Math 519 – B. Davis
Qualifying Examination
January, 2011

Do not do arithmetic or simplify answers.

(15) 1. Let X and Y be independent random variables each uniform on $[0,1]$. Give a function

$$H(x,y) = (u(x,y), v(x,y))$$

such that $H(X,Y)$ has a uniform distribution on the parallelogram with vertices $(0,0), (0,2), (3,3)$, and $(3,5)$.

(20) 2. A pond contains M golden fish and K silver fish. The fish are removed one at a time at random until all the fish remaining in the pond are the same color. Find the expectation and the variance of the number of fish remaining in the pond.

(15) 3. Let X_i, $1 \leq i \leq 10$, be independent exponential random variables. Le Y_1 and Y_2 be independent random variables which are both discrete uniform on $\{0,1,2\}$ and which are also independent of the exponentials. Find the probability that of these twelve random variables, the Y_j's are the second and third order statistics, that is the second and third smallest of the twelve numbers.

(25) 4. Sunny, partly cloudy, and cloudy days each have probability 1/3 of occurring and from day to day what happens is independent. On cloudy days a certain restaurant loses two dollars with probability one half and makes two dollars with probability one half. On sunny days it makes an amount of money which is normally distributed with mean 0 and variance seven. And on partly cloudy days it makes a dollar with probability one half and loses a dollar with probability one half.

4a. Estimate the probability that in one hundred days the restaurant makes a net profit of at least ten dollars. Give your answer in terms of the standard normal distritution function.

4b. Find the conditional probability that there were exactly ninety nine cloudy days out of one hundred days if in those hundred days the restaurant's net profit was exactly 197 dollars.

(25) 5. Ten points are chosen independently according to the uniform distribution on the interval $[0,1]$.

5a. Find the probability that at least 8 of the other 9 points are within .01 of the minimum point.

5b. Find the probability that there is an interval of length .01 which contains at least 9 of the 10 points.