1. Compute \( \int_{-\infty}^{\infty} \frac{e^{ix}}{(1+x^2)^2} \, dx \).

2. Find a one-to-one conformal map from the eighth disc
\[ \{ z = re^{i\theta} : 0 < r < 1, 0 < \theta < \pi/4 \} \]
on onto the strip \( \{ z : 0 < \text{Im } z < 1 \} \).
(You may express your answer as a composition of more elementary maps.)

3. Let \( \log z \) denote the Principal Branch of the complex log function.
   a) Show that the Taylor series for \( \log \left( \frac{1}{1-z} \right) \) is
\[ \sum_{n=1}^{\infty} \frac{z^n}{n}. \]
   b) Show that the Taylor series for \( \left[ \log \left( \frac{1}{1-z} \right) \right]^2 \) is
\[ \sum_{n=2}^{\infty} \frac{2}{n} \left( 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} \right) z^n. \]

4. a) State Liouville’s Theorem.
   b) Prove Liouville’s Theorem using only the fact that
\[ f'(a) = \frac{1}{2\pi i} \int_{C_R} \frac{f(z)}{(z-a)^2} \, dz, \]
where \( C_R \) is the counterclockwise boundary curve of a circle of radius \( R \), \( a \) is a point inside the circle, and where \( f \) is analytic on a bigger disc containing the circle.
   c) Characterize the entire functions \( f \) that satisfy the estimate
\[ |f(z)| \leq |z|^2 \]
for all \( z \). Explain.

5. Prove that \( z^2 + z \) has an analytic square root on \( \{ z : |z| > 1 \} \).