QUALIFYING EXAM COVER SHEET

August 2017 Qualifying Exams

Instructions: These exams will be "blind-graded" so under the student ID number please use your PUID

ID #: ____________________________
(10 digit PUID)

EXAM (circle one) 519 523 **530** 544 553 554 562 **571**

For grader use:

Points _________ / Max Possible_________ Grade _________

MATH 530 Qualifying Exam

August 2017 (S. Bell)

Each problem is worth 25 points

1. Suppose the power series centered about zero for an entire function converges *uniformly* on the whole complex plane. What can you say about the entire function? Explain.

2. Suppose that $u(z,s)$ is a continuous real valued function on $\mathbb{C}\times\mathbb{R}$ such that $u(z,s)$ is harmonic in z for each fixed s. Define

$$U(z)=\int_{-1}^{1} u(z,s)\ ds.$$

a) Give an ϵ-δ proof that U is continuous on $\mathbb{C}$.
b) Prove that U is harmonic on $\mathbb{C}$ without taking derivatives.

3. Suppose that $f(z)$ is an entire function such that $f(z+\pi)=f(z)$ for all z and $f(z+i\pi)=f(z)$ for all z. Prove that f must be a constant function.

4. Suppose that $R(z)=P(z)/Q(z)$ where P and Q are complex polynomials and the degree of $Q(z)$ is at least two greater than the degree of $P(z)$. Show that the sum of the residues of $R(z)$ in the complex plane must be zero.

5. Show that the family of one-to-one conformal mappings of the horizontal strip $\{z : 0 < \text{Im } z < 1\}$ onto itself is such that given any two points z_1 and z_2 in the strip, there is a mapping in the family that maps z_1 to z_2.

6. Explain why

$$\frac{\sin z^2}{(z-1)(z+1)}$$

has an analytic antiderivative on $\mathbb{C}-[-1,1]$.

7. Compute

$$\int_\gamma \frac{\sin z}{z^{10}}\ dz,$$

where γ denotes an ellipse with one focus at the origin parameterized in the *clockwise* direction.

8. Prove that every harmonic function $u(z)$ on a simply connected domain Ω can be expressed as $u(z)=\text{Ln}\,|f(z)|$ where $f(z)$ is a nonvanishing analytic function on Ω. Is the function $f(z)$ unique? Explain.