1. Compute
\[ \int_{-\infty}^{\infty} \frac{\cos x}{\cosh x} \, dx. \]

Hint: Integrate \( f(z) = \frac{e^{iz}}{e^z + e^{-z}} \) around the boundary of the rectangle with vertices at \( \pm R, \pm R + i\pi \) and let \( R \to \infty \).

2. Suppose that \( u \) is a real valued harmonic function on the unit disc. Show that there is a sequence of real valued harmonic polynomials that converges uniformly on compact subsets of the unit disc to \( u \). Given the real valued harmonic function \( u = \ln |z| \) on \( A = \{ z : 1 < |z| < 2 \} \), is it possible to find a sequence of harmonic polynomials which converges uniformly on compact subsets of \( A \) to \( u \)? Explain.

3. Let \( \mathcal{F} \) denote the family of all analytic functions \( f \) on the unit disc that map the unit disc into itself with \( f(1/2) = 0 \). Find \( \sup \{ \text{Im } f(0) : f \in \mathcal{F} \} \).

4. a) State Rouche’s Theorem for the unit disc.

b) Use Rouche’s Theorem to prove that a polynomial of degree \( N \geq 1 \) has \( N \) roots (counted with multiplicities) in the complex plane.

5. Prove that
\[ e^z + \frac{1}{(z - 1)^3} + \frac{1}{(z + 1)^6} \]
has an analytic cube root on \( \mathbb{C} - \{ \pm 1 \} \).