Math 519 – Prof. Sellke
Qualifying Examination
August, 2010

1. A city has n families which have at least 3 children in the family. Give a good estimate of the minimal value of n so that there is a probability of at least $\frac{1}{2}$ that, for some pair of families, the firstborns will have a common birthday, the secondborns have a common birthday, and the thirdborns will have a common birthday. As usual in birthday problems, you should ignore leap days and assume that birthdays are independent for different people and uniformly distributed on the 365 possible days.

2. There are 30 chairs around a (very large) circular table. People arrive one–by–one. As each person arrives, he or she takes one of the empty seats at random (with all empty seats having equal probability, of course). After 7 people have arrived and seated themselves, what is the probability that no two people are adjacent?

3. Let $X_1, \ldots, X_{10^6}$ and $Y_1, \ldots, Y_{10^6}$ be *iid* standard normal. Let $T = \max \sqrt{(X_k)^2 + (Y_k)^2}$, $1 \leq k \leq 10^6$, (= maximum distance from origin for points (X_k, Y_k).)

   About how big will T typically be? Estimate the median value of T.

4. Let X and Y be independent unit exponential random variables, and let $W = \dfrac{X}{Y}$. Find the density of W.

5. A straight stick of length one is marked at two independent random (i.e. uniformly distributed) places, chosen one after the other, the first colored red and the second colored blue. The stick is then placed on a board and a nail is driven through the red mark, and the stick is spun around the nail twice independently and randomly. Find the expected square of the distance between the two places the blue mark ended up after the first and second spins.