QUALIFYING EXAMINATION
AUGUST 2004
MATH 519 - Profs. Davis and Viens

Do not do arithmetic.
$I(A)$ is the random variable which equals one on the set A and zero on the complement of A.
Each problem is worth twenty points.

1. Customers entering a store spend nothing half the time and the other half the time they spend a continuous uniform on the interval $(0,1)$ number of dollars. Use Chebyshev's inequality to bound the probability that the average number of dollars spent by the first one hundred customers to enter the store is between .23 and .27.

2. A die is rolled repeatedly. Let A_k be the event that the sum of the k^{th} and the $(k+1)^{\text{st}}$ rolls is two. Let B_k be the event that the sum of the k^{th} and the $(k+1)^{\text{st}}$ rolls is seven. Let $X = \sum_{k=1}^{100} I(A_k)$ and let $Y = \sum_{k=1}^{100} I(B_k)$.
   (a) Are the events A_k, $k = 1, 2, \cdots$ independent? Are the events B_k, $k = 1, 2, \cdots$ independent? Briefly justify these answers.
   (b) Find the mean and variance of X and Y.
   (c) Find ONE of $P(X = 20)$, $\quad P(Y = 20)$.

3. Suppose that A and B are two points chosen at random in the unit circle in $\mathbb{R}^2$. Find the probability that the circle centered at A with radius AB (i.e., radius equals the distance between A and B) is completely contained within the unit disc.

4. Two points are picked at random from the unit square $\{(x,y) : 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$. Find the probability that the (infinite) straight line through the two points intersects the line segment $\{(x,y) : x = 1,\ 0 \leq y \leq 1\}$.

5. Ten white and ten black balls are divided into two boxes. What is the division that maximizes the probability that if one of the boxes is picked at random and then one ball is drawn from the box at random, the chosen ball is white? Justify your answer.