Math 519 Qualifying Exam-January 10, 1998

Each part of each problem is worth 17 points. A table of the standard normal distribution is attached.

1. Ten points are selected independently and at random(i.e. according to a uniform (0,1) distribution) from the interval (0,1). Let D be the minimum of the ten distances from these points to the complement of (0,1). Find the expectation of D.

2. Six cards are dealt from a shuffled deck, one at a time.

   (a) Find the probability of "two triples," that is, only two of the demominations ace,2,...J,Q,K, are present in the hand, and exactly three of each of these denominations occurs.

   (b) Find the expected number of suits among the six cards. (The suits are hearts, clubs, spades, diamonds.)

3. A balanced six sided die numbered 1-6 is rolled 1000 times. Estimate the probability that the sum of all the even numbers rolled exceeds the sum of all the odd numbers rolled by at least 480.

4. A point P is picked at random (i.e. uniformly with respect to area) on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ of Euclidean three space. Then a point Q is picked at random (uniformly with respect to length) on the line which connects the origin and P.

   (a) Let (X, Y, Z) be the rectangular coordinates of Q. Find the joint density of (X, Y, Z).

   (b) Show that the random variables X, Y, Z of part (a) are not independent. Suppose P is picked as above, and that a point R is picked so that it is on the half line starting at the origin and going through P on out to infinity, with the distance of R from the origin being a random variable with a density function f. (So, the special case where f equals one between 0 and 1 gives the point in part (a).) Let (X', Y', Z') be the rectangular coordinates of R. Is there any choice of f which makes X', Y', Z' independent?