QUALIFYING EXAM COVER SHEET

August 2017 Qualifying Exams

Instructions: These exams will be “blind-graded” so under the student ID number please use your PUID

ID #: __________________________
(10 digit PUID)

EXAM (circle one)  519  523  530  544  553  554  562  571

For grader use:

Points ________ / Max Possible________  Grade _________
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T.T.Moh  Math 554 Qualifying Examination  August 8th. 2017

• This is a two hour test.
• Write your answers on the test paper!
• For decimal approximations, it is enough to give 2 decimal places.
• Show your work such that your reasoning can be followed.
• There are 10 pages, 10 questions, 20 points each and 200 points on this test.

1. Let $A \in M_{n \times n}(K)$ (the vector space of $n \times n$ matrices over a field $K$). Show that the monic minimal polynomial of $A$ is a factor of the characteristic polynomial of $A$ and all roots of the characteristic polynomial of $A$ are roots of the minimal polynomial of $A$. :
2. Let \( M = (f_1, f_2, f_3)^T \) be a matrix over \( R[x] \) where \( R[x] \) is the ring of real polynomials and 
\( f_1 = (x - 3, 1, 0) \), \( f_2 = (1, x - 3, 0) \), \( f_3 = (0, 0, x - 2) \) be the three row vectors of \( M \). Express 
\( M \) as a diagonal matrix with diagonals \( (c_i) \) for \( i = 1, 2, 3 \) and \( c_i | c_{i+1} \). (The Smith theorem of matrices over P.I.D.)
3. Find the area of the parallelogram spanned by two vectors $[1, 2, 3, 4, 5]^T$ and $[5, 4, 3, 2, 1]^T$. 
4. Find the singular value decomposition (SVD) of the following matrix

\[ A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}. \]
5. Find the best straight line fit (least square approximation) to the measurement \(b = 2 \text{ at } t = 0, \ b = 1 \text{ at } t = 1, \ b = 3 \text{ at } t = 2.\)
6. Find an orthonormal basis for $P_3$, the vector space of all real polynomials of degree $\leq 3$ under the inner product defined as

$$< f | g > = \int_0^1 fg \, dx$$
7. Let $V$ be an inner product space (finite or infinite dimensional), show that every isometry $T$, i.e., $<Tv, Tu> = <v, u>$ for all $u, v \in V$, is injective.
8. Show that a reflection matrix $A$ of $\mathbb{R}^3$ (the real 3-dimensional space), i.e., a $3 \times 3$ matrix $A$ is reflection, iff $A$ is orthogonal and $\det(A) = -1$, must have $-1$ as an eigenvalue.
9. Let $A$ be the matrix over complex numbers as follows,

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$ 

Find the Jordan canonical form of $A$. 
10. Decide type of the following function, a ellipse? a hyperbola? a parabla?

\[ 2x^2 + 6xy + 2y^2 + x = 0 \]