1. Convert the integral
   \[ \int_0^{2\pi} \frac{d\theta}{2 + \sin \theta} \]
   into a contour integral of the form \( \int_C f(z) \, dz \) where \( C \) is the unit circle and \( f \) is a rational function, and then use the Residue Theorem to compute the integral.

2. Find the radius of convergence about \( z = 0 \) of the power series
   \[ \sum_{n=1}^{\infty} \frac{n^n}{n!} z^{2n}. \]

3. Find a one-to-one conformal map from the quarter disc
   \[ \{ z = re^{i\theta} : 0 < r < 1, 0 < \theta < \pi/2 \} \]
   onto the unit disc. You may express your solution as a composition of simpler mappings.

4. Compute
   \[ \int_0^\infty \frac{1}{x^3 + 1} \, dx \]
   by integrating \( f(z) = 1/(z^3 + 1) \) around the contour that follows the real line from zero to \( R \), then follows the circle \( Re^{it} \) from \( t = 0 \) to \( t = 2\pi/3 \), and then follows the line \( te^{i2\pi/3} \) from \( t = R \) back to \( t = 0 \). Use the Residue Theorem and let \( R \to \infty \).

5. Suppose that \( f(z) \) is an analytic function that maps the unit disc into itself with two distinct fixed points, i.e., with points \( z_1 \) and \( z_2 \) in the unit disc, \( z_1 \neq z_2 \), such that \( f(z_j) = z_j \) for \( j = 1, 2 \). Prove that \( f(z) = z \) for all \( z \).