1. (10 pts) Let $f$ be analytic in the punctured disk $D' = \{z : 0 < |z - z_0| < r\}$. Prove that if
   \[ |f''(z)| \leq \frac{1}{|z - z_0|^2} \]
   for all $z \in D'$, then $f$ has a removable singularity at $z_0$.

2. (20 pts) Adapt the proof of Schwarz’s lemma to prove the following. Let $f$ be analytic in the unit disk $U = \{z : |z| < 1\}$ having zeros at $z = 1/2$ and $z = -1/2$, and such that $|f(z)| < 1$ for all $z \in U$. Prove that $|f(i/2)| \leq 8/17$.

3. i) (10 pts) Suppose that the functions $f_1, f_2, f_3, \ldots$ are analytic in a domain $\Omega$, and the sequence converges uniformly on all compact subsets of $\Omega$ to a function $f$. Prove that the sum $\sum_{n=1}^{\infty} \frac{f_n(z)}{2^n}$ converges to a function $g$ which is analytic on $\Omega$.

   ii) (10 pts) Suppose that the functions in i) satisfy the condition
   \[ |f_n(z)| \leq 10, \quad n = 1, 2, \ldots \]
   throughout $\Omega$, and recall that $g(z)$ is the sum in i). Prove that if $g(z_0) = 10$ for some $z_0 \in \Omega$, then $g(z) \equiv 10$ in $\Omega$.

4. i) (10 pts) Determine the set of points in the complex plane where $\sin z$ is pure real.

   ii) (10 pts) Determine the set of points in the complex plane where $|e^{z^2}| > 1$.

5. (10 pts) Let $C = \{z : |z| = 5\}$, traversed once counterclockwise. Compute the contour integral
   \[ \int_C \frac{e^{1/z} + \sin z}{z} \, dz. \]

6. (10 pts) Let $f$ be analytic in a domain which contains the closed disk $\{z : |z| \leq 2\}$, and let $C$ be its boundary $\{z : |z| = 2\}$ traversed once counterclockwise.
   Prove that $\text{Re} f(0) = \frac{1}{2\pi i} \int_C \frac{\text{Re} f(z)}{z} \, dz$.

7. (10 pts) Evaluate
   \[ \int_{1-i\infty}^{1+i\infty} \frac{e^z}{1+z^2} \, dz, \]
   i.e., along the line $\{z : \text{Re} z = 1\}$ in the direction of increasing imaginary part.