QUALIFYING EXAMINATION

JANUARY 1995

MATH 530

1. Let $f(z) = a_1 z + a_2 z^2 + a_3 z^3 + \dots$ be an analytic function at 0 and $a_2 \neq 0$. Express the residue of $1/f^2$ at 0 in terms of a_i.

   <u>Remark</u>: Don't forget the case $a_1 = 0$.

2. Find an analytic function f such that

$$|f(x+iy)| = e^{xy}.$$

3. Find all complex solutions of the equation $\cos z = 2$.

4. Find the conformal mapping φ of the following domain onto the unit disk with $\varphi(0) = 0, \quad \varphi(\pm\frac{1}{2}) = \pm 1$.

5. a) How many roots does this equation

$$z^4 + z + 5 = 0$$

have in the first quadrant.

b) How many of them have argument between $\frac{\pi}{4}$ and $\frac{\pi}{2}$?

6. Compute

$$\int_{|z|=1} e^z z^{-n} dz,$$

where n is an integer.

7. Show that an isolated singularity of f cannot be a pole of $\sin f$.