1. Find the solution to the Cauchy problem

\[ x z_x + y z_y = z \]

with \( z = 1 \) on the parabola \( y = x^2 \) \((y > 0)\).
2. Describe the characteristic curves for

\[(\sin y)^2 u_{xx} + 2u_{yy} - x^2 u_y + 2u + xy = 0.\]

Explain the importance of characteristic surfaces in the study of partial differential equations.
3. State and prove a maximum principle for solutions $u(x, t)$ to the one dimensional heat equation for $0 \leq x \leq L$, $0 \leq t < T$. 
4. State carefully the Neumann problem for Laplace’s equation in a bounded domain $\Omega$ in $\mathbb{R}^3$ with smooth boundary $\partial \Omega$.

State and prove the necessary condition for the existence of a solution to the Neumann problem for Laplace’s equation.
5. Solve by Fourier series the following one dimensional wave equation with initial–boundary value conditions:

\[
\begin{align*}
    u_{tt} - u_{xx} &= 0 & 0 < x < \pi, & t > 0, \\
    u(0, t) &= u(\pi, t) = 0 & t > 0, \\
    u(x, 0) &= \sin x & 0 \leq x \leq \pi, \\
    u_t(x, 0) &= \sin 2x & 0 \leq x \leq \pi
\end{align*}
\]

Compute the value of the energy integral \((1/2) \int_0^\pi (u_x^2 + u_t^2) \, dx\) when \(t = 100\).
6. Find the solution by Fourier series to the Dirichlet problem in the annulus

\[ A = \{(r, \theta) : 1/2 < r < 1\}, \]

\[ \Delta u = 0 \]

\[ u(1/2, \theta) = \sin \theta \quad -\pi \leq \theta \leq \pi, \]

\[ u(1, \theta) = \cos 2\theta \quad -\pi \leq \theta \leq \pi. \]