operators. Applications to ordinary and partial differential equations, as well as to integral equations, will be discussed. If time permits, we will cover the Hille-Yosida theorem and its applications to evolution equations.

Text: A. Friedman, Foundations of Modern Analysis, Dover

References: H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer.

Prerequisite: MA 51100 and MA 54400, or instructor's approval.

Several Complex Variables

Instructor: Laszlo Lempert

Course Number: MA 63100

Credits: Three

Time: MWF 12:30–1:20 P.M.

Description

Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. The inhomogeneous Cauchy?Riemann equations. Local theory of holomorphic functions and of analytic sets (Weierstrass preparation theorem and consequences). Complex manifolds, meromorphic functions, analytic cohomology.

Prerequisite: MA 53000.

Methods of Linear & Nonlinear Partial Differiatial Equations

Instructor: Donatella Danielli

Course Number: MA 64200

Credits: Three

Time: MWF 9:30–10:20 A.M.

Description

This is the first semester of a one-year course in the theory of second order elliptic and parabolic PDEs. The aim of the course is to study the solvability of boundary value problems and regularity properties of solutions. The first semester will focus on linear elliptic equations, both in divergence and nondivergence form. The starting point for the study of classical solutions will be the theory of Laplace?s and Poisson's equations. The emphasis here will be on: 1. Existence of solutions to the Dirichlet problem for harmonic functions via the Perron method (based on the maximum principle); 2. Holder estimates for Poisson's equation derived from the analysis of the Newtonian potential. The crowning achievement of the theory of classical solutions is Schauder's theory, which extends the results of potential theory to a general class of non-divergence form equations with Holder-continuous coefficients. In the second part of the semester we will consider a more general - and modern - approach to linear problems, based not on potential theory, but on Hilbert space methods for so-called "weak" solutions. Our main goal will be to prove the celebrated De Giorgi-Nash-Moser theorem on the regularity of weak solutions. The relevant tools from the theory of Sobolev spaces will be developed concurrently.

Text: D. Gilbarg and N. S. Trudinger, Elliptic Partial Differential Equations of Second Order, Second Edition, Springer.

Prerequisite: MA 54400 and MA 61100, or instructor's approval.

Methods Of Linear And Nonlinear Partial Differential Equations II
Instructor: Daniel Phillips
Course Number: MA 64300
Credits: Three
Time: MWF 1:30–2:20 P.M.

Description

This course will be a continuation of MA 642 which included theories for classical solutions to linear second-order elliptic equations with smooth coefficients and for weak solutions to linear equations in divergence form with bounded measurable coefficients.

MA 643 builds on these theories and includes introductions to:

1) existence results for solutions to quasi-linear equations,
2) strong solutions for linear second order elliptic equations with bounded measurable coefficients,
3) Galerkin's method and the continuous semigroup method for parabolic and hyperbolic time dependent problems,
4) viscosity solutions and fully nonlinear applications.

Modern Differential Geometry
Instructor: Prof. Harold Donnelly
Course Number: MA66100
Credits: Three
Time: MWF 2:30–3:20 pm

Description:

A foundational course in Riemannian geometry. Topics include the Levi–Civita connection, geodesics, normal coordinates, Jacobi fields. Emphasis on curvature and its relation with topology. Familiarity with differentiable manifolds, tensor fields, and differential forms is assumed. Text: John M. Lee, Riemannian Manifolds: An Introduction to Curvature, Springer Verlag, 1997

Prerequisite: MA 56200.

Class Field Theory
Instructor: Prof. Edray Goins
Course Number: MA68400
Credits: Three
Time: MWF 1:30–2:20 P.M.

Description:

It was first conjectured by the German mathematician David Hilbert (1862 - 1943) in his 1897 work Zahlbericht (literally ?number report?) that if K is a number field with class number h, then there exists an extension L of degree h over K such that every ideal of K becomes principal, even though L itself might not have class number 1. The Japanese mathematician Teiji Takagi (1875 - 1960) proved in 1920 what we now call the Existence Theorem: that is, he showed that the field

L which Hilbert conjectured does indeed exist. This field is now called the Hilbert Class Field. However, Takagi proved a stronger result: this field is an unramified extension of K such that its Galois group is abelian, and moreover it is the largest such extension. The Austrian mathematician Emil Artin (1898 - 1962) extended the results of Takagi in 1923 in his paper Uber eine neue Art von L-Reihen by considering properties of L-series. (The title translates as ?On a new type of L-Series.?) This paper relied on existing reciprocity laws. He turned in a different direction in 1927 by using ideas of Russian mathematician Nikolai Grigorievich Cebotarov (1894 - 1947) in a 1924 paper on density of prime ideals in number fields. This new approach proved the existing reciprocity laws! This result is now called the Artin Reciprocity Theorem.

In this course, we will focus on Class Field Theory from both the classical (i.e., ideal class group) and modern (i.e., idele class group) points of view. We will study study Quadratic and Cubic Reciprocity via the Artin automorphism; the Hilbert Class Field via the Artin Reciprocity Theorem; the Ray Class Field via the Existence Theorem; and then conclude with a discussion of the modern (i.e. idele class) point of view via adeles and ideles. Topics that will be covered include Ray Class Group; Special Values of L-Series; Density of Primes; Galois Cohomology; Artin Reciprocity Theorem; Class Groups and Class Fields; Existence and Classification Theorems; Hilbert Class Fields; and Extended Class Group.

Prerequisite: MA584

Representation Theory with Applications

Instructor: Prof. Freydoon Shahidi

Course Number: MA69000A

Credits: Three

Time: MWF 9:30–10:20 A.M.

