1. (10 points) Suppose $X$ is $N(0, 1)$ (i.e., standard normal) and $Y \mid X = x$ is $N(x + 1, 1)$.
   
   (a) (3 points) Find the marginal distribution of $Y$.
   
   (b) (4 points) Find the correlation between $X$ and $Y$.
   
   (c) (3 points) Find $E[X \mid Y = y]$.

2. (10 points) Let $X$ and $Y$ be independent standard exponential rv’s, with density $f(t) = e^{-t}, t \geq 0$. Let $U = \exp(X^2 - Y)$ and $V = \exp(Y^2 - X)$. Find the value at $(1, 1)$ of the continuous joint density of $U$ and $V$.

3. (10 points) A one dimensional nonhomogeneous Poisson process has the intensity (rate) function $\lambda(t) = ct$, where $c$ is a positive constant. Find the density of the $n$th arrival time for a general integer $n \geq 1$.

4. (10 points) A man has had much too much to drink, but is still strong enough to walk and to see where he is trying to go. He starts at the origin in $\mathbb{R}^2$. He takes iid steps, each step equally likely to be "up", "down", or "right", always of length 1: in other words, the three steps $(0, 1)$, $(0, -1)$, and $(1, 0)$ are of probability 1/3 each. At the first time that his horizontal position equals 100, what is the approximate numerical probability that his vertical position is greater than or equal to 10?

5. (10 points) Let $X_1, \ldots, X_5$ and $Y_1, \ldots, Y_5$ be independent $N(0, 1)$ (i.e., standard normal) random variables. Consider the 5 points in $\mathbb{R}^2$ with coordinates $(X_k, Y_k), 1 \leq k \leq 5$. Let $D_1 < D_2 < \cdots < D_5$ be the ordered distances of those 5 points to the origin. Find the joint density of $D_1$ and $D_2$. 