QUALIFYING EXAMINATION
AUGUST 2002
MATH 554 - Prof. Moh

You have to show your work and reasonings.

(10 points) (1) Let A be an invertible square matrix. Prove that there is a set of elementary matrices $E_1, ..., E_k$ such that $E_k ... E_1 A$ is the identity.

(10 points) (2) Show that the number of invertible $n \times n$ matrices over $\mathbf{F}_p$ is $\prod_{i=0}^{i=n-1}(p^n - p^i)$ where p is a prime number and $\mathbf{F}_p$ is Z/pZ, the finite field of p elements.

(10 points) (3) Find all linear transformation $T : R^2 \mapsto R^2$ which carry the line y=2x to the line y=3x.

(20 points) (4) Let A, B be two $n \times n$ matrices with $AB = 0$. Show that $rank(A) + rank(B) \leq n$.

(10 points) (5) Find all non-isomorphic commutative group of order 50.

(10 points) (6) Show that a matrix A is similar to its transpose A^t.

(20 points) (7) An $n \times n$ matrix A is said to be a rotation matrix if it is orthogonal and $\det A = 1$. Let A be a 3×3 real rotation matrix. Show that 1 is an eigen value.

(10 points) (8) Find the invariant factors of the following matrix;

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

(10 points) (9) Show that an $n \times n$ matrix A is c for some constant c iff it commutes with every $n \times n$ matrix B

(10 points) (10) Let A be an $n \times n$ complex matrix. Suppose that $A^2 = A$. Show that A is self-adjoint iff $A^*A = AA^*$

(10 points) (11) Find the Jordan canonical form of the following matrix over the complex numbers C;

$$\begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

(10 points) (12) Describe all bilinear forms f on R^3 which satisfy $f(\alpha, \beta) = -f(\beta, \alpha)$ for all α, β.