QUALIFYING EXAMINATION
AUGUST 2003
MATH 519 - Prof. Eremenko

NAME:

1. An engine of certain type fails with probability p. An rocket having several engines of this type crashes when at least half of its engines fail, and they fail independently of each other. Suppose that two such rockets, one with two engines and another with four engines, have the same probability of crash. Find p.

2. Let X_1, X_2, X_3 and X_4 be independent uniformly distributed points on a circle. What is the probability that the chords $[X_1, X_2]$ and $[X_3, X_4]$ intersect.

3. A ball of radius R contains N random points which are independently and uniformly distributed.

   a) Find the distribution of the distance from the center to the closest point.

   b) Find the limit of this distribution when $N/R^3 \to 4\pi\lambda/3$, where $\lambda > 0$ is a given number.

4. A particle arrives to a random point x uniformly distributed on $[-1, 1]$. There are two devices independently detecting this particle; one detects the particle with probability $p(x)$ and another with probability $q(x)$. What is the conditional distribution of the point x of arrival, given that the particle was detected by both devices.

5. A man sells newspapers on the corner. Suppose that every one who passes by buys a newspaper with probability 1/3. Let N be the number of people who passed by until the 100-th newspaper was sold. Find the expectation and variance of N.

   More precisely, let $X_1, \ldots,$ be a sequence of Bernoulli trials with probability of success 1/3. Find the expectation and variance of

$$N = \min\{k : X_1 + \ldots + X_k = 100\}.$$

6. Let $X_1, X_2, \ldots, X_{n+1}$ be a sequence of independent identically distributed random variables taking the value 1 with probability p and 0 with probability $1-p$. Let $Y_k = 0$ if $X_k + X_{k+1}$ is even and $Y_k = 1$ if $X_k + X_{k+1}$ is odd. Find the expectation and variance of $Y_1 + \ldots + Y_n$.

7. According to Maxwell, the speed of a molecule in an ideal gas has density

$$f(s) = 4\sqrt{\frac{a^3}{\pi}} s^2 e^{-as^2},$$

where $a > 0$ is physical constant.

a) Find the average speed of molecules.

b) Find the density of the kinetic energy $E = ms^2/2$.

8. Let X and Y be independent standard normal variables. Prove that $X^2 + Y^2$ and X/Y are independent.