QUALIFYING EXAMINATION

August 1998

MATH 530 - Profs. Bell/Catlin

Notation: $D_r(a)$ denotes the disk, $\{z \in \mathbb{C} : |z-a| < r\}$.

1. *(10 pts)* Find all entire functions f such that the real part of $f'(z)$ is non-negative at every point $z \in \mathbb{C}$.

2. *(15 pts)* Evaluate the integral

$$\int_0^\infty \frac{\sqrt{x}}{x^2+1}\,dx.$$

3. *(15 pts)* Suppose that f is a continuous complex valued function on the unit disk that is holomorphic on the sets $\{\text{Im } z > 0\} \cap D_1(0)$ and $\{\text{Im } z < 0\} \cap D_1(0)$. Prove f is holomorphic on all of $D_1(0)$. Is the analogue of this problem for *harmonic* functions true?

4. *(15 pts)* Find a one-to-one analytic map from $\{x+iy : 2 < y < 3,\ x < 1\}$ onto $\{x+iy : 5 < y < 8\}$.

5. *(15 pts)* Let f be a non-constant entire function such that $f(n) = 1998$ for every $n \in \mathbb{Z}$. Can f have at ∞:

a) an essential singularity,
b) a pole,
c) a removable singularity?

6. *(15 pts)* Suppose that f is analytic on $D_1(0)$ and that $|f(z)| < 1$ for all $z \in D_1(0)$. Prove that if $f(0) = a \neq 0$, then f has no zeroes in the disk $D_{|a|}(0)$.

7. *(15 pts)* Show that a *one-to-one* entire function must be of the form $az + b$ for some complex constants a and b with $a \neq 0$.