1. Find the solution to the Cauchy problem
\[ yu_x - xu_y = 2xyu, \quad u|_{x=y} = x^2. \]

2. Consider the Cauchy problem
\[ u_{tt} - u_{tx} - 2u_{xx} + xu_t + 5u = 0 \]
\[ u|_{\gamma} = e^x, \quad u|_{\gamma} = 3 - 2x, \]
where \( \gamma \) is the curve \( t = 1 - \cos x \) in the \( xt \) plane.

   (a) Is there a solution of this problem near \( (0, 0) \)? Why? What is the regularity of the solution?
   
   (b) Find the truncated Taylor expansion of \( u(x, t) \) of order 2 near the point \( (0, 0) \) (i.e., find the quadratic approximation \( u_2 \) of \( u \) near \( (0, 0) \) such that \( u(x, t) = u_2(x, t) + O(|x|^3 + |t|^3)) \).

3. Find an explicit solution \( u(x, y) \) to the following problem
\[
\begin{align*}
  &u_{xx} + u_{yy} = 0 \quad \text{for } 1 < x^2 + y^2 < 4, \\
  &u = x \quad \text{for } x^2 + y^2 = 1, \\
  &u = 1 + xy \quad \text{for } x^2 + y^2 = 4.
\end{align*}
\]

4. Find an explicit solution to the problem
\[
\begin{align*}
  &u_t - u_{xx} = 0, \quad 0 < t, \quad x \in \mathbb{R}, \\
  &u|_{t=0} = e^{3x}, \quad x \in \mathbb{R}
\end{align*}
\]

5. Let \( \Omega \subset \mathbb{R}^n \) be a bounded domain with smooth boundary. Let \( u_1 \in C^1(\overline{\Omega}) \) be harmonic in \( \Omega \), and let \( u_2 \in C^1(\mathbb{R}^n \setminus \Omega) \) be harmonic outside \( \Omega \). Prove that the function \( u(x), \ x \in \mathbb{R}^n \) defined by
\[
  u(x) = \begin{cases} 
    u_1(x), & x \in \Omega, \\
    u_2(x), & x \in \mathbb{R} \setminus \Omega
  \end{cases}
\]
is harmonic in \( \mathbb{R}^n \) if and only if
\[
  u_1|_{\partial \Omega} = u_2|_{\partial \Omega} \quad \text{and} \quad \partial_{\nu} u_1|_{\partial \Omega} = \partial_{\nu} u_2|_{\partial \Omega}.
\]

As usual, \( \nu \) denotes the exterior normal to \( \partial \Omega \), that is also the interior normal to \( \mathbb{R}^n \setminus \Omega \).

**Hint:** Prove first that \( u \) is a weak solution to \( \Delta u = 0 \) using Green’s formula.
6. Use Hadamard’s method of descent to derive the formula for the solution of initial value problem for the 1D wave equation on the whole line

\[ u_{tt} = u_{xx}, \quad u|_{t=0} = f(x), \quad u_t|_{t=0} = g(x) \]

from the known formula for the solution of the 2D wave equation in the whole plane

\[ u_{tt} = u_{xx} + u_{yy}. \]

Assume that \( f \) and \( g \) are as smooth as needed.