Description

The subjects to be covered:

1. Topics from Fulton-Harris with connections to combinatorics (in connection with the theory of γ–factors attached to the L–functions of representations of reductive groups and arbitrary representations of their L–groups via Braverman–Kazhdan/Ngo).
2. Some representation theory of real Lie groups
3. Applications to number theory through automorphic forms and L-functions

Suggested references:

Fulton-Harris, Representation Theory, GTM 129, Springer, 1991.

F. Shahidi, Eisenstein Series and L-functions, AMS Colloquium Publications, Vol. 58, AMS, 2010.

A. Knapp, Representations of Semisimple Groups, Princeton University Press, 1986

Introduction to Kinetic Theory

Instructor: Jingwei Hu

Course Number: MA69200

Credits: Three

Time: T Th 3:00–4:15 P.M.

Description

In multiscale modeling hierarchy, kinetic theory serves as a basic building block that bridges atomistic and continuum models. It describes the non-equilibrium dynamics of a gas or system comprised of a large number of particles. On one hand, kinetic descriptions are more efficient (requiring fewer degrees of freedom) than molecular dynamics; on the other hand, they provide rich information at the mesoscopic level when the well-known fluid mechanics laws of Navier-Stokes and Fourier become inadequate, and have proved to be reliable in many fields such as rarefied gas/plasma dynamics, radiative transfer, semiconductor modeling, or even social and biological sciences. This course will constitute an introduction to the kinetic theory with the focus on the Boltzmann equation and related kinetic models. We will discuss basic mathematical theory, numerical methods, and various applications. Specific topics will include: derivation and properties of the Boltzmann equation (collision integral, conservation laws, H-theorem, etc.), connection to molecular dynamics, connection to fluid equations, Chapman-Enskog expansion, moment closures, deterministic numerical methods (e.g., discrete-velocity methods, spectral methods, fast summation methods), stochastic numerical methods (e.g., direct simulation Monte Carlo methods), asymptotic-preserving schemes (multiscale methods coupling kinetic and fluid equations). Other possible topics depending on the interests of the audience: Fokker-Planck-Landau equation for plasmas, quantum Boltzmann equation for bosons and fermions, inelastic Boltzmann equation for granular media, multi-species Boltzmann equation for gaseous mixtures, semiconductor Boltzmann equation for electron transport, kinetic models for collective behavior of swarming and flocking, etc. No textbook is required. The material will be based on the lecture notes by the instructor, and some papers and book chapters. No exams. Students are expected to present course-related material in class or work on small research projects.

Topics in Analysis — von Neumann Algebras

Instructor: Thomas Sinclair

Course Number: MA 693

Credits: Three

Time: MWF 11:30–12:20 P.M.

Description

This course is intended to serve as an introduction to von Neumann algebras, specifically the theory and classification of II_1 factors. The course will also strive to treat some advanced topics, acquainting students with active research areas and problems within the field. Basic knowledge of functional analysis will be assumed. The theory of von Neumann algebras was initiated by Murray and von Neumann in the 1930s and 40s. The last several decades have seen a wealth of remarkable applications of von Neumann algebras to many other mathematical fields including: Jones' theory of subfactors and the discovery of new knot invariants; Voiculescu's theory of free probability with connections to the theory of random matrices; Popa's deformation/rigidity theory with applications to Mostow-Margulis type superrigidity theorems for ergodic actions of discrete groups; and connections of work of Connes and Kirchberg with Quantum Information Theory and violations of Bell's inequality. Topics covered will include:

A) The basic theory - the spectral theorem; von Neumann's double commutant theorem; Kaplanksy's density theorem; the predual; projections and type classification; construction of the trace on a II_1 factor; group von Neumann algebras; subfactors, the basic construction, and index; correspondences and approximation properties.

B) Advanced topics (a selection of these) — amenability and Connes' classification of amenable factors; Popa's intertwining theorem and the modern theory of the classification of II1 factors (2002-present) using Popa's deformation/rigidity theory; solid von Neumann algebras; basics of Voiculescu's free probability theory; Connes' embedding problem, sofic groups, and Kirchberg's conjecture.

Grades: No exams will be given. Optional homework problems will be assigned but will not be collected. Grades will be determined by active involvement in the course and a short presentation.

Resources:

[1] Zhu, Kehe, An Introduction to Operator Algebras, CRC Press, 1993.

[2] Jones, Vaughan, Von Neumann Algebras, online lecture notes, http://www.math.berkeley.edu/ vfr/MATH20909/VonNeumann2009.pdf

[3] Peterson, Jesse, Notes on von Neumann Algebras, online lecture notes, http://www.math.vanderbilt.edu/ peters10/teaching/spring2013/vonNeumannAlgebras.pdf

In addition, many other secondary resources in the form of articles?

Introduction to Microlocal Analysis

Instructor: Professor Plamen Stefanov

Course Number: MA 693

Credits: Three

Time: T Th 4:30-5:45 PM

Description

This course is an introduction to some of the most important topics in Microlocal Analysis. I will start with a review of distributions, the Fourier Transform and introduction of Wave Front Sets. Then I will introduce the pseudo-differential operators in an open domain and develop the calculus of such operators: sums, compositions, adjoints, boundedness in Sobolev spaces, etc. Then I will show how to build a parametrix of an elliptic operator, and what it is good for: local solvability of elliptic PDEs, elliptic regularity and Fredholm properties of elliptic operators. The second part of the course will deal with hyperbolic equations, like the wave equation with variable coefficients. I will explain the geometric optics construction and that will serve as an example of a Fourier Integral Operator (FIO) even though the theory of FIOs is beyond the scope of this course. One of the fundamental results of the theory is Hormander's theorem of propagation of singularities, which will be presented in the context of the wave equation first.

Prerequisites for the course are: familiarity with distributions and the Fourier Transform (that we will review anyway), linear operators in Banach and Hilbert spaces, basic PDE theory.